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AN ANALYSIS OF CONDITIONAL DEPENDENCIES OF COVARIANCE MATRICES FOR ECONOMIC PROCESSES IN SELECTED EU COUNTRIES

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Abstract

The paper looks at the issues related to the research on and assessment of the contagion effect. Based on several examinations of two selected EU countries, Poland paired with one of the EU member states; it presents the interaction between their economic development. A DCC-GARCH model constructed for the purpose of the study was used to generate a covariance matrix H_t, which enabled the calculation of correlation matrices R_t. The resulting variance vectors were used to present a linear correlation model on which a further analysis of the contagion effect was based. The aim of the study was to test a contagion effect among selected EU countries in the years 2000–2014. The transmission channel under study was the GDP of a selected country. The empirical studies confirmed the existence of the contagion effect between the economic development of the Polish and selected EU economies.

Keywords: GARCH model, contagion effect, economic development

JEL classification: C01, C32, C51

Introduction

Socio-economic systems undergo continuous transformations and consequently change over time. The causes of those transformations have been analysed for many years in an attempt to enrich scientific research and observations. In the world economic literature, there are various theories analysing the relationship between globalization, technological advancement or political changes and economic growth. Economists M. Wynne and J. Koo (2000) prove that the economic fluctuations in the old EU member states are more synchronised with one another than in the countries that subsequently joined the EU. There is also extensive literature about economic growth, as well as about the disappearance of the business cycles in periods of long-term prosperity (Drozdowić-Bieć, 2006). M. Kaiser (2010) shows in his studies that the German, Austrian, French, Dutch and Belgian economies have strongly synchronised business cycles. Research into business fluctuations in the EU countries conducted by J. Azevedo (2002) reveals that they are closely correlated with economic activity fluctuations. M. Bergman (2004) claims that the business cycles across Europe strongly fit one another. The same views are expressed by A.P. Dickerson, H.D. Gibson, E. Tsakalotos (1998). The OECD experts' latest forecasts for Poland predict an acceleration in the real GDP growth due to increased exports and a steady rise in the domestic demand. On the other hand, the unfavourable situation in Ukraine may negatively affect Poland by weakening its exports and causing increases in prices, including energy prices. Business cycle synchronisation, dependencies between business activity fluctuations and a variety of factors which affect the economy hinder the construction of models and universal tools for conducting scientific studies of economies.

Economic development shocks are understood as situations where there are blurred boundaries between the economic phenomena which contribute to rapid changes in the trends determining the directions of economic development. If the components of economic development become divergent, they can lead to its collapse.

The crises of the 1990s and of the 21stcentury were mainly financial ones, connected with problems in the banking sector. It was then noticed that there was a significantly increased correlation between the majority of world stock markets, which, unfortunately, is a characteristic feature of the current crises. So, "a considerable growth in a correlation among various financial markets due to shocks is called a contagion effect" (Fiszeder, Razik, 2003). A shock in economic development can be defined as a situation where there is a clash between economic phenomena which causes rapid changes in the trends determining its direction. It is a moment when the memory of an economic development process is being obliterated (Wyciślak, 2013).

Theoretical and empirical studies strive to explain all kinds of causes, mechanisms and sources of contagion. The World Bank proposes three definitions of contagion: broad, restrictive and a very restrictive one (World Bank, 2012):

1. Contagion is the cross-country transmission of shocks or the general cross-country spill over effects.
2. Contagion is the transmission of shocks to other countries or the cross-country correlation, beyond any fundamental link among the countries and beyond common shocks.
3. Contagion occurs when cross-country correlations increase during "crisis times" relative to correlations during "tranquil times."

Any economic and financial linkages usually open up a channel for the transmission of fluctuations (Dach, 2011). A channel is understood as "a cross-country link between two or more national economies, through which there can happen mutual transmission of economic impulses, particularly various economic crises, no matter what kind of link it is or how it manifests itself, as long as it produces cross-country effects". The World Bank indicates four kinds of links which can constitute transmission channels: real, financial, behavioural and political.

The contagion effect has been studied by numerous researchers, who take two different approaches to the problem (Alvarez, Barlevy, 2015). Some of them, e.g. R. Glick and A.K. Rose, B. Eichengreen analyse the transmission of contagion. Others, including P. Masson, Forbes and R. Rigobon, I. Pritsker focus on the source of contagion. The most commonly adopted method is a correlation analysis, proposed by Forbes and Rigobon (Fiszeder, 2009), but there are also other ones, such as the VAR models, used by C. Favero and F. Giavazzi, or probability models, employed by B. Eichengreen, A. Rose and C. Wyplosz. The subject literature comprises also an event study analysis, presented by G. Kaminsky, S. Lizondo, C. Reinhart (2003) or a hidden factor analysis, developed by M. Dungey, G. Bekaert, A. Ng and R. Hodrick, G. Corsetti, M. Pericoli, M. Sbracia.

A number of other methods can be found in Masson's works and G. Marais, Babes, C. Cailleteau, Vidon, D. Diamond, P. Dybving (Naoui, Khemiri, Liuoane, 2010), M. Fratzscher (2003).

The aim of this analysis is to identify patterns of change in the dependencies that occur over a period of time in the mutual impact of selected factors affecting a country's economic development. An analysis based on a DCC-GARCH model is used to work towards comprehensive solutions (Fiszeder, 2009). The study covers Poland, France, Great Britain, Germany, Denmark, the Netherlands, Belgium and Greece. Greece has recently sparked concerns in the financial markets – despite a positive economic growth recorded there for the first time since 2007, economists doubt this positive change will be maintained as in 2015 growth already started waning.

1. A multivariate DCC-GARCH model (*p, q*)

GARCH models were first used by R.F. Engle to study inflation volatility in Great Britain. GARCH-type models, particularly the multivariate ones, fairly accurately reflect the phenomenon under observation. One of their greatest advantages is the possibility for extending equations with, i.e. various kinds of exogenous variables. It is also possible to include additional structural parameters in a volatility equation or transform the form of an equation. Engle's DCC-GARCH model has undergone multiple modifications; the literature employs an asymmetric DCC-GARCH model, a factor DCC-GARCH model, a DECO model, a DCC model of Y.K. Tse and A.K. Tsu, or a SDCC-GARCH model (Fiszeder, 2009).

The models play a key role in the research on the information flow between the populations under study. They also prove useful when development volatility in particular populations is examined, both separately in each one as well as in the entire group of populations.

A multivariate GARCH model requires simultaneous presentation of the execution of expected values and variance volatility. A model of the expected values dynamics will be featured in a direct relationship, in accordance with the formula:

$$y_t = \mu_t + \varepsilon_t \tag{1}$$

where:

- y_t – a synthetic variance related to the description of economic development,
- μ_t – defined by means of the expected value of the variance referring to the economic development y_t, taking into account the determinants of the process in the previous periods:

$$\mu_t = E(y_t | \psi_{t-1}) \tag{2}$$

If the current values of phenomenon η_t are under statistically significant influence of the values from the previous period, model (3) can be applied:

$$y_t = \mu_t + \alpha_1 y_{t-1} + \varepsilon_t \tag{3}$$

The residual component ε_t is related by its similarity to vector η_t, where:

$$\varepsilon_{it} = \sqrt{h_{it}}\eta_{it} \tag{4}$$

The components of vector η_{it} can be also presented as a subtraction (4).

$$\eta_{it} = y_{it}^2 - h_{ii,t} \tag{5}$$

Using the relationship (5), variable η_{it} and variable h_{it} can be applied to determine some theoretical values of variable y_t, according to formula (6),

$$y_{it} = \sqrt{\eta_{it} + h_{ii,t}} = \sqrt{\frac{\varepsilon_{it}}{\sqrt{h_{ii,t}}} + h_{ii,t}} \tag{6}$$

where η_t denotes the subtraction between the square of standardised synthetic variable y_t and variance h_{it} for period t.

As for residuals ε_t and η_t, their probability distributions have to be the same and that is guaranteed by the similarity of these variables.

Next, a DCC-GARCH (Bauwens and Laurent, 2006) model is constructed, using process y_t, represented by formula (6). The DCC-GARCH model can be presented as follows (Fiszeder, 2009):

$$H_t = D_t R_t D_t^T \tag{7}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{8}$$

$$Q_t = (1 - \sum_{i=1}^{q} \alpha_i - \sum_{j=1}^{p} \beta_j) S + \sum_{i=1}^{q} \alpha_i (y_{t-i} y_{t-i}^T) + \sum_{j=1}^{p} \beta_j Q_{t-j} \tag{9}$$

The values of variances and covariances used to calculate the above correlation coefficients make up the generalised matrix H_t, similar to a correlation matrix. Matrix H_t is not clearly defined; it just has to be a positive definite matrix of degree m:

$$H_t = D_t R_t D_t^T \tag{10}$$

The construction of the DCC-GARCH model starts with generating a constant covariance matrix, which is denoted by S.

In the covariance matrix, a fixed part and a dependent part are singled out for each of its elements. Fixed and invariant levels of the correlation coefficients are placed in the matrix. Matrix S from equation (9) is calculated by means of the formula (Fiszeder, 2009):

$$S = S_{kl} = \mathrm{cov}(y_k, y_l) = \overline{\varsigma_{kl}} \tag{11}$$

These are fixed covariance values, which do not change over time t. Delays are taken into account, where $p = q = 1$, and then, model Q_t from the DCC-GARCH model takes the form:

$$Q_t = (1-\alpha-\beta)S + \alpha y_{t-1} y_{t-1}^T + \beta Q_{t-1} \tag{12}$$

where y_t is denoted as (1).

Taking into consideration the above-mentioned dependencies, the elements of matrix Q_t are calculated in the following way:

$$q_{kl,t} = (1-\alpha-\beta)\operatorname{cov}(y_k, y_l) + \alpha y_{t-1} y_{t-1}^T + \beta q_{kl,t-1} \tag{13}$$

By properly ordering the elements in equation (12) we get:

$$q_{kl,t} = \overline{\varsigma_{kl}} + \alpha(y_{t-1} y_{t-1}^T - \overline{\varsigma_{kl}}) + \beta(q_{kl,t-1} - \overline{\varsigma_{kl}}) \tag{14}$$

We determine delays of the covariance matrix for dependent variables and use relationship:

$$\begin{gathered} \eta_{k,t-1} \cdot \eta_{l,t-1}^T = \frac{\varepsilon_{k,t-1}}{\sqrt{h_{k,t-1}}} \times \frac{\varepsilon_{l,t-1}}{\sqrt{h_{l,t-1}}} \\ y_{k,t-1} \cdot y_{l,t-1}^T = \sqrt{\eta_{k,it-1} + h_{kk,t-1}} \times \sqrt{\eta_{l,t-1}^T + h_{ll,t-1}^T} \end{gathered} \tag{15}$$

When two countries are compared, the matrix takes the form:

$$Q = \begin{bmatrix} y_{k,t-1} y_{k,t-1}^T & y_{k,t-1} y_{l,t-1}^T \\ y_{l,t-1} y_{k,t-1}^T & y_{l,t-1} y_{l,t-1}^T \end{bmatrix} \tag{16}$$

The variances of these matrices should be constant over time or differ insignificantly from one another, whereas covariances can change with the passage of time. If covariances are positive, a stimulating impact of one country's economy on another country's economy can be observed in a proper period of time. And if the covariance is negative, the economies destimulate each other. Matrix Q is symmetric and positive definite. Next, the diagonal matrix Q^* is built, with the main diagonal consisting of the roots of the values from the principal diagonal of matrix Q.

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \sqrt{q_{mm,t}} \end{bmatrix} \tag{17}$$

Matrix R_t is a correlation matrix whose elements are defined as follows:

$$\rho_{ijt} = \frac{q_{ijt}}{\sqrt{q_{ii} q_{jj}}} \tag{18}$$

Matrix R_t, similarly to matrix Q, is positive definite since both matrices are alike. Matrices D_t and R_t are used to construct variance model H_t Matrix D_t, occurring in the equation, is a similarity matrix of matrix H_t and correlation matrix R_t. It is defined by the formula:

$$D_t = \begin{bmatrix} \sqrt{h_{11,t}} & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & \sqrt{h_{mm,t}} \end{bmatrix} \tag{19}$$

Matrix H_t, as similar to a positive definite correlation matrix, is also a positive definite matrix. In addition, it is a diagonal matrix of degree m. In the analysed DCC-GARCH model, matrix R_t is time-dependent (Fiszeder, 2009). The random component ε_{kt} is conditionally subordinate to random components ε_{lt-i}, $l < k$, bound on conditional variance $h_{kk,t}$ by the equality of the expected values:

$$E\,h_{kk\ t} = E\varepsilon_{kt}^2 \tag{20}$$

Acting on Engle's recommendation, ML is applied to produce a DCC-GARCH model of a generalised dynamic variance. The maximum likelihood estimator is as follows:

$$L = -\frac{1}{2}\sum_{t=1}^{T}(m\log(2\pi) + 2\log|D_t| + \log(|R_t| + \varepsilon_t R_t^{-1}\varepsilon_t) \qquad \varepsilon_t \sim N(0, I_t) \tag{21}$$

The final model will be determined by the following estimator:

$$h_{it} = \omega_i + \sum_{p=1}^{P_i}\alpha_{ip} y_{it-p}^2 + \sum_{q=1}^{Q_i}\beta_{iq} h_{it-p} \tag{22}$$

In the case when $p = q = 1$, the model takes the form:

$$h_{it} = \omega_i + \alpha_i y_{it-1}^2 + \beta_i h_{it-1} \tag{23}$$

Based on model (23), the theoretical variances h_{it} are determined, which are used to estimate, by means of the Maximum Likelihood Estimation method, model (24).

2. An analysis of the contagion

An analysis of the contagion effect in the economic development of the countries under study by means of selected GARCH models can by performed in various ways. Two of them are presented below. One can be used when the economies of two countries are compared. The other, which applies a simple multivariate model of economic development, allows for a parallel comparison of any number of developing economies. In the study, the first of the presented methods is applied. Matrix *S* is generated for each pair of the counties under study in order to analyse the level of dependence between the development of their economies and the contagion occurrence. In the previous chapter, a DCC-GARCH model was presented, in which the relationship coefficient for economies was emphasised. In the constructed matrix *S*, we obtain two values on the main diagonal. One of them, e.g. lower, represents one economy, whereas the other, higher, is assigned to another economy. That means that the economy with the higher value affects the economic development of that with the lower value. When we examine the strength of impact and contagion, we can devise an equation put forward by e.g. R. Chou and D. Li (2009). As it can be employed for two economies only, an appropriate equation has to be estimated for each pair of economies.

The sequences of variances h_t, calculated earlier in accordance with model (4), constitute a basis for the estimation of this model:

$$\rho_{ij,t} = \alpha_0 + \beta_i h_{ti} + \beta_j h_{tj} + \varepsilon_{ij,t} \tag{24}$$

The conditional correlation equation is time-dependent, where α_0, β_i, β_j are the model parameters estimated by ML, while h_{ti}, h_{tj} are variances in the time period t for the i^{th} and j^{th} economy. If all coefficient values are positive, α_0 denotes a fixed correlation, invariable over time (in the case when $\alpha_0 \leq 1$). Then the contagion effect may, or may not, occur.

If $\alpha_0 > 1$, one economy exerts a substantial impact on the other (along with the contagion effect). In the case when $\alpha_0 > 1$, additional verification has to be carried out by means of matrix H_t from the DCC-GARCH model. Coefficients β are negative in such a situation. The model provides a basis for the examination of a hypothesis on the existence of the contagion effect between the analysed economies.

3. An empirical analysis

In order to conduct analyses for the selected countries of (the United Kingdom, Belgium, Denmark, France, Poland, the Netherlands, Germany and Greece) empirical data were prepared based on the data on annual GDP published by the Central Statistical Office and Eurostat. The analysis covers the years 2000–2014. It presents a comparison of Poland's economy with the economies of selected countries, because they enjoy stable development and economic growth. The GDP ratio is a fundamental determinant of changes in economies and is also a factor affecting economic fluctuations. GDP may be treated as a synthetic representation of a country's economic situation. Its value and volatility depend on a number of factors which determine the economic development of a country. When related to the population size, it becomes a key measure of the condition of the economy. GDP (Hellwig, 1997) is highly valued as the broadest and most comprehensive indicator of the overall economic performance of a country.

It has been already shown that the random component tends to be auto correlated. It is also heteroscedastic, and there are periods of aggregated variance. A Ljung-Box test was applied to discover the existence of the ARCH effect (Janiga-Ćmiel, 2013). For the selected EU economies and for the initial calculations for Poland and France, matrix S takes the form:

Table 1. Fixed levels of variances and covariances calculated for the economic development of Poland and France

S	France	Poland
France	4.12	0.35
Poland	0.35	1.27

Source: based on own research.

Based on the established variance and covariance matrix, we find out that a fixed, time-independent correlation coefficient stands at 0.15. Matrix Q_0 calculated in accordance with formula (14) takes the form:

$$Q_0 = \begin{bmatrix} 4.42 & 0.35 \\ 0.35 & 1.38 \end{bmatrix} \tag{25}$$

The elements of the matrix are used to determine parameters α, β, affecting the course of an iterative process. They fulfil the following condition:

$$\alpha + \beta < 1 \tag{26}$$

The estimated values of the parameters are as follows:

$$\alpha = 0.68 \text{ and } \beta = 0.21 \tag{27}$$

Under the condition (17), the sum of the parameters is less than 1. Moreover, α denotes a parameter determined by the dependent variables, while β – by the correlation matrix dynamics. In the next stage of the study, the following model was built:

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = \begin{bmatrix} 4.12 & 0.35 \\ 0.35 & 1.25 \end{bmatrix} + 0,68\left(\begin{bmatrix} y_{k,t-1}y_{k,t-1}^T & y_{k,t-1}y_{l,t-1}^T \\ y_{l,t-1}y_{k,t-1}^T & y_{l,t-1}y_{l,t-1}^T \end{bmatrix} - \begin{bmatrix} 4.12 & 0.35 \\ 0.35 & 1.25 \end{bmatrix}\right) +$$
$$+ 0.21\left(\begin{bmatrix} y_{k,t-1}y_{k,t-1}^T & y_{k,t-1}y_{l,t-1}^T \\ y_{l,t-1}y_{k,t-1}^T & y_{l,t-1}y_{l,t-1}^T \end{bmatrix} - \begin{bmatrix} 4.12 & 0.35 \\ 0.35 & 1.25 \end{bmatrix}\right) =$$
$$= \begin{bmatrix} 4.12 & 0.35 \\ 0.35 & 1.25 \end{bmatrix} + 0.68\left(\begin{bmatrix} \eta_{1t} + h_{11,t} & \sqrt{\eta_{1t} + h_{11,t}}\sqrt{\eta_{2t} + h_{22,t}} \\ \sqrt{\eta_{1t} + h_{11,t}}\sqrt{\eta_{2t} + h_{22,t}} & \eta_{2t} + h_{22,t} \end{bmatrix} - \begin{bmatrix} 4.12 & 0.35 \\ 0.35 & 1.25 \end{bmatrix}\right) + \tag{28}$$
$$+ 0.21\left(\begin{bmatrix} \eta_{1t} + h_{11,t} & \sqrt{\eta_{1t} + h_{11,t}}\sqrt{\eta_{2t} + h_{22,t}} \\ \sqrt{\eta_{1t} + h_{11,t}}\sqrt{\eta_{2t} + h_{22,t}} & \eta_{2t} + h_{22,t} \end{bmatrix} - \begin{bmatrix} 4.12 & 0.35 \\ 0.35 & 1.25 \end{bmatrix}\right)$$

Based on the model, we can calculate matrix Q_1:

$$Q_1 = \begin{bmatrix} 5.15 & 2.26 \\ 2.26 & 8.34 \end{bmatrix} \tag{29}.$$

Next, we generate matrix Q_t^* and a matrix inverse:

$$Q_1^* = \begin{bmatrix} \sqrt{5.15} & 0 \\ 0 & \sqrt{8.34} \end{bmatrix} = \begin{bmatrix} 2.27 & 0 \\ 0 & 2.89 \end{bmatrix}$$
$$Q_1^{*-1} = \begin{bmatrix} 0.44 & 0 \\ 0 & 0.35 \end{bmatrix} \tag{30}$$

Where 0.44 > 0.35 means that the economy of France affects the Polish economy.

Afterwards, we construct matrix R_t $(t = 1)$:

$$R_t = \begin{bmatrix} 1 & 0.17 \\ 0.17 & 1 \end{bmatrix} \tag{31}$$

and matrix H_t from the DCC-GARCH model:

$$H_t = \begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix}\begin{bmatrix} 1 & 0.17 \\ 0.17 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{bmatrix} =$$
$$= \begin{bmatrix} h_{11,t} & 0.17\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 0.17\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{32}$$

The value of an influence coefficient of one economy on the other amounts to 0.17. For France, the equation takes the form:

$$\rho_{ij,t} = 0.17 + 0.14 h_{t_F} + 0.32 h_{tp} + \varepsilon_{ij,t} \tag{33}$$

Matrix H_t, calculated for a period of time t, indicates an increasing dependence between the development of the Polish and French economies as the correlation coefficient of 0.17 in matrix R_t is higher than the fixed correlation level, with its coefficient of 0.15.

The results for Poland and Belgium.

Table 2. Fixed levels of variances and covariances calculated for the economic development of Poland and Belgium

S	Belgium	Poland
Belgium	2.33	0.17
Poland	0.17	0.95

Source: based on own research.

$$H_t = \begin{bmatrix} h_{11,t} & 0.13\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 0.13\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{34}$$

$$\rho_{ij,t} = 0.13 + 0.014 h_{t_B} + 0.027 h_{tp} + \varepsilon_{ij,t} \tag{35}$$

The calculated value $\alpha_0 = 0.3$ indicates the moderate impact of one economy on the other. Inequality $\beta_B < \beta_P$ means the Belgian economy exerts a modest influence on the Polish economy. A similar analysis is conducted for Great Britain, obtaining respectively:

Table 3. Fixed levels of variances and covariances calculated for the economic development of Poland and Great Britain

S	Great Britain	Poland
Great Britain	5.18	0.33
Poland	0.33	2.47

Source: based on own research.

$$H_t = \begin{bmatrix} h_{11,t} & 1.96\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 1.96\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{36}$$

$$\rho_{ij,t} = 1.96 - 0.35h_{t_{WB}} - 0.941h_{tp} + \varepsilon_{ij,t} \tag{37}$$

The results show that the British economy strongly affects Poland's economy. For the Netherlands, the results are as follows:

Table 4. Fixed levels of variances and covariances
calculated for the economic development of Poland and the Netherlands

S	The Netherlands	Poland
The Netherlands	1.24	0.17
Poland	0.17	1.01

Source: based on own research.

$$H_t = \begin{bmatrix} h_{11,t} & 0.44\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 0.44\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{38}$$

$$\rho_{ij,t} = 0.44 + 0.001h_{t_H} + 0.003h_{tp} + \varepsilon_{ij,t} \tag{39}$$

The results show that the Polish economy influences the Dutch economy. When the economies of Poland and Denmark are compared, we receive:

Table 5. Fixed levels of variances and covariances
calculated for the economic development of Poland and Denmark

S	Denmark	Poland
Denmark	0.64	0.03
Poland	0.03	0.15

Source: based on own research.

$$H_t = \begin{bmatrix} h_{11,t} & 0.11\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 0.11\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{40}$$

$$\rho_{ij,t} = 0.11 + 0.111h_{t_D} + 0.214h_{tp} + \varepsilon_{ij,t} \tag{41}$$

This indicates a moderate impact of one economy on the other. For Poland and Germany the levels are:

Table 6. Fixed levels of variances and covariances calculated for the economic development of Poland and Germany

S	Germany	Poland
Germany	6.21	0.99
Poland	0.99	2.43

Source: based on own research.

$$H_t = \begin{bmatrix} h_{11,t} & 1.99\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 1.99\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{42}$$

$$\rho_{ij,t} = 1.99 - 0.75h_{t_N} - 0.92h_{tp} + \varepsilon_{ij,t} \tag{43}$$

This proves that there is a strong mutual influence. The last pair of countries in the study are Poland and Greece.

Table 7. Fixed levels of variances and covariances calculated for the economic development of Poland and Greece

S	Greece	Poland
Greece	2.05	–0.05
Poland	–0.05	1.04

Source: based on own research.

$$H_t = \begin{bmatrix} h_{11,t} & 0.01\sqrt{h_{11,t}}\sqrt{h_{22,t}} \\ 0.01\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} \end{bmatrix} \tag{44}$$

$$\rho_{ij,t} = 0.01 + 0.03h_{t_G} - 0.05h_{tp} + \varepsilon_{ij,t} \tag{45}$$

The results reveal the weak influence of the Greek economy on the Polish economy and a negligible decreasing correlation. Greece records a negative value of the coefficient, which is a sign of a divergence of the economic development of the two countries and a weak correlation. The estimation of the model parameters are presented with the p-value not exceeding 0.05.

Summing up the results of the variance dynamics matrix, we can order the impact of the individual economies on Poland's economic development taking into consideration a mutual influence coefficient.

Table 8. Influence coefficients for the economies under study

Country	Influence coefficients
Denmark	1.99
Belgium	0.13
Great Britain	1.96
the Netherlands	0.44
Denmark	0.11
France	0.17
Greece	0.01

Source: based on own research.

It can be noticed that the influence coefficients for all economies, except those of Great Britain and Germany, are less than 1. This indicates a decreased interdependence of their economic development. As far as the British and German economies are concerned, an increased correlation can be observed.

Conclusions

The paper has looked at the dependencies occurring in the economic development of selected EU countries, analysing two countries, i.e. Poland and another EU member state, at a time. The DCC-GARCH model led to the generation of the covariance matrix H_t, which enabled the creation of correlation models for the successive intervals of the time period under study; they feature the values of the development correlation coefficient of two economies over time. The results show that the development variability of the economies under study is strongly dependent on the variance of the DCC-GARCH model. The analysis of the pairs of economies including France and Poland, Belgium and Poland, the Netherlands and Poland, Denmark and Poland resulted in correlation models with positive coefficients not exceeding the value of 1, which indicates the moderate development interdependence of these economies. However, the contagion effect can be observed between them as at least one of the obtained coefficients exceeds the value of coefficient α. In the case of pairs consisting of Germany and Poland and Great Britain and Poland, the independent variable in both models exceeds 1 and the values of coefficient β are negative. This means that the development of these economies is strongly correlated and thus a significant contagion can be observed. The model created as a result of the analysis conducted for Greece and Poland confirmed the lack of development interdependence of the two economies and a lack of the contagion effect (one of the variance coefficients in the GARCH model is positive, the other is negative).

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