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A DATA PRE-PROCESSING MODEL FOR THE TOPSIS METHOD

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Abstract

TOPSIS is one of the most popular methods of multi-criteria decision making (MCDM). Its fundamental role is the establishment of chosen alternatives ranking based on their distance from the ideal and negative-ideal solution. There are three primary versions of the TOPSIS method distinguished: classical, interval and fuzzy, where calculation algorithms are adjusted to the character of input rating decision-making alternatives (real numbers, interval data or fuzzy numbers). Various, specialist publications present descriptions on the use of particular versions of the TOPSIS method in the decision-making process, particularly popular is the fuzzy version. However, it should be noticed, that depending on the character of accepted criteria – rating of alternatives can have a heterogeneous character. The present paper suggests the means of proceeding in the situation when the set of criteria covers characteristic criteria for each of the mentioned versions of TOPSIS, as a result of which the rating of the alternatives is vague. The calculation procedure has been illustrated by an adequate numerical example.

Keywords: multi-criteria decision making, the TOPSIS method

JEL classification: M10, M20

Introduction

TOPSIS, that is a similarity method for an ideal solution, is a well-known, classical MCDA method, first developed in the work of Hwang and Yoon (1981). The method is based on the principle of establishing a synthetic rating which aims at determining the distance of each alternative from the ideal solution and negative-ideal solution. The ideal solution is defined on the basis of values which among the whole set of available values, within the frames of each criteria, are considered the best (Hwang, Yoon, 1981). On the other hand, a negative-ideal solution is defined on the basis of the worst values. Criteria can have the character of benefits criteria (set $C_{benefits}$) and costs criteria (set C_{costs}). Therefore, depending on the character of the criterion while defining an ideal solution there is a maximum value chosen in the case of benefits criteria and a minimum value in the case of costs criteria. However, while defining a negative-ideal solution there are inverse values assumed. The application potential of the TOPSIS method is considerable which can be proved by numerous works on this subject matter, which is published in specialist literature (Behzadian et al., 2012).

The literature distinguishes three types of the TOPSIS method:

- classical, where the input data are precisely defined real data (Hwang, Yoon, 1981),
- interval, where the output data are interval numbers and where the beginning of an interval is defined by the minimum value and the ending is defined by the maximum value (Jahanshahloo et al., 2006a),
- fuzzy, where the output data are not precisely defined, however, they are expressed by the means of linguistic variable levels (e.g. negative, medium and positive ratings) (Chen, 2000; Jahanshahloo et al., 2006b; Kahraman et al., 2007).

What is genuinely popular is the fuzzy version of the TOPSIS method which can be proved by numerous publications concerning the subject matter (Behzadian et al., 2012). Among the prolific work concerning the fuzzy version of the TOPSIS method it is possible to enumerate the following articles: (Afshar et al., 2011; Amiri, 2010; Amiri et al., 2009; Awasthi et al., 2011; Aydogan, 2011; Bottani, Rizzi, 2006; Erkayman et al., 2011; Ertugrul, 2010; Kannan et al., 2009; Krohling, Campanharo, 2011; Sun, 2010; Sun, Lin 2009; Wang et al., 2009; Wang, Chang, 2007; Wang, Lee, 2009; Wang 2009). An extension of the TOPSIS method to decision problems, in the conditions of interval data of a fuzzy character is described in the work of Park et al. (2011). In practice, there are frequently situations where the set of criteria is ambiguous and covers various groups of criteria where the ratings have a character of both real numbers as well as interval and fuzzy data. In the result, the decision making problem cannot be considered strictly according to the procedures defined for the particular version of the TOPSIS method (classical, interval and fuzzy). Below, the means of conducting the analysis in such conditions have been presented.

1. Analysis procedures by the use of the TOPSIS method

In the TOPSIS method, in the progress of rating particular alternatives and comparing them with others, there is a distance expressed in the n-dimensional Euclidean distance (n – number of criteria) between the value vectors describing particular alternatives and vectors responding to ideal and negative-ideal variants. The most reasonable alternative is the one with the value vector of simultaneously the shortest distance from the vector of the ideal solution and the longest distance from the vector of a negative-ideal solution.

The decisive steps, covered by the analysis of the use of the TOPSIS method are the following:

- step 1 – creation of a decision matrix,
- step 2 – creation of a normalized decision matrix,
- step 3 – creation of a weight, normalized decision matrix,
- step 4 – indication of the ideal and negative-ideal solution,
- step 5 – calculation of the distance of each alternative for the ideal and negative-ideal solution,
- step 6 – calculation of the similarity indicators of particular alternatives for the ideal solution,
- step 7 – creation of the final alternatives ranking in the decreasing order of the similarity value indicator.

1.1. Classical version of the TOPSIS method

As it was mentioned in the previous part of the paper, the foundations of the TOPSIS method were presented in the work of (Hwang, Yoon, 1981). The basis of the analysis is the decision matrix $\mathbf{Q}_{m,n}$ including ratings of considered alternatives $i = 1, 2, ..., m$ in the context of the accepted criteria $j = 1, 2, ..., n$:

$$\mathbf{Q}_{m,n} = \begin{bmatrix} Q_{1,1} & Q_{1,2} & \cdots & Q_{1,n} \\ Q_{2,1} & Q_{2,2} & \cdots & Q_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{m,1} & Q_{m,2} & \cdots & Q_{m,n} \end{bmatrix} \quad (1)$$

On the basis of which there have been calculated normalized ratings of particular alternatives:

$$n_{i,j} = \frac{Q_{i,j}}{\sqrt{\sum_{i=1}^{m} Q_{i,j}^2}} \tag{2}$$

In the phase of normalized rating it is possible to use the formulas (Ishizaka, Nemery, 2013):

– for the benefits criterion

$$n_{i,j} = \frac{Q_{i,j}}{Q_{\max}} \tag{3}$$

– for the cost criterion

$$n_i = \frac{Q_{\min}}{Q_{i,j}} \tag{4}$$

normalization causes all the criteria to have the character of a benefits criterion. Corrected ratings (with the use of weights of the assigned criteria) are calculated as:

$$v_{i,j} = w_j n_{i,j} \tag{5}$$

Then, there is an identification of the ideal solution conducted (V^+) and negative-ideal solution (V^-) with the use of corrected assessments. The ideal solution is defined as:

$$V^+ = \left\{ v_1^+, v_2^+, ..., v_n^+ \right\} \tag{6}$$

where $v_j^+ = \left(\left(\max_i \ v_{i,j} \,\middle|\, j \in \mathrm{C}_{\text{benefits}} \right), \left(\min_i \ v_{i,j} \,\middle|\, j \in \mathrm{C}_{\text{costs}} \right) \right)$, $i = 1, 2, ..., m$, whereas the negative-ideal solution is defined as:

$$V^- = \left\{ v_1^-, v_2^-, ..., v_n^- \right\} \tag{7}$$

where $v_j^- = \left(\left(\min_i \ v_{i,j} \,\middle|\, j \in \mathrm{C}_{\text{benefits}} \right), \left(\max_i \ v_{i,j} \,\middle|\, j \in \mathrm{C}_{\text{costs}} \right) \right)$, $i = 1, 2, ..., m$. In the above equations v_j^+ and v_j^- are the values defining ideal and negative-ideal solutions in the context of criterion (j), however, $\mathrm{C}_{\text{benefits}}$, $\mathrm{C}_{\text{costs}}$ are respectively benefits and costs criteria subsets.

After indication of the ideal and negative-ideal solution there are the distances calculated d_i^+ and d_i^- between them and consecutive alternatives:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{i,j} - v_j^+ \right)^2} \tag{8}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{i,j} - v_j^-\right)^2} \tag{9}$$

On the basis of d_i^+ and d_i^- there is a ranking the coefficient of the particular alternatives indicated:

$$R_i = \frac{d_i^-}{d_i^- + d_i^+} \tag{10}$$

The procedure ends with the establishment of the alternatives ranking in the decreasing order of the R_i value rating.

1.2. Interval version of the TOPSIS method

The interval version of the TOPSIS method was developed in the work of (Jahanshahloo et al., 2006a). The primary steps of the calculating procedure in the case of this version of the TOPSIS method are similar to the steps in the classical version. The interval decision matrix has the following form:

$$\overline{\mathbf{Q}} = \begin{bmatrix} [q_{1,1}^L, q_{1,1}^U] & [q_{1,2}^L, q_{1,2}^U] & \dots & [q_{1,n}^L, q_{1,n}^U] \\ [q_{2,1}^L, q_{2,1}^U] & [q_{2,2}^L, q_{2,2}^U] & \dots & [q_{2,n}^L, q_{2,n}^U] \\ \dots & \dots & \dots & \dots \\ [q_{m,1}^L, q_{m,1}^U] & [q_{m,2}^L, q_{m,2}^U] & \dots & [q_{m,n}^L, q_{m,n}^U] \end{bmatrix} \tag{11}$$

where $q_{i,j}^L$, $q_{i,j}^U$ are respectively the bottom and top border of the interval being the rating of the i alternative in the context of the j criterion. First of all, the normalized decision matrix is calculated, where the bottom and top border of each interval value is:

$$\overline{n}_{i,j}^L = \frac{q_{i,j}^L}{\sqrt{\sum_{j=1}^{m} \left[\left(q_{i,j}^L\right)^2 + \left(q_{i,j}^U\right)^2\right]}} \tag{12}$$

$$\overline{n}_{i,j}^U = \frac{q_{i,j}^U}{\sqrt{\sum_{j=1}^{m} \left[\left(q_{i,j}^L\right)^2 + \left(q_{i,j}^U\right)^2\right]}} \tag{13}$$

Interval $[\overline{n}_{i,j}^L, \overline{n}_{i,j}^U]$ is a normalized interval $[\overline{q}_{i,j}^L, \overline{q}_{i,j}^U]$. In the result of normalization, the elements of the decision matrix take the values from the interval [0, 1]. Subsequently, the weight and normalized interval decision matrix with the elements are calculated:

$$\overline{v}_{i,j}^{L} = w_j \times \overline{n}_{i,j}^{L} \tag{14}$$

$$\overline{v}_{i,j}^{U} = w_j \times \overline{n}_{i,j}^{U} \tag{15}$$

On the basis of weight and the normalized interval ratings there is an ideal and negative-ideal solution identified:

$$\overline{V}^{+} = \left\{\overline{v}_1^{+}, \overline{v}_2^{+}, ..., \overline{v}_n^{+}\right\} = \left\{\max_i \overline{v}_{i,j}^{U} \mid j \in \mathrm{C}_{\text{benefits}}, \min_i \overline{v}_{i,j}^{L} \mid j \in \mathrm{C}_{\text{costs}}\right\} \tag{16}$$

$$\overline{V}^{-} = \left\{\overline{v}_1^{-}, \overline{v}_2^{-}, ..., \overline{v}_n^{-}\right\} = \left\{\min_i \overline{v}_{i,j}^{L} \mid j \in \mathrm{C}_{\text{benefits}}, \max_i \overline{v}_{i,j}^{U} \mid j \in \mathrm{C}_{\text{costs}}\right\} \tag{17}$$

Next the distance of each alternative to the ideal solution is calculated:

$$\overline{d}_i^{+} = \sqrt{\sum_{j=1}^{n}\left(\overline{v}_{i,j}^{L} - \overline{v}_j^{+}\right)^2 + \sum_{j=1}^{n}\left(\overline{v}_{i,j}^{U} - \overline{v}_j^{+}\right)^2} \tag{18}$$

And to the negative-ideal solution:

$$\overline{d}_i^{-} = \sqrt{\sum_{j=1}^{n}\left(\overline{v}_{i,j}^{U} - \overline{v}_j^{-}\right)^2 + \sum_{j=1}^{n}\left(\overline{v}_{i,j}^{L} - \overline{v}_j\right)^2} \tag{19}$$

On the basis of d_i^{+} and d_i^{-} the ranking coefficient of particular alternatives is calculated:

$$\overline{R}_i = \frac{\overline{d}_i^{-}}{\overline{d}_i^{-} + \overline{d}_i^{+}} \tag{20}$$

On the basis of $\overline{R}_i$ the alternatives ranking is created in the decreasing $\overline{R}_i$ value order.

1.3. Fuzzy version of the TOPSIS method

The fuzzy version of the TOPSIS method was presented, among others, in the works of (Chen 2000; Jahanshahloo et al., 2006b; Kahraman et al., 2007). In the case of this version of the TOPSIS method the starting point is the decision matrix which is comprised of ratings in a fuzzy form, for example from triangular fuzzy numbers $\tilde{Q}_{i,j} = \left(q_{i,j}, L_{i,j}, U_{i,j}\right)$:

$$\tilde{\mathbf{Q}} = \begin{bmatrix} \tilde{Q}_{1,1} & \tilde{Q}_{1,2} & \cdots & \tilde{Q}_{1,n} \\ \tilde{Q}_{2,1} & \tilde{Q}_{2,2} & \cdots & \tilde{Q}_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{Q}_{m,1} & \tilde{Q}_{m,2} & \cdots & \tilde{Q}_{m,n} \end{bmatrix} \tag{21}$$

In the general sense, the character of fuzzy numbers can also have a weight criterion $\tilde{w}_j$.

Firstly there is a normalized, fuzzy decision matrix calculated. For each fuzzy number set of α-cross-section is calculated as

$$\tilde{Q}_{i,j} = \left[\left(\tilde{Q}_{i,j}\right)_\alpha^L, \left(\tilde{Q}_{i,j}\right)_\alpha^U \right] \tag{22}$$

where $\alpha \in [0,\ 1]$. Therefore, each fuzzy value can be transformed to the form of interval values. Then in order to normalize the interval values there are the following formulas used:

$$\left(\tilde{n}_{i,j}\right)_\alpha^L = \frac{\left(\tilde{Q}_{i,j}\right)_\alpha^L}{\sqrt{\sum_{j=1}^{m} \left[\left(\left(\tilde{Q}_{i,j}\right)_\alpha^L \right)^2 + \left(\left(\tilde{Q}_{i,j}\right)_\alpha^U \right)^2 \right]}} \tag{23}$$

$$\left(\tilde{n}_{i,j}\right)_\alpha^U = \frac{\left(\tilde{Q}_{i,j}\right)_\alpha^U}{\sqrt{\sum_{j=1}^{m} \left[\left(\left(\tilde{Q}_{i,j}\right)_\alpha^L \right)^2 + \left(\left(\tilde{Q}_{i,j}\right)_\alpha^U \right)^2 \right]}} \tag{24}$$

In the result interval $\left[\left(\tilde{n}_{i,j}\right)_\alpha^L, \left(\tilde{n}_{i,j}\right)_\alpha^U \right]$ is the normalized interval $\left[\left(\tilde{Q}_{i,j}\right)_\alpha^L, \left(\tilde{Q}_{i,j}\right)_\alpha^U \right]$. According to the fuzzy numbers algebra rules (Zadeh, 1965), a normalized interval can be in return transformed to the triangular fuzzy number form $\tilde{N}_{i,j} = \left(n_{i,j}, l_{i,j}, u_{i,j}\right)$.

In the next step there is a weight; normalized, fuzzy decision matrix calculated where the elements are expressed as:

$$\tilde{v}_{i,j} = w_j \times \tilde{N}_{i,j} \tag{25}$$

or (if weights are also fuzzy numbers):

$$\tilde{v}_{i,j} = \tilde{w}_j \times \tilde{N}_{i,j} \tag{26}$$

On the basis of the above, it is possible to identify the fuzzy ideal and negative-ideal solution:

$$\tilde{V}^+ = \left\{ \tilde{v}_1^+, \tilde{v}_2^+, ..., \tilde{v}_n^+ \right\} \tag{27}$$

$$\tilde{V}^- = \left\{ \tilde{v}_1^-, \tilde{v}_2^-, ..., \tilde{v}_n^- \right\} \tag{28}$$

where:

- in the case of the benefits criteria $\tilde{v}_j^+ = (1,\ 0,\ 0)$ and $\tilde{v}_j^- = (0,\ 0,\ 0)$,
- in the case of the costs criteria $\tilde{v}_j^+ = (0,\ 0,\ 0)$ and $\tilde{v}_j^- = (1,\ 0,\ 0)$.

In the next phase there is a distance defined for each alternative of the ideal and negative-ideal solution:

$$\tilde{d}_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{i,j}, \tilde{v}_j^+) \tag{29}$$

$$\tilde{d}_i^- = \sum_{j=1}^{n} d(\tilde{v}_{i,j}, \tilde{v}_j^-) \tag{30}$$

Similarly as in the classical and interval versions of the TOPSIS method, the last step is based on the calculation of the ranking coefficient for particular alternatives. Relative closeness, in an optional alternative, in relation to the ideal solution is defined as:

$$\tilde{R}_i = \frac{\tilde{d}_i^-}{\tilde{d}_i^- + \tilde{d}_i^+} \tag{31}$$

Obviously an alternative is closer to the solution $\tilde{V}^+$ and further in relation to the solution $\tilde{V}^-$, the more coefficient $\tilde{R}_i$ is closer to the unity. The procedure ends with the establishment of a final ranking of alternatives which occurs according to the decreasing value of the ranking coefficient $\tilde{R}_i$.

2. Proposed procedure

As it was earlier mentioned, TOPSIS is an immensely popular multi-criteria method of supporting decision making. In practice, decision making problems appear frequently when the set of criteria is ambiguous. Heterogeneous sets of criteria should be understood as the situation when this set embraces the criteria of values for particular decision alternatives of a character not allowing for the problem analysis, according to the procedure defined for the chosen version of the TOPSIS method (classical, interval or fuzzy). Among the criteria there are values being real numbers (as in the classical version of the method), as well as interval data (as in interval data) or fuzzy numbers (as in the fuzzy version). It can be assumed that:

– n_{clasical} – number of criteria where values are real numbers,
– n_{interval} – number of criteria where values are interval data,
– n_{fuzzy} – number of criteria where values are fuzzy numbers.

One condition is met:

$$n_{\text{classical}} + n_{\text{interval}} + n_{\text{fuzzy}} = n \tag{32}$$

where n – number of all criteria.

The issue of the suggested approach is based on the use of an integrated decision matrix, marked as $\mathbf{Q}_{m,n}^{(\text{all})}$. The matrix is defined as the sum of three component matrices: ${}_{m,n}^{(\text{classical})}$, $\mathbf{Q}_{m,n}^{(\text{interval})}$ and $\mathbf{Q}_{m,n}^{(\text{fuzzy})}$:

$$\mathbf{Q}_{m,n}^{(\text{all})} = \mathbf{Q}_{m,n}^{(\text{classical})} + \mathbf{Q}_{m,n}^{(\text{interval})} + \mathbf{Q}_{m,n}^{(\text{fuzzy})} \tag{33}$$

Each of the component matrices is a matrix which in the columns corresponding to the adequate criteria includes the values of those criteria for particular decision alternatives; however, elements in the remaining columns are zeroes. This means:

$$\mathbf{Q}_{m,n}^{(\text{classical})} = \begin{bmatrix} \underbrace{\begin{matrix} Q_{1,1}^{(\text{classical})} & Q_{1,2}^{(\text{classical})} & \cdots & Q_{1,n_{\text{classical}}}^{(\text{classical})} \\ Q_{2,1}^{(\text{classical})} & Q_{2,2}^{(\text{classical})} & \cdots & Q_{2,n_{\text{classical}}}^{(\text{classical})} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{m,1}^{(\text{classical})} & Q_{m,2}^{(\text{classical})} & \cdots & Q_{m,n_{\text{classical}}}^{(\text{classical})} \end{matrix}}_{n_{\text{classical}}} & \underbrace{\begin{matrix} 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & & 0 \end{matrix}}_{n_{\text{interval}}} & \underbrace{\begin{matrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{matrix}}_{n_{\text{fuzzy}}} \end{bmatrix} \tag{34}$$

$$\mathbf{Q}_{m,n}^{(\text{interval})} = \begin{bmatrix} \underbrace{\begin{matrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{matrix}}_{n_{\text{classical}}} & \underbrace{\begin{matrix} Q_{1,1}^{(\text{interval})} & Q_{1,2}^{(\text{interval})} & & Q_{1,n_{\text{interval}}}^{(\text{interval})} \\ Q_{2,1}^{(\text{interval})} & Q_{2,2}^{(\text{interval})} & & Q_{2,n_{\text{interval}}}^{(\text{interval})} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{m,1}^{(\text{interval})} & Q_{m,2}^{(\text{interval})} & & Q_{m,n_{\text{interval}}}^{(\text{interval})} \end{matrix}}_{n_{\text{interval}}} & \underbrace{\begin{matrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{matrix}}_{n_{\text{fuzzy}}} \end{bmatrix} \tag{35}$$

$$\mathbf{Q}_{m,n}^{(\text{fuzzy})} = \begin{bmatrix} \underbrace{\begin{matrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{matrix}}_{n_{\text{classical}}} & \underbrace{\begin{matrix} 0 & 0 & & 0 \\ 0 & 0 & & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & & 0 \end{matrix}}_{n_{\text{interval}}} & \underbrace{\begin{matrix} Q_{1,1}^{(\text{fuzzy})} & Q_{1,2}^{(\text{fuzzy})} & \cdots & Q_{1,n_{\text{fuzzy}}}^{(\text{fuzzy})} \\ Q_{2,1}^{(\text{fuzzy})} & Q_{2,2}^{(\text{fuzzy})} & \cdots & Q_{2,n_{\text{fuzzy}}}^{(\text{fuzzy})} \\ \cdots & \cdots & \cdots & \cdots \\ Q_{m,1}^{(\text{fuzzy})} & Q_{m,2}^{(\text{fuzzy})} & \cdots & Q_{m,n_{\text{fuzzy}}}^{(\text{fuzzy})} \end{matrix}}_{n_{\text{fuzzy}}} \end{bmatrix} \tag{36}$$

The integrated decision matrix is therefore a flow matrix and looks as follows:

$$\mathbf{Q}_{m,n}^{(\text{all})} = \begin{bmatrix} \underbrace{\begin{matrix} Q_{1,1}^{(\text{classical})} & \cdots & Q_{1,n_{\text{classical}}}^{(\text{classical})} \\ \cdots & \cdots & \cdots \\ Q_{m,1}^{(\text{classical})} & \cdots & Q_{m,n_{\text{classical}}}^{(\text{classical})} \end{matrix}}_{n_{\text{classical}}} & \underbrace{\begin{matrix} Q_{1,1}^{(\text{interval})} & \cdots & Q_{1,n_{\text{interval}}}^{(\text{interval})} \\ \cdots & \cdots & \cdots \\ Q_{m,1}^{(\text{interval})} & \cdots & Q_{m,n_{\text{iinterval}}}^{(\text{classical})} \end{matrix}}_{n_{\text{interval}}} & \underbrace{\begin{matrix} Q_{1,1}^{(\text{fuzzy})} & \cdots & Q_{1,n_{\text{fuzzy}}}^{(\text{fuzzy})} \\ \cdots & \cdots & \cdots \\ Q_{m,1}^{(\text{fuzzy})} & \cdots & Q_{m,n_{\text{fuzzy}}}^{(\text{fuzzy})} \end{matrix}}_{n_{\text{fuzzy}}} \end{bmatrix} \tag{37}$$

In case of matrix $\mathbf{Q}_{m,n}^{(\text{classical})}$ non-zero elements are real numbers $Q_{i,j}$, similarly as in the case of matrix **Q** defined by equation (1). In the case of matrix $\mathbf{Q}_{m,n}^{(\text{interval})}$ non-zero elements are respectively transformed elements of output, the interval matrix, defined analogically to (11). Moreover, non-zero elements of matrix $\mathbf{Q}_{m,n}^{(\text{fuzzy})}$ are respectively transformed elements of output, the fuzzy decision matrix being defined analogically to (21).

The true meaning in terms of output data in an interval or fuzzy form has the transformation of respective elements of the output matrix to the values included in the matrices (35) and (36). If output data were defined as interval data the rule which operates in most of the comparing methods of intervals it would be based on the comparison of its sources (Moore, 1979; Sengupta, Pal, 2000). Therefore, non-zero elements of the matrix $\mathbf{Q}_{m,n}^{(\text{interval})}$ are indicated as:

$$Q_{i,j}^{(\text{interval})} = \frac{1}{2}\left(q_{i,j}^{L} + q_{i,j}^{U}\right) \tag{38}$$

In the case of output values defined as fuzzy numbers one can base them on the indicator suggested by Yager (1981). This indicator responds to the gravity centre of the fuzzy number $\tilde{Q}_{i,j} = \left(q_{i,j}, L_{i,j}, U_{i,j}\right)$ and is defined as a gravity centre of the space representing its affiliation function (Yager, 1981):

$$F\left(q_{i,j}, L_{i,j}, U_{i,j}\right) = \frac{3q_{i,j} - L_{i,j} + U_{i,j}}{3} \tag{39}$$

Note that if $L_{i,j} = U_{i,j}$, then:

$$F\left(q_{i,j}, L_{i,j}, U_{i,j}\right) = q_{i,j} \tag{40}$$

Non-zero elements of the matrix $\mathbf{Q}_{m,n}^{(\text{fuzzy})}$ are assumed as values:

$$Q_{i,j}^{(\text{fuzzy})} = F\left(q_{i,j}, L_{i,j}, U_{i,j}\right) \tag{41}$$

After transformation of the output ratings of the suitable criteria according to the formulas (38) and (41) matrix $\mathbf{Q}_{m,n}^{(\text{all})}$ (37) constitutes the starting point of the further analysis of the decision-making problem which can be implemented analogically as in the case of the classical TOPSIS method.

3. Numerical example

It will be considered a decision-making problem concerning the choice of an optimal scenario for the development of a particular area. The presented example is strictly theoretical and based on fictitious data. This approach results from the fact that the presented example is intended to illustrate the proposed procedure when evaluations of considered alternatives are not homogeneous. In practice, it may happen that the set of decision criteria may include criteria characteristic for each version of the TOPSIS method (i.e. classic, interval and fuzzy).

As a criteria for the rating purpose of the accepted alternatives the following criteria have been approved:

– criterion C_1 – investment capital (costs criterion),
– criterion C_2 – annual running costs (costs criterion),
– criterion C_3 – utilization level of available area (benefits criterion),
– criterion C_4 – assumed annual return on the investment (benefits criterion),
– criterion C_5 – social needs (benefits criterion).

Assuming that the rating of criteria C_1 and C_2 are real numbers (as in the classical version of the TOPSIS method), the assessment of criteria C_3 and C_4 – interval data, whereas the assessment criterion C_5 – fuzzy numbers. The ratings of individual alternatives (scenarios) that are assigned to the assumed criteria are presented in Table 1. Assuming that the admitted criteria were assigned weights presented in Table 2 (where $\sum w_i = 1.00$).

Table 1. Ratings of alternatives assigned to the assumed criteria

Alternative	Criterion				
	C_1 (million euro)	C_2 (thousand euro)	C_3 (%)	C_4 (%)	C_5 (points)
A_1	16.2	890	31–35	0.30–0.34	(9, 0.5, 0.5)
A_2	17.8	740	27–29	0.35–0.39	(7, 1.1, 1.1)
A_3	14.3	810	40–44	0.37–0.41	(5, 1.5, 1.5)
A_4	15.6	920	48–54	0.41–0.47	(6, 1.3, 1.3)
A_5	18.4	690	33–37	0.33–0.35	(8, 0.9, 0.9)

Source: own work.

Table 2. Weights of the criteria

Criterion	C_1	C_2	C_3	C_4	C_5
Weight	$w_1 = 0.35$	$w_2 = 0.10$	$w_3 = 0.05$	$w_4 = 0.20$	$w_5 = 0.30$

Source: own work.

4. Solution

First of all, according to the formulas (38) and (41), the output ratings criteria will be transformed C_3 and C_4, as well as C_5 in order to achieve the matrix $\mathbf{Q}_{m,n}^{(\mathrm{all})}$. The outcome is the decision matrix presented in Table 3.

Table 3. Decision matrix

Alternative	Criterion				
	C_1	C_2	C_3	C_4	C_5
A_1	0.4384	0.4886	0.3820	0.3804	0.5636
A_2	0.4817	0.4062	0.3241	0.4399	0.4384
A_3	0.3870	0.4446	0.4862	0.4637	0.3131
A_4	0.4221	0.5050	0.5904	0.5231	0.3757
A_5	0.4979	0.3788	0.4051	0.4161	0.5010

Source: own work.

Subsequently, using the calculation procedure of the classical version of the TOPSIS methods, the following can be indicated:

- normalized ratings (Table 4),
- corrected ratings (Table 5),
- an ideal and negative-ideal solution:

$$V^{+} = \{0.1354,\ \ 0.0379,\ \ 0.0295,\ \ 0.1046,\ \ 0.1691\}$$

$$V^{-} = \{0.1743,\ \ 0.0505,\ \ 0.0162,\ \ 0.0761,\ \ 0.0939\},$$

- distance to the ideal and negative-ideal solution (Table 6),
- global ratings (Table 7).

Table 4. Normalized ratings

Alternative	Criterion				
	C_1	C_2	C_3	C_4	C_5
A_1	0.4384	0.4886	0.3820	0.3804	0.5636
A_2	0.4817	0.4062	0.3241	0.4399	0.4384
A_3	0.3870	0.4446	0.4862	0.4637	0.3131
A_4	0.4221	0.5050	0.5904	0.5231	0.3757
A_5	0.4979	0.3788	0.4051	0.4161	0.5010

Source: own work.

Table 5. Corrected ratings

Alternative	Criterion				
	C_1	C_2	C_3	C_4	C_5
A_1	0.1534	0.0489	0.0191	0.0761	0.1691
A_2	0.1686	0.0406	0.0162	0.0880	0.1315
A_3	0.1354	0.0445	0.0243	0.0927	0.0939
A_4	0.1477	0.0505	0.0295	0.1046	0.1127
A_5	0.1743	0.0379	0.0202	0.0832	0.1503

Source: own work.

Table 6. Distance to the ideal and negative-ideal solution

Alternative	Distance	
	d_i^+	d_i^-
A_1	0.0370	0.0781
A_2	0.0546	0.0411
A_3	0.0766	0.0435
A_4	0.0591	0.0453
A_5	0.0491	0.0584

Source: own work.

Table 7. Global ratings

Alternative	Rating R_i
A_1	0.6785
A_2	0.4299
A_3	0.3622
A_4	0.4339
A_5	0.5432

Source: own work.

The following are the ranking results from above:

$$A_1 \succ A_5 \succ A_4 \succ A_2 \succ A_3.$$

Conclusions

The TOPSIS method is known in three versions: classical, interval and fuzzy (depending on the character of the rating criteria). In the present discourse, the calculation procedure by the use of the TOPSIS method is suggested, in the situation – considering the heterogeneous character of the ratings – in the set of criteria three subsets can be distinguished, including the characteristic criteria for the particular, mentioned above versions of the TOPSIS method. The probability of the occurrence of such a situation in practice increases with the growth number of the criterion set. The suggested procedure aims at defining for each subgroup a separate criteria of the decision matrix in the dimension $m \times n$ ($\mathbf{Q}_{m,n}^{(\text{classical})}$, $\mathbf{Q}_{m,n}^{(\text{interval})}$ and $\mathbf{Q}_{m,n}^{(\text{fuzzy})}$). The next step is the transformation of the matrix ratings $\mathbf{Q}_{m,n}^{(\text{interval})}$ and $\mathbf{Q}_{m,n}^{(\text{fuzzy})}$ to the form of real numbers. As a result, after summing up the three matrices a further calculating procedure can be used as in the classical version of the TOPSIS. The presented calculation example has only a demonstrative character and does not illustrate any specific use. Its aim is to illustrate a suggested calculating procedure.

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