INTUITIONISTIC ASSESSMENT OF BEHAVIOURAL PRESENT VALUE[*]

Prof. Krzysztof Piasecki

Poznań University of Economics
Al. Niepodległości 10, 60-875 Poznań
e-mail: k.piasecki@ue.poznan.pl

Abstract

The article discussesd the impact of chosen behavioural factors on the imprecision of present value assessment. The formal model of behavioural present value is offered as a result of this discussion. The behavioural present value is described here as an intuitionistic fuzzy set. The significance of the replacement of a fuzzy set by an intuitionistic fuzzy set is proved.

Introduction

The considerations set out in this article had their genesis in the acceptance of a view that – without the interest theory – the present value of future cash flow may be imprecise. The natural consequence of this approach is the assessment of the present value (PV) by means of fuzzy numbers.

Ward defines fuzzy PV as discounted fuzzy cash flow[1]. The axiomatic definition of PV is generalized for the fuzzy case by Calzi[2]. The Ward's definition is generalized to the case of fuzzy duration by Greenhut at al.[3] Sheen generalizes the Ward's definition to the case of fuzzy interest rate[4]. Buckley, Gutierrez, Kuchta and Lesage discuss some problems connected with the application of fuzzy arithmetic for calculating fuzzy PV[5]. Huang generalizes the Ward's definition for the case when future cash flow is given as a fuzzy random variable[6]. A more general definition of fuzzy PV is proposed by Tsao who assumes that future cash flow is a fuzzy probabilistic set. All these authors present the PV as the discount of imprecisely estimated future cash flow value.

A different approach is presented Piasecki[7]where imprecisely estimated PV was based on the current market price of a financial asset. The lack of precision in the estimation of PV was justified there by behavioural premises. Hence, described there PV is called a behavioural present value. The defined there behavioural present value has been used to explain the paradox of maintaining the market equilibrium on a highly efficient financial market.

On the other hand, Atanassov defined an intuitionistic fuzzy set as an extension of the fuzzy set concept[8]. This extension will be described in the next chapter. We can find an example which shows a significant usefulness of intuitionistic fuzzy sets to the description of financial problems[9]. Thus the description of behavioural present value by means of the intuitionistic fuzzy sets will be proposed in this work.

1. Intuitionistic fuzzy sets – basic concepts

Objects of any cognitive-application activities are elements of the space $\mathbb{X}$. The basic tool for imprecise classification of these elements is the notion of the fuzzy set $\tilde{A}$ described by its membership function $\mu_A \colon \mathbb{X} \to [0; 1]$ as the set of ordered pairs:

$$\tilde{A} = \{(x, \mu_A(x)); x \in \mathbb{X}\} \tag{1}$$

In multi-valued logic terms, the value $\mu_A(x)$ of a membership function is interpreted as the logical value of the sentence $x \in \tilde{A}$.

Atanassov have defined the intuitionistic fuzzy set (IFS) A^* as the set of ordered triples[10]:

$$A^* = \{(x, \mu_A(x), v_A(x)); x \in \mathbb{X}\} \tag{2}$$

where the nonmembership function $v_A: \mathbb{X} \to [0; 1]$ identically fulfills the condition:

$$v_A(x) \leq 1 - \mu_A(x) \tag{3}$$

In multi-valued logic terms, the value $v_A(x)$ of nonmembership function is interpreted as the logical value of the sentence $x \notin A^*$.

We define the hesitation function $\pi_A: \mathbb{X} \to [0; 1]$ determined by identity:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \tag{4}$$

The value $\pi_A(x)$ indicates the degree of our hesitation in the assessment of the relationship between the element $x \in \mathbb{X}$ and IFS A^*. For this reason, the hesitation function π_A can be interpreted as a picture of knightian uncertainty[11].

Let us consider the fuzzy set $\tilde{B}$ described by its membership function $\pi_B: \mathbb{X} \to [0; 1]$. This fuzzy subset can be identified with IFS represented by the set of ordered triples:

$$B^* = \{(x, \mu_B(x), 1 - \mu_B(x)); x \in \mathbb{X}\} \tag{5}$$

The hesitation function of the above IFS identically fulfills the condition:

$$\pi_B(x) = 0 \tag{6}$$

It implies that the fuzzy sets application to create real object models is the implicit acceptance of a strong assumption proclaiming that we are always able to decide on the fulfillment of postulated all elementary state requirements. However, as we know from everyday observations, usually it is not, and our settlements are burdened with a noticeable hesitation margin. This means that the extension of the fuzzy sets class to the IFS class significantly extends the capabilities of a honest description of imprecision. The way to obtain this extension is quite easy as the IFS class currently has a rich and comprehensive theoretical study[12].

2. Behavioural present value

Let us consider any security understood as the right to receive future income. It is shown that PV is the utility function defining arrangement of the cash flows set. In this situation, any cash flow PV depends both on objective and subjective conditions[13]. The normative finance theory suggests that PV of analyzed security should be equal to the market price $\check{C}$. On the other hand, the substantially justified equilibrium price C_0 can influence the PV deviation from the observed market price $\check{C}$. This deviation is in large part dependent on an investor's susceptibility to internal and external behavioural factors. In their essence, behavioural environmental conditions are defined imprecisely. For this reason, PV deviation from the market price is at risk of imprecision.

Subjective assessment of PV is ambiguous. Each of the considered valuation alternatives will be called a potential present value (PPV). The set of all PPVs will be called a behavioural present value (BPV). The dependence of PV on subjective financial factors leads to the situation that each investor may appoint their own version of the BPV. Thus, we will conduct all further considerations for any fixed investor.

3. Interval representation of behavioural present value

The starting point for all further considerations is the presentation of the BPV as an interval. Our consideration of the BPV we begin by discussing the financial equilibrium case when the market price $\check{C}$ coincides with the equilibrium price C_0:

$$\check{C} = C_0 \qquad (7)$$

This equilibrium is momentary. This fact requires that PPV value is specified as a number approximating the market price. The assumed scope of PPV variability is characterized by a specific investor's susceptibility to behavioural factors. Therefore each investor determines the following values:

- $C_{min\ PPV}$ lower scope assumed under the conditions of financial equilibrium;
- C_{max} PPV upper scope assumed under the conditions of financial equilibrium.

In the case of the financial equilibrium an investor must take into account the possibility of declines and increases in quotations. In this situation, the scope of PPV variability satisfies the condition:

$$C_{min} < C_0 < C_{max} \qquad (8)$$

The numerical interval $[C_{min}, C_{max}]$ is the BPV image for the case of the financial equilibrium.

Further considerations on BPV we lead for the case where the quoted market price $\check{C}$ is arbitrary. It is obvious that BPV should be dependent on the deviation:

$$\Delta C = \check{C} - C_0 \tag{9}$$

of the market price from the equilibrium price. Then each investor determines the following values:

– $\check{C}_{min}$ PPV lower scope assumed for the case of the market price $\check{C}$,
– $\check{C}_{max}$ PPV upper scope assumed for the case of the market price $\check{C}$.

Both of these values are dependent on the number $\alpha \in [0; 1]$ which determines the degree of an investor's susceptibility to changes. This degree value informs us about intensity of the impact in which the deviation ΔC influences an investor's beliefs. This means that the value $1 - \alpha$ describes the degree of the cognitive conservatism phenomenon impact[14]. This phenomenon is taken into account in many behavioural models of the financial market. The discussion on this subject can be found, for example, in Barberis at.al.[15] The degree of susceptibility is an individual investor's characteristic having a behavioural base.

An investor determines the PPV lower scope, as the weighted average of the lower scope C_{min} and the corrected lower scope $C_{min} + \Delta C$. The weight of the corrected lower scope is equal of the degree of an investor's susceptibility to changes. In determining the PPV lower scope an investor must take into account the fact that this scope is always less or equal than the current market price. We have here:

$$\check{C}_{min} = min\ \{\alpha(C_{min} + \Delta C) + (1 - \alpha)\ C_{min}, \check{C}\} = min\{C_{min} + \alpha\Delta C, C_0 + \Delta C\} \tag{10}$$

An investor determines the PPV upper scope as the weighted average of the upper scope C_{max} and the corrected upper scope $C_{max} + \Delta C$. The weight of the corrected upper scope is equal to the degree of an investor's susceptibility to changes. In determining the PPV upper scope an investor must take into account the fact that this scope is always greater than or equal to the current market price. We have here:

$$\check{C}_{max} = max\ \{\alpha(C_{max} + \Delta C) + (1 - \alpha)\ C_{max}, \check{C}\} = max\{C_{max} + \alpha\Delta C, C_0 + \Delta C\} \tag{11}$$

It is easy to note that in the case:

$$\Delta C \leq (C_{min} + C_0)/(1 - \alpha) \tag{12}$$

the lower scope PPV is equal to the market price. It means that in the case of a large surplus of the equilibrium price over the market price, the BPV model excludes the possibility of quotation downtrend. Also in the case:

$$\Delta C \geq (C_{max} + C_0)/(1 - \alpha) \tag{13}$$

of a large surplus of the market price over the equilibrium price, the BPV model excludes the possibility of the rise in the quotation. Then the upper scope PPV is equal to the market price. Hence we conclude that only with considerable deviations of the market price from the equilibrium price investment decisions shall be made solely basing on rational reasons. The scope of the behavioural reasons impact is determined by the following condition:

$$(C_{min} + C_0)/(1 - \alpha) < \Delta C < (C_{max} + C_0)/(1 - \alpha) \tag{14}$$

Finally, for each investor we can determine a specific scope of PPV variability:

$$\mathbb{Z}(\Delta C) = \begin{cases} \left[\check{C}, C_{max} + \alpha \cdot \Delta C \right] & \text{for (12)} \\ \left[C_{min} + \alpha \cdot \Delta C, C_{max} + \alpha \cdot \Delta C \right] & \text{for (14)} \\ \left[C_{min} + \alpha \cdot \Delta C, \check{C} \right] & \text{for (13)} \end{cases} \tag{15}$$

forming the interval representation of BPV. In this way, we describe the impact of market conditions on the investor's beliefs. If the fixed PPV belongs to the interval representation of BPF, then it is called the acceptable PPV.

4. Fuzzy representation of behavioural present value

The interval image of BPV treats all the acceptable PPVs in the same way. On the other hand, we can suppose that an investor more absolutely accepts PPV closer to the price. It implies that individual PPVs differ in their degrees of acceptance. This means that the interval model of BPV describes the complexity of the behavioural effects in an insufficient way. This makes it necessary to build a BPV model taking into account the variability of an individual PPV importance. This leads directly to building a fuzzy image of BPV. Fuzzy representation of BPV boils down to determining its membership function by assigning degree of acceptance to each PPV.

We keep our further discussion for a given value ΔC of the market price deviation from the equilibrium price. Then, the interval $\mathbb{Z}(\Delta C) = [\check{C}_{min}; \check{C}_{max}]$ of the PPV variability is determined explicitly. To unify further considerations, this interval will be standardized. We use here the following transformation:

$$\beta = \begin{cases} \frac{x-\check{C}}{\check{C}-\check{C}_{min}} & x \in [\check{C}_{min}, \check{C}] \neq \{\check{C}\} \\ \frac{x-\check{C}}{\check{C}_{max}-\check{C}} & x \in [\check{C}, \check{C}_{max}] \neq \{\check{C}\} \end{cases} \tag{16}$$

In this way we transform the interval $[\check{C}_{min}; \check{C}_{max}]$ into the standardized interval $\mathbb{I}(\Delta C)$ given by the identityŁ

$$\mathbb{I}(\Delta C) = \begin{cases} [0; 1] & \text{for (12)} \\ [-1; 1] & \text{for (14)} \\ [-1; 0] & \text{for (13)} \end{cases} \tag{17}$$

The degree γ of PPV similarity to the market price we define by the identity:

$$\gamma = 1 - |\beta| \tag{18}$$

The defined above similarity degree of the similarity γ simultaneously determines the relative distance between PPV and the limit of the variability scope.

We define the standardized fuzzy BPV model as a fuzzy subset given by its membership function $\kappa(\cdot|\Delta C): \mathbb{I}(\Delta C) \to [0; 1]$. This function determines the acceptance degree of an individual PPV. For this reason it is called the acceptance distribution. We here assume that:

- the market price $\check{C}$ is a fully acceptable PPV,
- the increase in the degree of PPV similarity to the market price $\check{C}$ does not decrease the degree of PPV acceptance.

For this reason, any BPV should be a fuzzy number determined by its membership function $\kappa(\cdot|\Delta C): \mathbb{I}(\Delta C) \to [0; 1]$ fulfilling the conditions[16]:

$$\kappa(0|\Delta C) = 1 \tag{19}$$

$$\forall x, y, z \in \mathbb{I}(\Delta C): \ x \leq y \leq z \Rightarrow \kappa(y|\check{C}) \geq min\{\kappa(x|\check{C}), \kappa(z|\check{C})\} \tag{20}$$

We will assume further that for the case of the financial equilibrium (7) a standardized fuzzy BPV model will be defined as a triangular number supported on the interval $\mathbb{I}(0)$. The membership function of this number is given by the identity:

$$\kappa(\beta|0) = 1 - |\beta| \tag{21}$$

This function describes the balanced acceptance distribution which will be a reference point to determining the acceptance distribution for the case of the financial disequilibrium ($\Delta C \neq 0$).

The second reference point for determining any acceptance distribution will be rational forecast changes in quotation. It is known, that:

- if the disequilibrium condition ($\Delta C < 0$) is fulfilled, then rational premises dictate to expect a rise in the quotation,
- if the disequilibrium condition ($\Delta C > 0$) is fulfilled, then rational premises dictate to expect a quotation downtrend,
- if the balance condition (7) is fulfilled, then rational premises dictate to expect a quotation downtrend or a rise in the quotation.

Thus, the function $\Theta(\cdot|\Delta C): \mathbb{I}(\Delta C) \to \{0; 1\}$ describing the rational forecast is given by the identity:

- for $\Delta C < 0$

$$\Theta(\beta | \Delta C) = \begin{cases} 0 & \beta < 0 \\ 1 & \beta \geq 0 \end{cases} \tag{22}$$

- for $\Delta C > 0$

$$\Theta(\beta | \Delta C) = \begin{cases} 1 & \beta \leq 0 \\ 0 & \beta > 0 \end{cases} \tag{23}$$

- for $\Delta C = 0$

$$\Theta(\beta|0) = 1 \tag{24}$$

For any deviation ΔC an investor assesses the acceptance degree as a weighted average of the rational forecast and the balanced acceptance distribution. The BPV model was built under the assumption that the weight of the rational forecast is directly proportional to the product $\gamma \cdot |\Delta C|$[17]. The acceptance of such a weight caused that the importance of the rational forecast was dependent on the currency used for the security assessment. With high market prices the importance of balanced acceptance distribution had depreciated. All this caused that an approved valuation method could have a significant impact on the form of the standardized BPV model.

In this situation, it is right to suggest that the importance of the rational forecast increases with the relative distance:

$$\delta C = |\Delta C|/\check{C} \tag{25}$$

of the market price from the equilibrium price and with the increasing distance γ between PPV and the limit of variability scope. Without loss of generality we can assume that the weight

of the rational forecast is directly proportional to the product $\gamma \cdot |\delta C|$. Then the acceptance distribution is described by the identity:

$$\kappa(\beta \mid \Delta C) = \frac{1}{1+\gamma \cdot \delta C} \cdot \kappa(\beta|0) + \frac{\gamma \cdot \delta C}{1+\gamma \cdot \delta C} \cdot \Theta(\beta \mid \Delta C) = \frac{(1-|\beta|) \cdot (1+\delta C \cdot \Theta(\beta \mid \Delta C))}{1+(1-|\beta|) \cdot \delta C} \qquad (26)$$

The above membership function describes the standardized BPV model. Using the inverse transformation (16) we determine now a fuzzy assessment of BPV. In general, this assessment is equal to a fuzzy number given by the membership function $\mu(\cdot|\Delta C)$: $\mathbb{Z}(\Delta C) \to [0; 1]$ defined as follows:

$$\mu(x \mid \Delta C) = \begin{cases} \kappa\left(\frac{x-\check{C}}{\check{C}-\check{C}_{min}} \Big| \Delta C\right) & x \in \left[\check{C}_{min}; \check{C}\right] \neq \{\check{C}\} \\ \kappa\left(\frac{x-\check{C}}{\check{C}_{max}-\check{C}} \Big| \Delta C\right) & x \in \left[\check{C}; \check{C}_{max}\right] \neq \{\check{C}\} \end{cases} \qquad (27)$$

In the end, we obtain here:

– for $\Delta C \leq 0$

$$\mu(x \mid \Delta C) = \begin{cases} \frac{x-\check{C}_{min}}{\check{C}-\check{C}_{min}+(x-\check{C}_{min}) \cdot \delta C} & x \in \left[\check{C}_{min}; \check{C}\right] \neq \{\check{C}\} \\ \frac{(\check{C}_{max}-x) \cdot (1+\delta C)}{\check{C}_{max}-\check{C}+(\check{C}_{max}-x) \cdot \delta C} & x \in \left[\check{C}; \check{C}_{max}\right] \neq \{\check{C}\} \end{cases} \qquad (28)$$

– for $\Delta C > 0$

$$\mu(x \mid \Delta C) = \begin{cases} \frac{(x-\check{C}_{min}) \cdot (1+\delta C)}{\check{C}-\check{C}_{min}+(x-\check{C}_{min}) \cdot \delta C} & x \in \left[\check{C}_{min}; \check{C}\right] \neq \{\check{C}\} \\ \frac{\check{C}_{max}-x}{\check{C}_{max}-\check{C}+(\check{C}_{max}-x) \cdot \delta C} & x \in \left[\check{C}; \check{C}_{max}\right] \neq \{\check{C}\} \end{cases} \qquad (29)$$

The fuzzy assessment of BPV was applied in (Piasecki)[18].

5. Intuitionistic representation of behavioural present value

The above standardized fuzzy BPV model was built under the assumption that rational forecast $\Theta(\cdot|\Delta C)$ of market price changes would be accurate. But is that so, in practice, that rational prediction estimates are accurate? In this situation, we get a complete picture of BPV after the consideration of alternative assessment of BPV obtained by means of the negation of rational forecast changes in market prices. This negation is described by its membership function $\Xi(\cdot|\Delta C)$: $\mathbb{I}(\Delta C) \to \{0; 1\}$ given by the identity:

$$\Xi(\beta|\Delta C) = 1 - \Theta(\beta|\Delta C) \qquad (30)$$

Other premises determining the PPV acceptance distribution remain unchanged. In this situation, the standardized acceptance distribution $\varphi(\cdot|\Delta C): \mathbb{I}(\Delta C) \to [0; 1]$ is described by the identity:

$$\varphi(\beta|\Delta C) = \frac{1}{1+\gamma\cdot\delta C}\cdot\kappa(\beta|0) + \frac{\gamma\cdot\delta C}{1+\gamma\cdot\delta C}\cdot\Xi(\beta|\Delta C) = \frac{(1-|\beta|)\cdot(1+\delta C\cdot\Xi(\beta|\Delta C))}{1+(1-|\beta|)\cdot\delta C} \tag{31}$$

Thus obtained acceptance distribution is a membership function of the standardized alternative BPV model. The complement of the standardized BPV model is defined as the product of the standardized alternative BPV model and the negation of the standardized BPV model. This complement of BPV is uniquely described by the membership function $\lambda(\cdot|\Delta C): \mathbb{I}(\Delta C) \to [0; 1]$ given by the identity:

$$\lambda(\beta|\Delta C) = min\{\varphi(\beta|\Delta C),\ 1-\kappa(\beta|\Delta C)\} \tag{32}$$

It is easy to see that the membership functions $\kappa(\cdot|\Delta C)$ and $\lambda(\cdot|\Delta C)$ fulfill the condition (3). In this situation, the second of these functions can be the nonmembership function of IFS:

$$(BPV)^* = \{(\beta, \kappa(\beta|\Delta C), \lambda(\beta|\Delta C));\ \beta \in \mathbb{I}(\Delta C)\} \tag{33}$$

The above set $(BPV)^*$ is called a standardized intuitionistic BPV model. The replacement of a fuzzy model by an intuitionistic one is justified only when it contributes additional information to the description of BPV. This will happen only when IFS:

$$B^* = \{(\beta, \kappa(\beta|\Delta C),\ 1 - \kappa(\beta|\Delta C));\ \beta \in \mathbb{I}(\Delta C)\} \tag{34}$$

representing the standardized fuzzy BPV model will differ from $(BPV)^*$. This condition is fulfilled only when the inequality:

$$\varphi(\beta|\Delta C) < 1-\kappa(\beta|\Delta C) \tag{35}$$

has a solution beta $\beta \in \mathbb{I}(\Delta C)$. From (26) and (30) we obtain:

$$1 - \kappa(\beta|\Delta C) = \frac{1}{1+\gamma\cdot\delta C}\cdot\left(1 - \kappa(\beta|0)\right) + \frac{\gamma\cdot\delta C}{1+\gamma\cdot\delta C}\cdot\left(1 - \Theta(\beta|\Delta C)\right) =$$
$$= \frac{1}{1+\gamma\cdot\delta C}\cdot\left(1 - \kappa(\beta|0)\right) + \frac{\gamma\cdot\delta C}{1+\gamma\cdot\delta C}\cdot\Xi(\beta|\Delta C).$$

Together with (31) it shows that the inequality (35) is equivalent to the inequality:

$$\frac{1}{1 + \gamma \cdot \delta C}\cdot\kappa(\beta|0) < \frac{1}{1 + \gamma \cdot \delta C}\cdot\left(1 - \kappa(\beta|0)\right),$$

which finally leads to:

$$|\beta| > \frac{1}{2}.$$

Hence we conclude that the standardized intuitionistic BPV model does not represent any fuzzy set, and the more standardized fuzzy BPV model. The replacement of a fuzzy model by an intuitionistic one is justified.

The hesitation function $\varpi(\cdot|\Delta C)$: $\mathbb{I}(\Delta C) \to [0; 1]$ associated with the standardized intuitionistic BPV model takes the form here:

$$\varpi(\beta|\Delta C) = \begin{cases} 0 & |\beta| \le \frac{1}{2} \\ 1 - \kappa(\beta|\Delta C) - \varphi(\beta|\Delta C) & |\beta| > \frac{1}{2} \end{cases} = \begin{cases} 0 & |\beta| \le \frac{1}{2} \\ \frac{2\cdot|\beta|-1}{1+(1-|\beta|)\cdot\delta C} & |\beta| > \frac{1}{2} \end{cases} \tag{36}$$

The above hesitation function shows that the standardized intuitionistic BPV model is also under the knightian uncertainty[19]. The characteristic of this uncertainty can enrich the multidimensional image of risk endangering the considered security[20]. Using the inverse transformation (16) we determine now the intuitionistic assessment of BPV. In general case, it is the IFS:

$$(BPV)^* = \{(\beta, \mu(\beta|\Delta C), v(\beta|\Delta C)); \beta \in \mathbb{Z}(\Delta C)\} \tag{37}$$

where the membership function $\mu(\cdot|\Delta C)$: $\mathbb{Z}(\Delta C) \to [0; 1]$ is determined by (28) and (29). Defined by IFS (37) the hesitation function $\pi(\cdot|\Delta C)$: $\mathbb{Z}(\Delta C) \to [0; 1]$ is given as follows:

$$\pi(x|\Delta C) = \begin{cases} \frac{\check{C}+\check{C}_{min}-2\cdot x}{\check{C}-\check{C}_{min}+(x-\check{C}_{min})\cdot\delta C} & x \in \left[\check{C}_{min}, \frac{\check{C}_{min}+\check{C}}{2}\right[\\ 0 & x \in \left[\frac{\check{C}_{min}+\check{C}}{2}, \frac{\check{C}+\check{C}_{max}}{2}\right] \\ \frac{2\cdot x-\check{C}_{max}-\check{C}}{\check{C}_{max}-\check{C}+(\check{C}_{max}-x)\cdot\delta C} & x \in \left]\frac{\check{C}+\check{C}_{max}}{2}, \check{C}_{max}\right] \end{cases} \tag{38}$$

We determine the nonmembership function $v(\cdot|\Delta C)$: $\mathbb{Z}(\Delta C) \to [0; 1]$ by means of the identity:

$$v(\beta|\Delta C) = 1 - \mu(\beta|\Delta C) - \pi(x|\Delta C) \tag{39}$$

Conclusions

The paper presents a significant generalization of fuzzy assessments of BPV[21]. Fuzzy description of any PV leads to presenting the return rate as a fuzzy probabilistic set[22]. That observation allowed us to determine the three-dimensional image of risk endangering the security. That risk was composed of the quantified uncertainty risk, the ambiguity risk and

the indistinctness risk. That model allowed the formulation of new methods of securities management.

The presented here generalization of the BPV description makes it advisable to consider an intuitive description of PV. The consequences of such generalization are easy to foresee. Then the return rate will be presented as an intuitionistic probabilistic set. The return risk will be composed of the knightian risk, the quantified uncertainty risk, the ambiguity risk and the indistinctness risk. This perception of risk endangering the security will lead to the formulation of new approaches to risk management. All of these assumptions must be confirmed by carrying out solid theoretical and empirical research. Thus, the proposed extension indicates a new direction for theoretical research which is promising for future practical applications.

Notes

[1] Ward (1985).

[2] Calzi (1990).

[3] Greenhut at al. (1995).

[4] Sheen (2005).

[5] Buckley (1987, 1992); Gutierrez (1989); Kuchta (2000); Lesage (2001).

[6] Huang (2007).

[7] Piasecki (2011a, b).

[8] Atanassov (1983).

[9] Piasecki, Ziomek (2011).

[10] Atanassov (1983).

[11] Knight (1921).

[12] For example Atanassov (1999).

[13] (Piasecki, 2012 a, b).

[14] Cognitive conservatism has been described by W. Edwards (1968).

[15] Barberis at.al. (1998).

[16] Dubois, Prade (1979).

[17] Piasecki (2011a).

[18] Piasecki (2011a, b, c).

[19] Knight (1921).

[20] Piasecki (2011b, c).

[21] Piasecki (2011 a, b).

[22] Piasecki (2011b, c).

References

Atanassov, K. & Stoeva, S. (1983). *Intuitionistic fuzzy sets*. In: J. Albrycht, H. Wiśniewski (Eds.). *Proceedings of Polish Symposium on Interval and Fuzzy Mathematics*, (pp. 23–26) Poznań.

Atanassov, K. (1999). *Intuitionistic Fuzzy Sets*. Heidelberg: Springer-Verlag.

Barberis, N., Shleifer, A. & Vishny R. (1998). *A* model of investor sentiment. *Journal of Financial Economics*, 49, pp. 307–345.

Buckley, I.J. (1987). The fuzzy mathematics of finance. *Fuzzy Sets and Systems*, 21, pp. 257–273.

Calzi, M.L. (1990). Towards a general setting for the fuzzy mathematics of finance. *Fuzzy Sets and Systems*, 35, pp. 265–280.

Dubois, J. & Prade, H. (1979). Fuzzy real algebra: some results. *Fuzzy Sets and Systems*, 2, pp. 327–348.

Edwards, W. (1968). *Conservatism in human information processing*. In: B. Klienmutz (Ed.). *Formal representation of human judgment*, (pp 17–52), New York: Wiley.

Greenhut, J.G., Norman, G. & Temponi, C.T. (1995). *Towards a fuzzy theory of oligopolistic competition,* IEEE Proceedings of ISUMA-NAFIPS (pp. 286–291), College Park, IEEE.

Gutierrez, I. (1989). Fuzzy numbers and Net Present Value. *Scandinavian Journal of Management*, 5 (2), pp. 149–159.

Huang, X. (2007). Two new models for portfolio selection with stochastic returns taking fuzzy information. *European Journal of Operational Research*, 180 (1), pp. 396–405.

Knight, F.H. (1921). Risk, Uncertainty, and Profit. Boston, MA, Hart, Schaffner & Marx; Houghton Mifflin Company.

Kuchta, D. (2000). Fuzzy capital budgeting. *Fuzzy Sets and Systems*, 111, pp. 367–385.

Lesage, C. (2001). Discounted cash-flows analysis. An interactive fuzzy arithmetic approach. *European Journal of Economic and Social Systems*, 15 (2), pp. 49–68.

Peccati, L. (1972). *Su di una caratterizzazione del principio del criterio dell'attualizzazione*. Parma: Studium Parmense.

Piasecki, K. (2011a). Behavioural Present Value. *Behavioral & Experimental Finance eJournal*, 4, Retrieved August 15, 2013 from Social Science Research Network http://ssrn.com/abstract=1729351, DOI:10.2139/ssrn.1729351.

Piasecki, K. (2011b). *Rozmyte zbiory probabilistyczne, jako narzędzie finansów behawioralnych*. Poznań: Wydawnictwo Uniwersytetu Ekonomicznego w Poznaniu.

Piasecki, K. (2011c). Effectiveness of securities with fuzzy probabilistic return. *Operations Research and Decisions*, 2, pp. 65–78.

Piasecki, K. (2012a). *Podstawy arytmetyki finansowej w świetle teorii użyteczności*. In: *Księga Jubileuszowa Profesora Edwarda Smagi*, A. Malawski, J. Tatar (Eds.), (pp. 43–59). Kraków: Wydawnictwo Uniwersytetu Ekonomicznego w Krakowie.

Piasecki, K. (2012b). Basis of Financial Arithmetic from the Viewpoint of the Utility Theory, *Operations Research and Decisions*, 3, pp. 37–53.

Piasecki, K. & Ziomek, R. (2011). Intuitionistic sets in financial market analysis – case study. *European Finance eJournal*, 1, Retrieved August 15, 2013 from Social Science Research Network http://ssrn.com/abstract=1729377. DOI: 10.2139/ssrn. 1729377.

Tsao, C.-T. (2005). Assessing the probabilistic fuzzy Net Present Value for a capital, Investment choice using fuzzy arithmetic. *Journal of Chine Institute of Industrial Engineers*, 22 (2), pp. 106–118.

Ward, T.L. (1985). *Discounted fuzzy cash flow analysis*. 1985 Fall Industrial Engineering Conference Proceedings, (pp. 476–481) Berkeley.