FORMALIZED MATHEMATICS
2007, Vol. 15, No. 3, Pages 73–79
DOI: 10.2478/v10037-007-0009-4

Several Differentiation Formulas of Special Functions. Part V

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Summary. In this article, we give several differentiation formulas of special and composite functions including trigonometric, polynomial and logarithmic functions.

MML identifier: `FDIFF_9`, version: `7.8.05 4.84.971`

The articles [13], [15], [1], [16], [2], [4], [10], [11], [17], [5], [14], [12], [3], [7], [6], [9], and [8] provide the notation and terminology for this paper.

The partial function sec from $\mathbb{R}$ to $\mathbb{R}$ is defined as follows:

(Def. 1) $\sec = \frac{1}{\text{the function } \cos}$.

The partial function cosec from $\mathbb{R}$ to $\mathbb{R}$ is defined by:

(Def. 2) $\mathrm{cosec} = \frac{1}{\text{the function } \sin}$.

For simplicity, we follow the rules: x, a, b, c are real numbers, n is a natural number, Z is an open subset of $\mathbb{R}$, and f, f_1, f_2 are partial functions from $\mathbb{R}$ to $\mathbb{R}$.

One can prove the following propositions:

(1) If $(\text{the function } \cos)(x) \neq 0$, then sec is differentiable in x and $(\sec)'(x) = \frac{(\text{the function } \sin)(x)}{(\text{the function } \cos)(x)^{\mathbf{2}}}$.

(2) If $(\text{the function } \sin)(x) \neq 0$, then cosec is differentiable in x and $(\mathrm{cosec})'(x) = -\frac{(\text{the function } \cos)(x)}{(\text{the function } \sin)(x)^{\mathbf{2}}}$.

(3) $(\frac{1}{x})^{n}_{\mathbb{Z}} = \frac{1}{x^{n}_{\mathbb{Z}}}$.

(4) Suppose $Z \subseteq \mathrm{dom}\sec$. Then sec is differentiable on Z and for every x such that $x \in Z$ holds $(\sec)'_{\restriction Z}(x) = \frac{(\text{the function } \sin)(x)}{(\text{the function } \cos)(x)^{\mathbf{2}}}$.

ISSN 1426–2630

(5) Suppose $Z \subseteq \operatorname{dom}\operatorname{cosec}$. Then cosec is differentiable on Z and for every x such that $x \in Z$ holds $(\operatorname{cosec})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(6) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) $\sec \cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot f)'_{\restriction Z}(x) = \frac{a \cdot (\text{the function sin})(a \cdot x + b)}{(\text{the function cos})(a \cdot x + b)^{\mathbf{2}}}$.

(7) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot f)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then

(i) $\operatorname{cosec} \cdot f$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot f)'_{\restriction Z}(x) = -\frac{a \cdot (\text{the function cos})(a \cdot x + b)}{(\text{the function sin})(a \cdot x + b)^{\mathbf{2}}}$.

(8) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot \frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\sec \cdot \frac{1}{f}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot \frac{1}{f})'_{\restriction Z}(x) = -\frac{(\text{the function sin})(\frac{1}{x})}{x^{\mathbf{2}} \cdot (\text{the function cos})(\frac{1}{x})^{\mathbf{2}}}$.

(9) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot \frac{1}{f})$ and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\operatorname{cosec} \cdot \frac{1}{f}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot \frac{1}{f})'_{\restriction Z}(x) = \frac{(\text{the function cos})(\frac{1}{x})}{x^{\mathbf{2}} \cdot (\text{the function sin})(\frac{1}{x})^{\mathbf{2}}}$.

(10) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (f_1 + c\, f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$. Then

(i) $\sec \cdot (f_1 + c\, f_2)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot (f_1 + c\, f_2))'_{\restriction Z}(x) = \frac{(b + 2 \cdot c \cdot x) \cdot (\text{the function sin})(a + b \cdot x + c \cdot x^{\mathbf{2}})}{(\text{the function cos})(a + b \cdot x + c \cdot x^{\mathbf{2}})^{\mathbf{2}}}$.

(11) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (f_1 + c\, f_2))$ and $f_2 = \frac{2}{\mathbb{Z}}$ and for every x such that $x \in Z$ holds $f_1(x) = a + b \cdot x$. Then

(i) $\operatorname{cosec} \cdot (f_1 + c\, f_2)$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot (f_1 + c\, f_2))'_{\restriction Z}(x) = -\frac{(b + 2 \cdot c \cdot x) \cdot (\text{the function cos})(a + b \cdot x + c \cdot x^{\mathbf{2}})}{(\text{the function sin})(a + b \cdot x + c \cdot x^{\mathbf{2}})^{\mathbf{2}}}$.

(12) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (\text{the function exp}))$. Then

(i) $\sec \cdot (\text{the function exp})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot (\text{the function exp}))'_{\restriction Z}(x) = \frac{(\text{the function exp})(x) \cdot (\text{the function sin})((\text{the function exp})(x))}{(\text{the function cos})((\text{the function exp})(x))^{\mathbf{2}}}$.

(13) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (\text{the function exp}))$. Then

(i) $\operatorname{cosec} \cdot (\text{the function exp})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{cosec} \cdot (\text{the function exp}))'_{\restriction Z}(x) = -\frac{(\text{the function exp})(x) \cdot (\text{the function cos})((\text{the function exp})(x))}{(\text{the function sin})((\text{the function exp})(x))^{\mathbf{2}}}$.

(14) Suppose $Z \subseteq \mathrm{dom}(\sec \cdot (\text{the function ln}))$. Then

(i) $\sec \cdot (\text{the function ln})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot (\text{the function ln}))'_{\restriction Z}(x) = \frac{(\text{the function sin})((\text{the function ln})(x))}{x \cdot (\text{the function cos})((\text{the function ln})(x))^{\mathbf{2}}}$.

(15) Suppose $Z \subseteq \mathrm{dom}(\mathrm{cosec} \cdot (\text{the function ln}))$. Then

(i) $\mathrm{cosec} \cdot (\text{the function ln})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\mathrm{cosec} \cdot (\text{the function ln}))'_{\restriction Z}(x) = -\frac{(\text{the function cos})((\text{the function ln})(x))}{x \cdot (\text{the function sin})((\text{the function ln})(x))^{\mathbf{2}}}$.

(16) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp}) \cdot \sec)$. Then

(i) $(\text{the function exp}) \cdot \sec$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function exp}) \cdot \sec)'_{\restriction Z}(x) = \frac{(\text{the function exp})((\sec)(x)) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(17) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp}) \cdot \mathrm{cosec})$. Then

(i) $(\text{the function exp}) \cdot \mathrm{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function exp}) \cdot \mathrm{cosec})'_{\restriction Z}(x) = -\frac{(\text{the function exp})((\mathrm{cosec})(x)) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(18) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln}) \cdot \sec)$. Then

(i) $(\text{the function ln}) \cdot \sec$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function ln}) \cdot \sec)'_{\restriction Z}(x) = \frac{(\text{the function sin})(x)}{(\text{the function cos})(x)}$.

(19) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln}) \cdot \mathrm{cosec})$. Then

(i) $(\text{the function ln}) \cdot \mathrm{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function ln}) \cdot \mathrm{cosec})'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x)}{(\text{the function sin})(x)}$.

(20) Suppose $Z \subseteq \mathrm{dom}((^{n}_{\mathbb{Z}}) \cdot \sec)$ and $1 \leq n$. Then

(i) $(^{n}_{\mathbb{Z}}) \cdot \sec$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((^{n}_{\mathbb{Z}}) \cdot \sec)'_{\restriction Z}(x) = \frac{n \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{n+1}_{\mathbb{Z}}}$.

(21) Suppose $Z \subseteq \mathrm{dom}((^{n}_{\mathbb{Z}}) \cdot \mathrm{cosec})$ and $1 \leq n$. Then

(i) $(^{n}_{\mathbb{Z}}) \cdot \mathrm{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((^{n}_{\mathbb{Z}}) \cdot \mathrm{cosec})'_{\restriction Z}(x) = -\frac{n \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{n+1}_{\mathbb{Z}}}$.

(22) Suppose $Z \subseteq \mathrm{dom}(\sec - \mathrm{id}_Z)$. Then

(i) $\sec - \mathrm{id}_Z$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function sin})(x) - (\text{the function cos})(x)^{\mathbf{2}}}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(23) Suppose $Z \subseteq \mathrm{dom}(-\mathrm{cosec} - \mathrm{id}_Z)$. Then
 (i) $-\mathrm{cosec} - \mathrm{id}_Z$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(-\mathrm{cosec} - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function cos})(x) - (\text{the function sin})(x)^2}{(\text{the function sin})(x)^2}$.

(24) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp})\ \ \mathrm{sec})$. Then
 (i) (the function exp) sec is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $((\text{the function exp})\ \ \mathrm{sec})'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{(\text{the function cos})(x)} + \frac{(\text{the function exp})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^2}$.

(25) Suppose $Z \subseteq \mathrm{dom}((\text{the function exp})\ \ \mathrm{cosec})$. Then
 (i) (the function exp) cosec is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $((\text{the function exp})\ \ \mathrm{cosec})'_{\restriction Z}(x) = \frac{(\text{the function exp})(x)}{(\text{the function sin})(x)} - \frac{(\text{the function exp})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^2}$.

(26) Suppose $Z \subseteq \mathrm{dom}(\frac{1}{a}\,(\mathrm{sec} \cdot f) - \mathrm{id}_Z)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x$ and $a \neq 0$. Then
 (i) $\frac{1}{a}\,(\mathrm{sec} \cdot f) - \mathrm{id}_Z$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(\frac{1}{a}\,(\mathrm{sec} \cdot f) - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function sin})(a \cdot x) - (\text{the function cos})(a \cdot x)^2}{(\text{the function cos})(a \cdot x)^2}$.

(27) Suppose $Z \subseteq \mathrm{dom}((-\frac{1}{a})\,(\mathrm{cosec} \cdot f) - \mathrm{id}_Z)$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x$ and $a \neq 0$. Then
 (i) $(-\frac{1}{a})\,(\mathrm{cosec} \cdot f) - \mathrm{id}_Z$ is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $((-\frac{1}{a})\,(\mathrm{cosec} \cdot f) - \mathrm{id}_Z)'_{\restriction Z}(x) = \frac{(\text{the function cos})(a \cdot x) - (\text{the function sin})(a \cdot x)^2}{(\text{the function sin})(a \cdot x)^2}$.

(28) Suppose $Z \subseteq \mathrm{dom}(f\ \ \mathrm{sec})$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
 (i) f sec is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(f\ \ \mathrm{sec})'_{\restriction Z}(x) = \frac{a}{(\text{the function cos})(x)} + \frac{(a \cdot x + b) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^2}$.

(29) Suppose $Z \subseteq \mathrm{dom}(f\ \ \mathrm{cosec})$ and for every x such that $x \in Z$ holds $f(x) = a \cdot x + b$. Then
 (i) f cosec is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $(f\ \ \mathrm{cosec})'_{\restriction Z}(x) = \frac{a}{(\text{the function sin})(x)} - \frac{(a \cdot x + b) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^2}$.

(30) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ \ \mathrm{sec})$. Then
 (i) (the function ln) sec is differentiable on Z, and
 (ii) for every x such that $x \in Z$ holds $((\text{the function ln})\ \ \mathrm{sec})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)}}{x} + \frac{(\text{the function ln})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^2}$.

(31) Suppose $Z \subseteq \mathrm{dom}((\text{the function ln})\ \ \mathrm{cosec})$. Then

(i) (the function ln) cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds ((the function ln) cosec)$'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function sin})(x)}}{x} - \frac{(\text{the function ln})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(32) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{f}$ sec) and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\frac{1}{f}$ sec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{f}$ sec$)'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function cos})(x)}}{x^{\mathbf{2}}} + \frac{\frac{(\text{the function sin})(x)}{x}}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(33) Suppose $Z \subseteq \operatorname{dom}(\frac{1}{f}$ cosec) and for every x such that $x \in Z$ holds $f(x) = x$. Then

(i) $\frac{1}{f}$ cosec is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\frac{1}{f}$ cosec$)'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function sin})(x)}}{x^{\mathbf{2}}} - \frac{\frac{(\text{the function cos})(x)}{x}}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(34) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (\text{the function sin}))$. Then

(i) sec $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds (sec $\cdot$(the function sin))$'_{\restriction Z}(x) = \frac{(\text{the function cos})(x) \cdot (\text{the function sin})((\text{the function sin})(x))}{(\text{the function cos})((\text{the function sin})(x))^{\mathbf{2}}}$.

(35) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (\text{the function cos}))$. Then

(i) sec $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds (sec $\cdot$(the function cos))$'_{\restriction Z}(x) = -\frac{(\text{the function sin})(x) \cdot (\text{the function sin})((\text{the function cos})(x))}{(\text{the function cos})((\text{the function cos})(x))^{\mathbf{2}}}$.

(36) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (\text{the function sin}))$. Then

(i) cosec $\cdot$(the function sin) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds (cosec $\cdot$(the function sin))$'_{\restriction Z}(x) = -\frac{(\text{the function cos})(x) \cdot (\text{the function cos})((\text{the function sin})(x))}{(\text{the function sin})((\text{the function sin})(x))^{\mathbf{2}}}$.

(37) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (\text{the function cos}))$. Then

(i) cosec $\cdot$(the function cos) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds (cosec $\cdot$(the function cos))$'_{\restriction Z}(x) = \frac{(\text{the function sin})(x) \cdot (\text{the function cos})((\text{the function cos})(x))}{(\text{the function sin})((\text{the function cos})(x))^{\mathbf{2}}}$.

(38) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (\text{the function tan}))$. Then

(i) sec $\cdot$(the function tan) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds (sec $\cdot$(the function tan))$'_{\restriction Z}(x) = \frac{\frac{(\text{the function sin})((\text{the function tan})(x))}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function cos})((\text{the function tan})(x))^{\mathbf{2}}}$.

(39) Suppose $Z \subseteq \operatorname{dom}(\sec \cdot (\text{the function cot}))$. Then

(i) sec $\cdot$(the function cot) is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\sec \cdot (\text{the function cot}))'_{\restriction Z}(x) = -\frac{\frac{(\text{the function sin})((\text{the function cot})(x))}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function cos})((\text{the function cot})(x))^{\mathbf{2}}}$.

(40) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (\text{the function tan}))$. Then

(i) $\operatorname{cosec} \cdot (\text{the function tan})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot (\text{the function tan}))'_{\restriction Z}(x) = -\frac{\frac{(\text{the function cos})((\text{the function tan})(x))}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function sin})((\text{the function tan})(x))^{\mathbf{2}}}$.

(41) Suppose $Z \subseteq \operatorname{dom}(\operatorname{cosec} \cdot (\text{the function cot}))$. Then

(i) $\operatorname{cosec} \cdot (\text{the function cot})$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $(\operatorname{cosec} \cdot (\text{the function cot}))'_{\restriction Z}(x) = \frac{\frac{(\text{the function cos})((\text{the function cot})(x))}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function sin})((\text{the function cot})(x))^{\mathbf{2}}}$.

(42) Suppose $Z \subseteq \operatorname{dom}((\text{the function tan})\ \sec)$. Then

(i) $(\text{the function tan})\ \sec$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan})\ \sec)'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function cos})(x)} + \frac{(\text{the function tan})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(43) Suppose $Z \subseteq \operatorname{dom}((\text{the function cot})\ \sec)$. Then

(i) $(\text{the function cot})\ \sec$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cot})\ \sec)'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function cos})(x)} + \frac{(\text{the function cot})(x) \cdot (\text{the function sin})(x)}{(\text{the function cos})(x)^{\mathbf{2}}}$.

(44) Suppose $Z \subseteq \operatorname{dom}((\text{the function tan})\ \operatorname{cosec})$. Then

(i) $(\text{the function tan})\ \operatorname{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function tan})\ \operatorname{cosec})'_{\restriction Z}(x) = \frac{\frac{1}{(\text{the function cos})(x)^{\mathbf{2}}}}{(\text{the function sin})(x)} - \frac{(\text{the function tan})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

(45) Suppose $Z \subseteq \operatorname{dom}((\text{the function cot})\ \operatorname{cosec})$. Then

(i) $(\text{the function cot})\ \operatorname{cosec}$ is differentiable on Z, and

(ii) for every x such that $x \in Z$ holds $((\text{the function cot})\ \operatorname{cosec})'_{\restriction Z}(x) = -\frac{\frac{1}{(\text{the function sin})(x)^{\mathbf{2}}}}{(\text{the function sin})(x)} - \frac{(\text{the function cot})(x) \cdot (\text{the function cos})(x)}{(\text{the function sin})(x)^{\mathbf{2}}}$.

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Received July 9, 2007