DE GRUYTER OPEN

Folia Oeconomica Stetinensia
DOI: 10.1515/foli-2016-0004

WYDZIAŁ NAUK EKONOMICZNYCH I ZARZĄDZANIA
UNIWERSYTET SZCZECIŃSKI

ASSESSMENT OF THE DEGREE OF THE DIVERGENCE AND INEQUALITY OF HOUSEHOLD INCOME DISTRIBUTION IN POLAND IN THE YEARS 2005–2013

Filip Edmund Gęstwicki, M.Sc.

University of Warmia and Mazury in Olsztyn
The Department of Quantitative Methods, Faculty of Economics
M. Oczapowskiego 4, 10-710 Olsztyn, Poland
e-mail address: filip.gestwicki@uwm.edu.pl

Ewa Wędrowska, Ph.D., Associate Prof.

Nicolaus Copernicus University in Toruń
The Department of Econometrics and Statistics, Faculty of Economic Sciences and Management
Gagarina 13 A, 87-100 Toruń, Poland
e-mail: ewaw@umk.pl

Received 10 November 2015, Accepted 11 June 2016

Abstract

The increase in income and wealth inequality observed in the last decade of the twentieth century and the first decade of the twenty-first century is the subject of many analyses and discussions. Research shows that major changes in household incomes in Poland took place in the early years of transition (1990–1992), known as a 'revolution in income'. The article focuses on the assessment of the degree of household income inequality after the Poland's accession to the European Union. The most commonly used measures in income inequality studies are the measures of inequality based on the Lorenz function – a popular Gini coefficient and the Schutz ratio, measures using the concept of entropy, measures based on welfare function, or measures based on income distribution quantiles. The article proposes the possibility of broadening the measuring spectrum of income inequality analysis of the Csiszár's divergence measures. The main research objective of the article is to assess the divergence in the distribution of household equivalent disposable income in Poland in the years 2005–2013. The data used in the analysis come from the European Survey on Income and Living Conditions (EU-SILC).

Keywords: household income, income inequality, income distribution, divergence, Csiszár's divergence measures

JEL classification: C10, D14, D31, D33, D63

Introduction

Income inequality has long been and still is the subject of studies in Poland and abroad. The Polish and foreign subject literature undertakes the issue of growing inequality of household income.

The research conducted on the concentration, divergence and dispersion of salaries in Poland, as well as in other post-socialist countries, led to the conclusion that social sentiments about the increase in income inequality in the nineties of the twentieth century are not confirmed by the analyses performed on the level of income inequality measures. The subject literature hypothesizes that the difference in the perception of inequality stems from the increases in the level of the income of top earners, in particular within the private sector, though the poorest were not getting richer at a slower rate than the group of those from the mid-part of the social structure. Moreover, the most affluent group did not gain distinct advantages relative to other groups (Cichomski, 2001). Szymańska (2013) points to the increase by 1% of the wealthiest citizens as a cause of growing income disparities in OECD countries.

As follows from the research conducted, major changes in household incomes in Poland occurred in the early years of its transition (1990–1992), referred to as a 'revolution in income' (Szopa, 2006). The studies on the income obtained in the time period 1998–2005 also found that there was a slow but steady increase in inequality between different types of households (Ulman, Wałęga, 2006).

In the context of the available outcome of research regarding the analysis of household income, the article attempts to assess the degree of the divergence of this income in Poland in the time period from 2005 to 2013. Furthermore, the authors propose to use Csiszár's divergence measures to quantify the intensity of the changes occurring in the distribution of income by deciles, which can be a valuable contribution to the methodological apparatus applied.

The research objective of the article is to examine the degree of the changes in the distribution of income of Poland's households in the years 2005–2013.

1. Data source

The article uses data supplied by the European Union Statistics on Income and Living Conditions (EU-SILC). This is a study carried out under the Regulation of the Parliament and the Council of the European Union from 16 June 2003, whose aim is to provide comparable data for EU member states regarding the living conditions of the population (the Central Statistical Office [CSO], 2014). In Poland this study has been performed since 2005 by the

Central Statistical Office. It takes the form of a questionnaire survey conducted by means of a rotational panel on a four-year basis. The respondents are asked questions relating to such issues as, for instance, demographic characteristics, education, health, housing conditions, detailed information on economic activity, as well as the level and sources of earnings.

For the purpose of comparing the income situation of households, equivalent income taking into account economies of scale is applied. Eurostat uses the OECD modified equivalence scale (OECD50/30). The equivalence scale is composed of the parameters that allow the comparison of the situation of households representing different demographic structures. The OECD50/30 scale assigns to the first adult the weight of 1, every subsequent person aged 14 or more is assigned 0.5, and every person in a household under 14 is assigned the weight of 0.3 (CSO, 2014).

Increasingly, there are doubts related to the use of the data obtained from the survey questionnaires made on households. Some researchers suggest that these surveys are marked by a high rate of refusals within the group of affluent and wealthy persons, and, as a result, the income inequality measures provide underestimated values. Therefore, they propose using the Personal Income Tax returns (Brzeziński, 2015). In addition, the significantly higher variance in the group of the wealthiest citizens is indicated, which may also affect the reliability of the obtained results, particularly in the cases with relatively small samples (Cichomski, 2001). Jagielski, Duczmal, Kutner (2015) supplemented the data from the EU-SILC with the ranking of the wealthiest people prepared by the Forbes magazine ('The World's Billionaires').

The present article assesses the degree of inequality and income divergence based on the data obtained from EU-SILC. It was assumed that the harmonization of these studies at the European Union level provides comprehensiveness as well as comparability between the EU member states.

The report published by the CSO after the completion of every subsequent study contains comprehensive analyses of the results obtained. This report describes such aspects as, for instance, the structure of the population according to various criteria, the average annual income per capita and per equivalent unit in a household, net disposable income, the data on the housing situation, social cohesion indicators, i.e. the poverty line, the Gini coefficient, and the poverty risk indicator. This article presents the results of the analysis of income inequality on the basis of the indicators derived from distribution deciles, which complement the results available in the aforementioned report by the CSO.

2. Household income inequality

Analyses of inequality in income distribution use different measures allowing for a variety of variables. The most commonly used measures include the Gini coefficient (a measure of concentration), ratios of deciles of income distribution, and, the less frequently used, the variance of income, the Atkinson measure, and generalized entropy measures. The Gini coefficient denotes the relation of a half of the average absolute difference between the income of a randomly selected pair of individuals to average income (Kasprzyk, Wojnar, 2010). Depending on whether the calculations take into account social transfers, such as annuities, pensions, benefits, taxes, subsidies for medicines, or child maintenance tax reliefs, three different approaches to the Gini coefficient can be applied: the Gini coefficient before social transfers (all), the Gini coefficient before social transfers but including annuities and pensions, and the Gini coefficient after all social transfers. In this work the authors present the results for the Gini coefficient obtained after all social transfers and before social transfers but including annuities and pensions. It is obvious, therefore, that the coefficient values allowing for transfers reached lower values than those not taking into account social transfers.

The measures of inequality based on distribution deciles include the quintile and decile differentiation ratios of extreme distribution parts, the decile differentiation, and the maximum equality ratio.

The decile differentiation ratio of extreme distribution parts $K_{10/1}$ is the ratio of the total of equivalent incomes earned by the wealthiest 10% of households to the total income of the poorest 10%. However, as pointed out by Cichomski (2001), the last decile group is marked by high variance, which may adversely affect the adequacy of the data obtained through the surveys based on relatively small samples. While referring to this comment, Bochenek and Mikołajewska (2013) applied another measure, i.e. the decile differentiation ratio $D_{9/1}$, defined by the following formula:

$$D_{9/1} = \frac{D_9}{D_1} \times 100\% \tag{1}$$

where D_i is i-th decile.

In addition to the decile rate, the study also used the quintile differentiation ratio of extreme income parts:

$$Q_{5/1} = \frac{Q_5}{Q_1} \times 100\% \tag{2}$$

where Q_i is the total income in the i-th quantile group.

Another measure of inequality that was applied is the maximum equality ratio (MER), which indicates what proportion of the total income of the whole society should be transferred from decile groups with more than 10% of income to the groups whose share is less than 10%, in order to obtain a complete household income equality (Panek, 2015).

The household income distribution by deciles presented in Table 1 indicates what portion of the total income is attributed to every subsequent 10% of the households, ranked by the increasing levels of disposable income per equivalent unit.

Table 1. The household income distribution by deciles (%) in Poland in the years 2005–2013

Decile groups	2005	2006	2007	2008	2009	2010	2011	2012	2013
The first	2.2	2.7	2.9	3.0	3.1	3.1	3.0	3.1	3.1
The second	4.2	4.5	4.7	4.8	4.8	4.8	4.9	4.9	4.9
The third	5.5	5.7	5.9	5.9	6.0	6.0	6.0	6.0	6.1
The fourth	6.6	6.7	6.9	6.9	6.9	7.0	7.0	7.0	7.1
The fifth	7.7	7.8	7.9	7.9	8.0	8.1	8.1	8.1	8.2
The sixth	8.9	9.0	9.1	9.0	9.1	9.2	9.2	9.2	9.2
The seventh	10.3	10.5	10.3	10.3	10.4	10.5	10.5	10.5	10.5
The eighth	12.3	12.3	12.1	12.1	12.1	12.2	12.2	12.2	12.2
The ninth	15.4	15.3	15.0	14.8	14.7	14.9	14.8	14.8	14.9
The tenth	26.9	25.5	25.1	25.3	24.8	24.3	24.4	24.2	24.0

Source: EU-SILC, Eurostat.

In 2005 the households in the lowest quintile group reached a total of 6.4% of the income of the entire population of the households, while the households in the highest quintile group had 42.3% of the total income at their disposal. In 2013 the percentage of the poorest households that already achieved 8% of the total income was 20%, and 20% of the richest households had 38.9% of the income of the entire population. This demonstrates the existence of significant, but declining in the period considered, disparities in the distribution of the total household income between the poorest households and the wealthiest ones.

The impact of social transfers on the value of the Gini coefficient is the subject of research, the results of which are published *inter alia* in Zwiech's work (2013).

The analysed period exhibits a small but progressive transfer of income from the highest decile groups to the groups with the lowest income. The income inequality measured by the Gini coefficient did not show significant changes throughout the period, thus indicating the stabilization of the inequality level (Figure 1). What is marked is the difference between the level of income inequality including social transfers and the one excluding social transfers.

Moreover, the degree of income inequality of the households in Poland reached a level close to the level of EU15.[1]

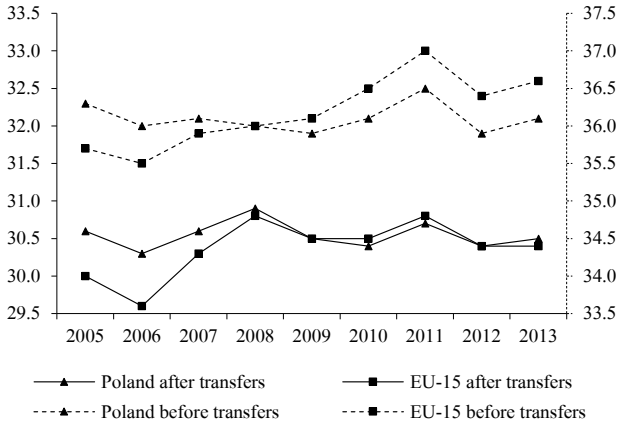

Figure 1. The value of the Gini coefficient in Poland and in the EU15 after social transfers (the main axis) and before transfers but including annuities and pensions (the secondary axis)

Source: elaborated by the authors based on EU-SILC, Eurostat data.

In 2005 the level of the income inequality in Poland measured by the decile and quintile divergence ratios of extreme distribution parts, the decile differentiation ratio, and by MER, significantly exceeded the level of the income inequality in the EU15. Then, the values of these measures for Poland fell, approaching the level of the indices for EU countries, and after 2009 the difference became insignificant.

Table 2 summarizes the values of the inequality measures analysed in Poland in the years 2005–2013 and their relative changes. The decile measures of inequality indicate in most cases a decrease in income divergence over the time period under study. The Gini coefficient after social transfers shows a slight reversible variation. A slight volatility confirms the continuing stabilization of the household inequality during the period considered.

[1] EU15 are the member states of the so-called 'Old EU' which made up the Community before the 2004 enlargement.

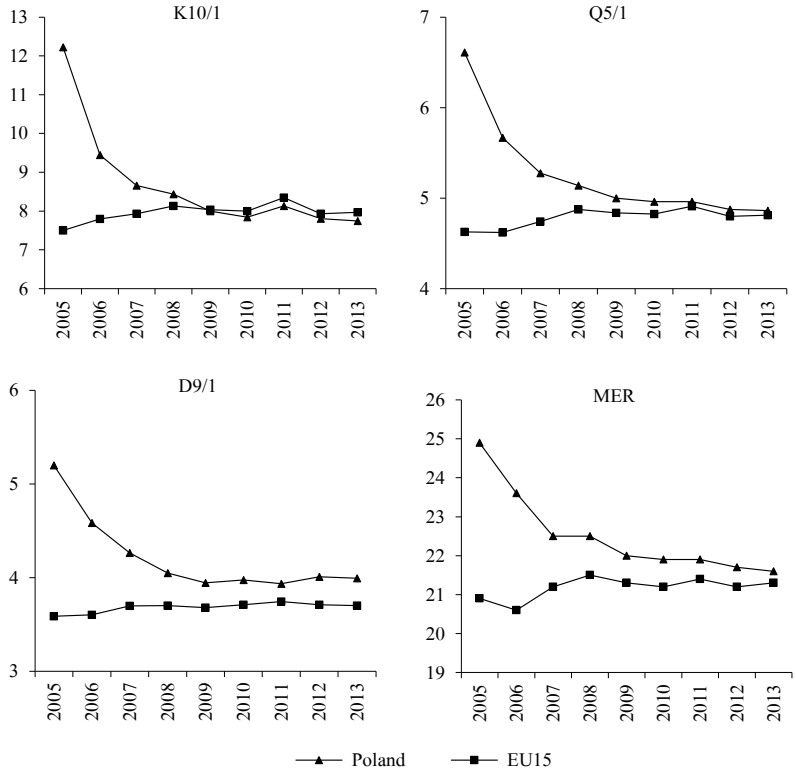

Figure 2. The values of selected measures of inequality in Poland and in the EU15 in the years 2005–2013

Source: elaborated by the authors.

Table 2. The values of selected measures of inequality in Poland in the years 2005–2013 and their changes year to year, expressed as a percentage

	$K_{10/1}$	± [%]	$Q_{5/1}$	± [%]	$D_{9/1}$	± [%]	MER	± [%]	Gini	± [%]
2005	12.23		6.61		5.20		24.9		30.6	
2006	9.44	–22.7	5.67	–14.2	4.58	–11.8	23.6	–5.2	30.3	–0.9
2007	8.66	–8.3	5.28	–6.8	4.26	–6.9	22.5	–4.6	30.6	+0.9
2008	8.43	–2.5	5.14	–2.5	4.05	–5.0	22.5	0	30.9	+0.9
2009	8.00	–5.1	5.00	–2.7	3.94	–2.6	22.0	–2.2	30.5	–1.2
2010	7.84	–2.0	4.96	–0.7	3.97	+0.7	21.9	–0.4	30.4	–0.3
2011	8.13	+3.7	4.96	0	3.94	–0.9	21.9	0	30.7	+0.9
2012	7.81	–4.0	4.88	–1.7	4.01	+1.8	21.7	–0.9	30.4	–0.9
2013	7.74	–0.8	4.86	–0.2	3.99	–0.4	21.6	–0.4	30.5	+0.3

Source: the authors' own calculations.

3. The assessment of the degree of divergence in the household income distribution by deciles

3.1. Csiszár's divergence measures

One of the important problems in solving of which measures of divergence are used is examining the distance, divergence, or discrimination between the distribution of random variables. Within the measures of divergence, one can distinguish Csiszár's divergence measures, referred to as *f*-divergence, or, more rarely, Csiszár-Ali-Silvey divergences. The concept of *f*-divergence as a measure of divergence between two probability distributions, was introduced at the same time by Csiszár (1967) in 1967, and Ali and Silvey (1966). Csiszár's divergence measures are a generalization of certain measures of divergence, and are defined by means of convex *f*-functions determined within the interval $[0,\infty)$ of real values.

The measure of divergence belonging to Csiszár's divergence measures (*f*-divergence) between a pair (S_t^n, S_τ^n) of *n*-element vectors (the elements that are indicative of the structure or share) from the set $\Gamma^n = \left\{ S_j^n = [\omega_{1j}, \omega_{2j}, ..., \omega_{nj}]^T \mid 0 \le \omega_{ij} \le 1, \sum_{i=1}^{n} \omega_{ij} = 1 \right\}$ for $j = 1, ..., m$, is defined by the following formula (Csiszár, 1967):

$$C_f(S_t^n, S_\tau^n) = \sum_{i=1}^{n} \omega_{i\tau} f\left(\frac{\omega_{it}}{\omega_{i\tau}}\right) \tag{3}$$

If a divergence measure is to be classified into Csiszár's divergence measures, the function $f(x)$ occurring in the formula (3) must be differentiable and convex, and for $x = 1$, $f(1) = 0$, $f''(1) \ge 0$ and for $x = 0$ the following holds (Menéndez, Pardo, Pardo, Zografos, 2003): $0 \times f\left(\frac{0}{0}\right) = 0$ and $0 \times f\left(\frac{\omega}{0}\right) = \lim_{x\to\infty} \frac{f(x)}{x}$.

The divergence C_f for a pair of vectors $(S_t^n, S_\tau^n) \in \Gamma^n \times \Gamma^n$ is convex and assumes the non-negative values for the convex function $f : [0,\infty) \to \Re$, such that $f(1) = 0$ (Taneja, 2005a), besides $C_f(S_t^n, S_\tau^n) = 0$ for all functions $f(x)$ that take a value of zero for the argument equal to unity (Dragomir, Gluščević, Pearce, 2001).

Among *f*-divergences, measures of various properties can be distinguished. These are the measures that meet property metrics, measures that do not meet the condition of symmetry, standardized and infinite. The examples of Csiszár's divergence measures include commonly known measures such as urban distance, squared Hellinger distance, triangular distance, χ^2-divergence or Kullback-Leibler divergence. In information theory and statistics, various methodologies of designing divergence measures are still being developed. The proposals made by Lin (1991), Taneja (1995), and Bhatia and Singh (2013) serve as examples. The list

of Csiszár's divergence measures along with their properties are contained in Taneja's (2005b, 2008) and Wędrowska's (2012) works.

The article focuses on the quantification of the intensity of distribution changes of household income over time, therefore, for this purpose, non-symmetric divergence measures were applied. The construction of the below-described non-symmetric *f*-divergence measures allows for the weight of the transformation of the vector S_τ^n into the vector S_t^n, assuming that the vector S_τ^n is marked by distribution by deciles of income over time τ preceding the moment *t*.

The first of the Csiszár's divergence measures is Lin's proposal referred to as *K*-divergence (Lin, 1991):

$$K(S_t^n, S_\tau^n) = \sum_{i=1}^{n} \omega_{it} \log_2 \frac{\omega_{it}}{\frac{1}{2}\omega_{it} + \frac{1}{2}\omega_{i\tau}} \tag{4}$$

For the divergence measure defined by Lin, the function $f(x)$ specified for $x \geq 0$ is a function of the following form:

$$f_K(x) = x \log_2 \frac{2x}{1+x} \tag{5}$$

For any pair of structures $(S_t^n, S_\tau^n) \in \Gamma^n \times \Gamma^n$ *K*-divergence takes values from the interval [0,1], assuming the value of zero for identical structures.

Another measure used is the Kullback-Leibler divergence (*KL*-divergence, relative entropy):

$$D_{KL}(S_t^n, S_\tau^n) = \sum_{i=1}^{n} \omega_{it} \log_2 \frac{\omega_{it}}{\omega_{i\tau}} \tag{6}$$

(6) The convex function $f : [0, \infty) \to \Re$ corresponding to the measure (6) is of the form:

$$f_{KL}(x) = x \log_2 x \tag{7}$$

The last of the Csiszár's divergence measures applied to assess the intensity of changes in the income distribution by deciles is χ^2-divergence. If the formula (3) adopts a convex function (Anwar, Hussain, Pečarić, 2009):

$$f_{\chi^2}(x) = (x-1)^2 \tag{8}$$

the divergence measure determined for $x \geq 0$ will take the form:

$$\chi^2(S_t^n, S_\tau^n) = \sum_{i=1}^{n} \frac{(\omega_{it} - \omega_{i\tau})^2}{\omega_{i\tau}} \tag{9}$$

The values of the Kullback-Leibler divergence and $2\chi^2$-divergence take non-negative values and are infinite, making it difficult to interpret the results. Both measures react unevenly to the differences that exist between the various components of the compared structures. For both measures, most variation interval is reserved for the structures close to full divergence (Wędrowska, 2012). For Kullback-Leibler divergence and 2χ-divergence inequality holds true (Taneja, Kumar, 2004):

$$0 \le D_{KL}(S_t^n, S_\tau^n) \le \chi^2(S_t^n, S_\tau^n) \tag{10}$$

In the analysis of the changes in income distribution by deciles, the measure of monotonic structural was used additionally (Czempas, Palica, 2007):

$$d_p = \frac{\sum_{i=1}^{n} \left| \omega_{ip} - \omega_{i0} \right|}{\sum_{t=1}^{m} \sum_{i=1}^{n} \left| \omega_{it} - \omega_{i(t-1)} \right|} \tag{11}$$

where ω_{ip} are the components of vector S_p^n of any observed period $p = 1, ..., m$, ω_{i0} – the components of vector S_0^n of the period taken as the base, ω_{it} and $\omega_{i(t-1)}$ – components of the vectors S_t^n and S_{t-1}^n from the time period *t* and the one immediately preceding it, respectively.

Measure (11) is regulated in the interval [0, 1] and takes the value of zero for the structures with identical components. It takes the value of one when the sequences of *i*-th shares of those constituent structures give monotonic sequences in *n* analysed periods.

3.2. The outcome of the research

The next stage of the research, following the analysis of the income inequality, is to assess the degree of changes in the household income distribution by decile groups in the years 2005–2013. To achieve this research objective, Csiszár's non-symmetric divergence measures were applied. Their values are given in Table 3. While examining the degree of divergence between the pairs of structures at two moments, two cases related to the choice of the distribution base were considered. In the first case, the base distribution is the income distribution by deciles from the year immediately preceding the year of the survey $\left(S_{(t-1)}^{10}\right)$, in the second – from 2005 $\left(S_{2005}^{10}\right)$.

Minor changes in the household income inequality were reflected in the low intensity of the transformation of the distribution of income by deciles. The values of each measure of divergence are different, which is obvious due to their different designs. However, the small values of each of the measures quantifying the degree of divergence between the income distribution in year *t* and $(t - 1)$ indicate that in the period from 2005 to 2013 only slight changes occurred in the structure of the income distribution for the total of households. The analysis of

Table 3. The values of Csiszár's divergence measures for the distribution of household income by deciles in Poland for the years 2005–2013

Year	K-divergence		KL-divergence		χ^2-divergence		d_p
	t/t–1	t/2005	t/t–1	t/2005	t/t–1	t/2005	
2006	0.000379	0.000379	0.001563	0.001563	0.002237	0.002237	0.3192
2007	0.000099	0.000770	0.000399	0.003214	0.000583	0.004688	0.5000
2008	0.000021	0.000877	0.000085	0.003715	0.000109	0.005475	0.5106
2009	0.000036	0.001168	0.000143	0.004936	0.000189	0.007292	0.6277
2010	0.000028	0.001373	0.000112	0.005732	0.000184	0.008267	0.6915
2011	0.000012	0.001300	0.000046	0.005387	0.000064	0.007686	0.6915
2012	0.000010	0.001461	0.000041	0.006090	0.000050	0.008845	0.7234
2013	0.000012	0.001636	0.000048	0.006783	0.000067	0.009643	0.7660

Source: the authors' own calculations.

the sequence of divergence measures allows to identify the moments of the occurrence of the above-average changes in the distribution, referred to as a caesura in phenomena periodization. The values of each of the applied measures indicate that the most significant changes in the income distribution by deciles occurred in 2006 as compared to 2005, then in 2007 as compared to 2006, and in 2009 as compared to 2008. In the subsequent years, the transformations occurring in the distribution of income were less significant.

In the assessment of the intensity of the changes in the income distribution by deciles, a measure of monotonic structural changes was also used. The sequence of calculated values d_p based on the income distribution by decile groups in the years 2005–2013 allows to specify the stability degree of the evolution direction of all the shares of the considered distributions. The measure values are rising monotonically from the value of 0.31915 (in 2006) to 0.76596 (in 2013). The approximation of the discussed measure to a unity indicates the evolution stability of the individual elements of the income structure under scrutiny. This results from a systematic increase in the share of the income attributable to the households of the first seven decile groups, with a simultaneous fall in the share of the household income of the three highest decile groups.

Conclusions

The article presents the outcome of the research conducted on the income inequality of households in Poland in the years 2005–2013. Some changes occurring over time in income distribution by deciles were also observed. On the basis of the analyses, the inhibition of the growth of income inequality could be observed, which was noted in the nineties of the last century. In addition, it was proposed to use Csiszár's divergence measures to assess the intensity

of changes in income distribution divergence in subsequent years. In particular, the following conclusions on the degree of the transformations of the distribution of household income by deciles were formulated:

– In the analysed period, there occurred slight but steadily declining divergences retaining a permanent direction of transformations in the household income distribution by deciles.
– There is a trend exhibiting a decrease in the divergence in the distribution structure of the total household income.

References

Ali, S.M., Silvey, S.D. (1966). A general class of coefficients of divergence one distribution from another. *Journal of the Royal Statistical Society*. *Series B*, *28* (1), 131–142.

Anwar, M., Hussain, S., Pečarić, J. (2009). Some inequalities for Csiszár-divergence measures. *Int. Journal of Math*. *Analysis*, *3* (26), 1295–1304.

Bhatia, P.K., Singh, S. (2013). On a new Csiszar's f-divergence measure. *Cybernetics and Information Technologies*, *13* (2), 43–57.

Bochenek, M., Mikołajewska, M. (2013). Dysproporcje dochodowe w Polsce przed i po wybuchu kryzysu. *Nierówności Społeczne a Wzrost Gospodarczy*, *30*, 162–171.

Brzeziński, W. (2015). *Nierówności większe, niż pokazują ankiety. Trzeba patrzeć w PIT-y*. Available at: http://wyborcza.biz/biznes/1,100897,18404115,nierownosci-wieksze-niz-pokazuja-ankiety-trzeba-patrzec-w-pity.html (access: 26.08.2015).

Cichomski, B. (2001). Płaszczyzny nierówności dochodów i wynagrodzeń w Polsce w latach dziewięćdziesiątych. In: B. Cichomski, W. Kozek, P. Morawski, W. Morawski (eds.), *Sprawiedliwość społeczna. Polska lat dziewięćdziesiątych* (pp. 56–58). Warszawa: Wydawnictwo Naukowe „Scholar".

Csiszár, I. (1967). Information-type measures of difference of probability distributions and indirect observation. *Studia Scientiarum Mathematicarum Hungarica*, *2*, 229–318.

Czempas, J., Palica, Z. (2007). Koncentracja aktywów netto otwartych funduszy inwestycyjnych. *Wiadomości Statystyczne*, *5*, 23–31.

Dragomir, S.S., Gluščević, V., Pearce, C.E.M. (2001). Csiszár f-divergence, Ostrowski's inequality and mutual information. *Nonlinear Analysis*, *47*, 2375–2386.

GUS (2014). *Dochody i warunki życia ludności Polski (raport z badania EU-SILC 2012)*. *Informacje i opracowania statystyczne*. Warszawa: GUS.

Jagielski, M., Duczmal, R., Kutner, R. (2015). Income distribution in the European Union versus in the United States. *Physica A: Statistical Mechanics and its Applications*, *433*, 36–41.

Kasprzyk, B. Wojnar, J. (2010). Nierówności dochodowe gospodarstw domowych w regionie Podkarpackim. In: M.G. Woźniak (ed.), *Nierówności społeczne a wzrost gospodarczy. Spójność społeczno-ekonomiczna a modernizacja gospodarki* (pp. 476–485). Rzeszów: Wydawnictwo Uniwersytetu Rzeszowskiego.

Lin, J. (1991). Divergence measures based on the Shannon entropy. *IEEE Transactions on Information Theory*, *37*, 145–151.

Menéndez, M.L., Pardo, J.A., Pardo, L., Zografos, K. (2003). On tests of homogeneity based on minimum ϕ-divergence estimator with constraints. *Computational Statistics and Data Analysis*, *43*, 215–234.

Panek, T. (2014). *Miary nierówności*. Collegium of Economic Analysis at The Warsaw School of Economics. Available at: http://kolegia.sgh.waw.pl/pl/KAE/struktura/ISiD/struktura/ZSS/zaklad/sklad/Documents/Statystyka_Tomasz_Panek/Statystyczne_metody_analizy_ubostwa/Miary_nierownosci1.pdf (access 13.08.2015).

Szopa, B. (2006). Zróżnicowanie dochodów w Polsce. *Zeszyty Naukowe AE w Krakowie*, *744*, 105–124.

Szymańska, A. (2013). Zróżnicowanie dochodów a efektywność rynku pracy na przykładzie wybranych krajów OECD, *Optimum. Studia Ekonomiczne*, *6* (66), 32–45.

Taneja, I.J. (1995). New developments in generalized information measures. In: P.W. Hawkes (ed.), *Advances in Imaging and Electron Physics*, *91*, 37–135.

Taneja, I.J. (2008). On Mean Divergence Measures. In: N.S. Barnett, S.S. Dragomir (eds.), *Advances in Inequalities from Probability Theory & Statistics* (pp. 169–186). Nova Science Publishers.

Taneja, I.J. (2005a). On Symmetric and Nonsymmetric Divergence Measures and Their Generalizations. In: P.W. Hawkes (ed.) *Advances in Imaging and Electron Physics*, *138*, 178–248.

Taneja, I.J. (2005b). Refinement Inequalities Among Symmetric Divergence Measures. *The Australian Journal of Mathematical Analysis and Applications*, *2* (1), 1–23.

Taneja, I.J., Kumar, P. (2004). Relative information of types, Csiszár's f-Divergence, and information inequalities. *Information Sciences*, *166*, 105–125.

Ulman, P., Wałęga, A. (2006). Nierówności dochodowe w Polsce i ich dekompozycja. *Zeszyty Naukowe Polskiego Towarzystwa Naukowego*, *4*, 77–96.

Wędrowska, E. (2012). *Miary entropii i dywergencji w analizie struktur*. Olsztyn: Wydawnictwo Uniwersytetu Warmińsko-Mazurskiego.

Zwiech, P. (2013). Nierówności dochodowe w państwach europejskich – analiza przy wykorzystaniu współczynnika Giniego. *"Studia Ekonomiczne" Zeszyty Naukowe Wydziałowe Uniwersytetu Ekonomicznego w Katowicach*, 89–101.