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CREATION AND VALUATION OF INSTRUMENTS COMPENSATING LOWER SHARE PRICES WITH THE HELP OF BLACK–SCHOLES FORMULA

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Abstract: In this paper, we present a 1-period model of the Polish financial market from the view point of KGHM, the Polish largest listed company that suffered huge declines in share prices from 125 PLN in August 2015 to 60 PLN in January 2015. Our goal is to show how KGHM might create a portfolio (with practically zero cost), which would fully compensate the abovementioned declines. The methodology presented below may be equally well employed by many other listed companies and investment funds, as well. We create here a matrix model of the Polish financial market and employ the Black–Scholes formula to valuate portfolios compensating potential declines of KGHM's shares prices. To give more insight to practitioners wishing to apply the results presented here to other listed companies, we distinguish two cases. In one of them, volatility of KGHM's share prices is 20%, and in the other case it equals 33%.

Keywords: approximate hedging, Black-Scholes formula, hedging, incomplete market, replication error, share prices.

Kody JEL: C02, C18, C54, C60.

1 Introduction

The motivation for writing this article came from observation that the Polish company KGHM, being one of the largest producers of cooper and silver in the world, suffered huge declines in share prices from 125 PLN in August 2015 to 60 PLN in January 2015. Our goal is to show how KGHM might create an extra income from a specifically created portfolio to fully compensate the abovementioned declines of its share prices with a very low cost.

We present two 1-period models of the Polish financial market (see also Zaremba, 2015) in which there are only two dates, today and tomorrow, or equivalently this week and next week, etc. All economic activity (consumption, trading and work) solely takes place "today" and "tomorrow". Despite these simplifications, such a model quite adequately represents the real financial market from the viewpoint of companies trying to manage their risks associated with uncertain share prices. It is also adequate for investment funds, which often hold shares of listed companies.

Following Cerny (2009) each vector (typically written in a column form), such as for example:

$$b = \begin{bmatrix} 65 \\ 80 \\ 95 \\ 110 \\ 125 \\ 140 \end{bmatrix}, \text{ features pay-offs resulting from a given}$$

security, for example, a share of a certain company, while matrix, say

$$P = \begin{bmatrix} 65 & 100 & 0 & 50 \\ 80 & 100 & 0 & 35 \\ 95 & 100 & 5 & 20 \\ 110 & 100 & 20 & 5 \\ 125 & 100 & 35 & 0 \\ 140 & 100 & 50 & 0 \end{bmatrix} \tag{1}$$

represents some financial market, say the Polish financial market, with b showing payments resulting from 1 share of KGHM in six different scenarios. Suppose that today is August 2015 and one share of KGHM costs 120 PLN. Since our investment horizon is a period of 6 months, matrix P shows all possible payments in six scenarios resulting from four different financial instruments in February

2016. Columns 2, 3, and 4 represent payments generated respectively by a treasury bill, a call option at strike 90 PLN, and a put option at strike 115 PLN.

2 Problem Statement

To make our model more realistic, we associate certain probabilities to six scenarios of the Polish financial market. Let them be given by the vector

$$p = \begin{bmatrix} 0.10 \\ 0.12 \\ 0.18 \\ 0.28 \\ 0.22 \\ 0.10 \end{bmatrix} \tag{2}$$

Our goal is to suggest and convince the management of KGHM as to what portfolio should they hold in order (i) to be able to compensate for all the potential declines of KGHM's share prices, and (ii) pay very little for such portfolio. We assume that KGHM's share prices may decline at least to 65 PLN from the current level of 120 PLN, as well as they may rise up at least to 140 PLN.

3 Theory

The underlying theory was first presented in (Cerny, 2009) and then employed in (Zaremba, 2015). Suppose a financial market is represented by a matrix

$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \dots & \dots & \dots & \dots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}$, and let vector

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \tag{3}$$

representing a desired financial instrument (so-called *focus* asset) compensating perfectly or almost perfectly potential declines of ABC's shares. If the market A is incomplete, that is, not all instruments (such as b) can be perfectly replicated from basis assets (columns of matrix A), then the natural question arises how one can build the *best approximate hedge* of b by means of a portfolio $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ consisting of columns of matrix A.

Such portfolio x should thus have the property that the replication error

$$\begin{aligned} &\text{SSRE} = \\ &\varepsilon_1{}^2 + \varepsilon_2{}^2 + \ldots + \varepsilon_m{}^2 = \\ &[(Ax)_1 - b_1]^2 + [(Ax)_2 - b_2]^2 + \ldots \\ &+ [(Ax)_m - b_m]^2 \end{aligned} \tag{4}$$

where $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_m) = Ax\text{-}b$,

is as small as possible. Here SSRE is the abbreviation for "sum of squared replication errors". In reality, however, some states of the world are less likely than others and consequently the company should be interested in the expected SSRE, ESSRE for short, where

$$\begin{aligned} &\text{ESSRE} = p_1\varepsilon_1{}^2 + p_2\varepsilon_2{}^2 + \ldots + p_m\varepsilon_m{}^2 = \\ &p_1[(Ax)_1 - b_1]^2 + p_2[(Ax)_2 - b_2]^2 + \ldots + \\ &p_m[(Ax)_m - b_m]^2, \end{aligned} \tag{5}$$

with $p_1 > 0$, $p_2 > 0$, … , $p_m > 0$ meaning objective probabilities of the individual states of the world; m denotes as usually the number of rows (scenarios) that may take place in our model.

Definition 1

A general *hedging (replication) problem* Ax = b consists in finding such portfolio $\hat{x}$ that $A\hat{x}$ is as close to b as possible in the sense of minimization of SSRE or ESSRE.

The following result can be found in (Cerny, 2009).

Theorem 1

Consider a general hedging problem Ax = b. Define a new matrix $\widetilde{A}$ and a new vector $\widetilde{b}$ by multiplying each row of A and b by the square root of the probability p_i for the corresponding state. The optimal hedging portfolio that minimizes ESSRE is of the form $\hat{x} = [\widetilde{A}^T\widetilde{A}]^{-1}\widetilde{A}^T\widetilde{b}$.

Its payments are given by vector:

$$A\hat{x} = A[\widetilde{A}^T\widetilde{A}]^{-1}\widetilde{A}^T\widetilde{b}$$

which replicates b in the best possible way.

4 Hedging instrument f $_1 = \begin{bmatrix} 55 \\ 40 \\ 25 \\ 10 \\ -5 \\ -20 \end{bmatrix}$

Suppose that today is September 2015. Let's ask the question: what financial instruments / portfolios will compensate all potential declines/movements in share prices of KGHM in the period of, say, nearest 6 months? One of them is for sure instrument f_1 because together with 1 share of KGHM paying

$\begin{bmatrix} 65 \\ 80 \\ 95 \\ 110 \\ 125 \\ 140 \end{bmatrix}$ in 6 states of the market,

it guarantees the risk-free income $\begin{bmatrix} 120 \\ 120 \\ 120 \\ 120 \\ 120 \\ 120 \end{bmatrix}$

for the holder of these two instruments. Let the role of A be played now by matrix P representing the Polish financial market (see (1)), with vector $b = f_1$ standing for the desired by KGHM security. Before finding portfolio $\hat{x}_1 = [\widetilde{P}^T\widetilde{P}]^{-1}\widetilde{P}^T\widetilde{f}_1$ and the resulting from it pay-offs in six scenarios by means of the methodology presented in Theorem 1, let's note that portfolio which perfectly replicates f_1 could be guessed!

In fact, letting $\hat{x}_1 = \begin{bmatrix} -1 \\ 1.2 \\ 0 \\ 0 \end{bmatrix}$ one sees that

$$P \cdot \hat{x}_1 = \begin{bmatrix} 65 & 100 & 0 & 50 \\ 80 & 100 & 0 & 35 \\ 95 & 100 & 5 & 20 \\ 110 & 100 & 20 & 5 \\ 125 & 100 & 35 & 0 \\ 140 & 100 & 50 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1.2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 55 \\ 40 \\ 25 \\ 10 \\ -5 \\ -20 \end{bmatrix} \tag{6}$$

We shall, however, determine portfolio

$$\hat{x}_1 = [\widetilde{P}^T\widetilde{P}]^{-1}\widetilde{P}^T\widetilde{f}_1$$

by means of the theory above because (i) we want to show that our theory works perfectly, also in this case, and because (ii) majority of calculations, particularly those leading to determination of matrices $\widetilde{P}$ and $[(\widetilde{P})^T\widetilde{P}]^{-1}$ is the same for all focus instruments, including all studied in this paper. Therefore, proceeding according to our theory, we arrive at

$$\widetilde{P} = \begin{bmatrix} 65\sqrt{0.1} & 100\sqrt{0.1} & 0 & 50\sqrt{0.1} \\ 80\sqrt{0.12} & 100\sqrt{0.12} & 0 & 35\sqrt{0.12} \\ 95\sqrt{0.18} & 100\sqrt{0.18} & 5\sqrt{0.18} & 20\sqrt{0.18} \\ 110\sqrt{0.28} & 100\sqrt{0.28} & 20\sqrt{0.28} & 5\sqrt{0.28} \\ 125\sqrt{0.22} & 100\sqrt{0.22} & 35\sqrt{0.22} & 0 \\ 140\sqrt{0.1} & 100\sqrt{0.1} & 50\sqrt{0.1} & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 20.55 & 31.62 & 0 & 15.81 \\ 27.71 & 34.64 & 0 & 12.12 \\ 40.31 & 42.43 & 2.12 & 8.49 \\ 58.21 & 52.92 & 10.58 & 2.65 \\ 59 & 47 & 16 & 0 \\ 44.27 & 31.62 & 15.81 & 0 \end{bmatrix} \tag{7}$$

and consequently

$$(\widetilde{P})^T \widetilde{P} = \begin{bmatrix} 11601 & 10550 & 2364 & 1157 \\ 10550 & 10000 & 1920 & 1420 \\ 2364 & 1920 & 636 & 46 \\ 1157 & 1420 & 46 & 476 \end{bmatrix} \text{ and}$$

$$[(\widetilde{P})^T \widetilde{P}]^{-1} = \begin{bmatrix} 0.16 & -0.16 & -0.11 & 0.11 \\ -0.16 & 0.17 & 0{,}11 & -0.11 \\ -0.11 & 0.11 & 0.09 & -0.07 \\ 0.11 & -0.11 & -0.07 & 0.08 \end{bmatrix}.$$

Since $f_1 = \begin{bmatrix} 55 \\ 40 \\ 25 \\ 10 \\ -5 \\ -20 \end{bmatrix}$, $\widetilde{f}_1 = \begin{bmatrix} 55\sqrt{0.1} \\ 40\sqrt{0.12} \\ 25\sqrt{0.18} \\ 10\sqrt{0.28} \\ -5\sqrt{0.22} \\ -20\sqrt{0.1} \end{bmatrix} = \begin{bmatrix} 17.39 \\ 13.86 \\ 10.61 \\ 5.29 \\ -2.35 \\ -6.32 \end{bmatrix}$

and $(\widetilde{P})^T \widetilde{f}_1 = \begin{bmatrix} 1060 \\ 1450 \\ -60 \\ 547 \end{bmatrix}$, the searched portfolio

$$\hat{x}_1 = \begin{bmatrix} 0.16 & -0.16 & -0.11 & 0.11 \\ -0.16 & 0.17 & 0.11 & -0.11 \\ -0.11 & 0.11 & 0.09 & -0.07 \\ 0.11 & -0.11 & -0.07 & 0.08 \end{bmatrix} \begin{bmatrix} 1060 \\ 1450 \\ -60 \\ 547 \end{bmatrix} = \begin{bmatrix} -1 \\ 1.2 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

is the same as the one we have guessed. Taking into account that the market price of one KGHM's stock equals 120 PLN and a 6-month treasury bill with face value of 100 PLN has the market price between 98.50 PLN and 99 PLN, the estimated cost of portfolio $\hat{x}$ is equal to

$$\begin{aligned} &-1 \cdot 120 + 1.2 \cdot 98.5 = -1.80 \text{ PLN or} \\ &-1 \cdot 120 + 1.2 \cdot 99 = -1.20 \text{ PLN} \end{aligned} \tag{9}$$

which is a fully satisfactory result.

Remark 1

The problem of eliminating risk of big declines of KGHM's stocks was solved in the mathematical model of the Polish market. The risk was eliminated in 100%. However, replication portfolio $\hat{x}_1$ requires the execution of the short sale with the same number of KGHM's shares, which have to be hedged (protected) against the loss of their market values. For that reason, the obtained theoretical solution may not be satisfactory in practice for many investors.

Fortunately, it appears that the methodology employed above gives also rise to many satisfactory results from practical point of view solutions.

5 Hedging instrument $f_2 = \begin{bmatrix} 51 \\ 43 \\ 27 \\ 9 \\ -6 \\ -17 \end{bmatrix}$

Let's therefore try another financial instrument, for example f_2, which only slightly differs from f_1, with the aim of determining its: (i) best approximate hedge (replica) $\hat{x}_2 = [\widetilde{P}^T \widetilde{P}]^{-1} \widetilde{P}^T \widetilde{f}_2$, (ii) pay-offs generated by $\hat{x}_2$, and (iii) market price of $\hat{x}_2$. Since matrices $\widetilde{P}$, $(\widetilde{P})^T \widetilde{P}$ as well as $[(\widetilde{P})^T \widetilde{P}]^{-1}$ remain the same as in case of f_1 (they have nothing to do with f_1 and f_2), we only need to calculate

$$\widetilde{f}_2 = \begin{bmatrix} 51\sqrt{0.1} \\ 43\sqrt{0.12} \\ 27\sqrt{0.18} \\ 9\sqrt{0.28} \\ -6\sqrt{0.22} \\ -17\sqrt{0.1} \end{bmatrix} = \begin{bmatrix} 16.13 \\ 14.90 \\ 11.46 \\ 4.76 \\ -2.81 \\ -5.38 \end{bmatrix} \text{ and}$$

$(\widetilde{P})^T \widetilde{f}_2 = \begin{bmatrix} 1080 \\ 1462 \\ -57 \\ 545 \end{bmatrix}$, which appeared to be almost the same vector as $(\widetilde{P})^T \widetilde{f}_1 = \begin{bmatrix} 1060 \\ 1450 \\ -60 \\ 547 \end{bmatrix}$.

The resulting optimal portfolio $\hat{x}_2$ is, however, quite different than $\hat{x}_1$, namely

$$\hat{x}_2 = \begin{bmatrix} 0.16 & -0.16 & -0.11 & 0.11 \\ -0.16 & 0.17 & 0.11 & -0.11 \\ -0.11 & 0.11 & 0.09 & -0.07 \\ 0.11 & -0.11 & -0.07 & 0.08 \end{bmatrix} \begin{bmatrix} 1080 \\ 1462 \\ -57 \\ 545 \end{bmatrix} = \begin{bmatrix} -0.18583 \\ 0.38409 \\ -0.59444 \\ 0.50913 \end{bmatrix} \quad (10)$$

and additionally it generates quite satisfactory pay-offs

$$P \cdot \hat{x}_2 = \begin{bmatrix} 65 & 100 & 0 & 50 \\ 80 & 100 & 0 & 35 \\ 95 & 100 & 5 & 20 \\ 110 & 100 & 20 & 5 \\ 125 & 100 & 35 & 0 \\ 140 & 100 & 50 & 0 \end{bmatrix} \begin{bmatrix} -0.18583 \\ 0.38409 \\ -0.59444 \\ 0.50913 \end{bmatrix} = \begin{bmatrix} 51.79 \\ 41.36 \\ 27.97 \\ 8.62 \\ -5.63 \\ -17.33 \end{bmatrix} \neq \begin{bmatrix} 51 \\ 43 \\ 27 \\ 9 \\ -6 \\ -17 \end{bmatrix} \quad (11)$$

with really small expected error, that is,

$$\text{ESSRE} = 0.1[0.79]^2 + 0.12[0.36]^2 + 0.18[0.97]^2 + 0.28[0.38]]^2 + 0.22[0.37]^2 + 0.1[0.33]^2 = 0.633$$

Corollary 1

If an investment fund in September 2015 possesses at least 100000 KGHM's shares or KGHM itself wished to protect 100000 of its own shares against their declines in the 6-month period, then each of them should short sale 18583 KGHM shares, buy 38409 six-month treasury bills, short sale 59444 six-month call options with strike price of 90 PLN, and buy 50 913 six-month put options with a strike price of 115 PLN.

We will show that the cost of acquiring portfolio $\hat{x}_2$ is close to zero, and depends on the dividend yield paid by KGHM to its shareholders. We discuss that issue in the following paragraphs.

6 Black–Scholes formula for options valuation ($\sigma = 33\%$)

We already know that the market price of 1 KGHM's share equals S = 120 PLN, and a 6-month treasury bill with face value of X = 100 PLN has the market price between 98.50 PLN and 99 PLN. What we do not know at the moment is the estimated market price of 6-month call option c_{90} with strike 90 PLN, and a 6-month put option p_{115} with strike 115 PLN.

According to Black–Scholes formula

$$c = S\exp(-qT)N(d_1) - X\exp(-rT)N(d_2) \quad (12)$$

where q is dividend yield (KGHM paid 2–4% in the last few years), T is the expiration date ($\frac{1}{2}$ of the year in the studied case), N(d) is the cumulative probability distribution function for the standard normal distribution N(0,1), r is the risk-free rate on Polish market (about 2,5% annually), with

$$d_1 = [\ln(S/X) + (r - q + 0{,}5\sigma^2]/\sigma\sqrt{T}$$
$$d_2 = [\ln(S/X) + (r - q - 0{,}5\sigma^2]/\sigma\sqrt{T} \quad (13)$$

For the sake of clarity, we have assumed in this paragraph a relatively high volatility σ of KGHM's stock prices, namely $\sigma = 33\%$. The case when $\sigma = 20\%$ will also be analyzed.

Let's see how different values of parameter q affect the valuation of our call option with strike price 90 PLN. When q = 2% then $d_1 = 1.3602$, $d_2 = 1.1269$ and consequently $N(d_1) = 0.9131$, $N(d_2) = 0.8701$ so that $c_{90} = 31.15$ PLN.

When dividend yield is higher, for example q = 3%, then $d_1 = 1.3388$, $d_2 = 1.1055$ and consequently

$N(d_1) = 0.9097,\ N(d_2) = 0.8655$ so that $c_{90} = 30.61$ PLN. Why does the call option cost now slightly less? It costs less because it gives the right to buy for the same price of 90 PLN a less valuable share of KGHM (due to a higher dividend payment from that share in the 6-month period of September 2015 to February 2016).

Finally, when q = 4% then $d_1 = 1.3174,\ d_2 = 1.0840$ and consequently $N(d_1) = 0.9061,\ N(d_2) = 0.8608$ so that the call option is even more cheaper, namely it costs $c_{90} = 30.07$ PLN. We have just proved the following.

Fact 1

The change of parameter q from 2% to 3% and next to 4% implies the corresponding change (decline) of c_{90} from 31.15 PLN to 30.61 PLN and next to 30.07 PLN by the same amount of 0.54 PLN in each of these two cases.

Now, let's see how different values of parameter q affect the value of our put option with strike price of 115 PLN for which we have a slightly different Black–Scholes valuation formula

$$p = S\exp(-qT)N(-d_1) + X\exp(-rT)N(-d_2) \quad (14)$$

When dividend q = 2% then $d_1 = 0.3098\ \ d_2 = 0.0764$ and consequently $N(d_1) = 0.6216,\ N(d_2) = 0.5305$ so that $p_{115} = 8.374$ PLN. When dividend is higher, for example q = 3%, then $d_1 = 0.2883,\ d_2 = 0.0550$ and consequently:

$$N(d_1) = 0.6135$$

$$N(d_2) = 0.5219$$

so that $p_{115} = 8.60$ PLN.

Why does the put option cost more when dividend yield is higher? It costs more because it gives the right to sell for 115 PLN a less valuable share of KGHM (due to a higher payment of dividend from that share in the period September 2015 to February 2016).

Finally, when q = 4% then $d_1 = 0.2669,\ d_2 = 0.0336$ and consequently $N(d_1) = 0.6052,\ N(d_2) = 0.5134$ so that $p_{115} = 8.83$ PLN. We have just proved the following.

Fact 2

The change of parameter q from 2% to 3% and next to 4% implies the corresponding rise of p_{115} from 8.37 PLN to 8.60 PLN and next to 8.83 PLN by the same amount of 0.23 PLN in each of these two cases.

6.1 Cost calculation of portfolio $\hat{x}_2$ replicating $f_2(\sigma = 33\%)$

Now, we are in a position to estimate the cost of portfolio $\hat{x}_2 = \begin{bmatrix} -0.186 \\ 0.384 \\ -0.594 \\ 0.509 \end{bmatrix}$.

Let P given by (1) be a matrix model of the Polish financial market from KGHM's point of view. When dividend yield q = 2%, then $S = \begin{bmatrix} 120 \\ 98.5 \\ 31.15 \\ 8.37 \end{bmatrix}$ designates the price vector for four basis assets (columns of matrix P). The cost of portfolio $\hat{x}_2$ is equal to

$$< S^T; \hat{x}_2 >= \begin{bmatrix} 120 \\ 98.5 \\ 31.15 \\ 8.37 \end{bmatrix}^T \cdot \begin{bmatrix} -0.186 \\ 0.384 \\ -0.594 \\ 0.509 \end{bmatrix} = 1.28 \text{ PLN}$$

When q = 4%, then the corresponding cost of $\hat{x}_2$ is higher, namely

$$\begin{bmatrix} 120 \\ 98.5 \\ 30.07 \\ 8.83 \end{bmatrix}^T \cdot \begin{bmatrix} -0.186 \\ 0.384 \\ -0.594 \\ 0.509 \end{bmatrix} = 2.15 \text{ PLN}$$

When q = 3% the cost of $\hat{x}_2$ lies in between the two above costs, it equals

$$\begin{bmatrix} 120 \\ 98.5 \\ 30.61 \\ 8.60 \end{bmatrix}^T \cdot \begin{bmatrix} -0.186 \\ 0.384 \\ -0.594 \\ 0.509 \end{bmatrix} = 1.72 \text{ PLN}$$

Corollary 2

The cost of acquiring the best approximate portfolio $\hat{x}_2$ is very small, depending on dividend yield (q) paid annually by KGHM. Specifically, when q is increasing from 2% to 4%, the price of $\hat{x}_2$ is going up from 1.28 PLN to 2.15 PLN.

The pay-offs $\begin{bmatrix} 55.31 \\ 39.36 \\ 24.46 \\ 11.65 \\ -4.53 \\ -22.41 \end{bmatrix}$ generated by $\hat{x}_2$, together with payments $\begin{bmatrix} 65 \\ 80 \\ 95 \\ 110 \\ 125 \\ 140 \end{bmatrix}$ resulting from holding one share of KGHM, guarantee an (almost risk-free) income of approximately 120 PLN, which is the market price of KGHM's stocks in September 2015 when this analysis might have been carried out.

7 Black–Scholes formula for options valuation when $\sigma = 20\%$

In this paragraph, we repeat the reasoning from paragraph 6 for lower volatility σ of KGHM's shares prices. Our first observation is that lower volatility implies lower values (premiums) for both call and put options due to smaller uncertainty of KGHM's stock prices. Once again, we will see how different values of parameter q affect the valuation of our call and put options.

When
$q = 2\%$ then $d_1 = 2.1226$, $d_2 = 1.9812$
and consequently
$N(d_1) = 0.9831$, $N(d_2) = 0.9762$
so that $c_{90} = 30.03$ PLN.

When dividend yield q is higher, for example
$q = 3\%$, then $d_1 = 2.0873$, $d_2 = 1.9458$
and consequently
$N(d_1) = 0.9816$, $N(d_2) = 0.9742$
so that $c_{90} = 29.45$ PLN.

The call option costs less for the same reason as in case $\sigma = 33\%$. Finally, when $q = 4\%$ then
$d_1 = 2.0519$ $d_2 = 1.9105$,
$N(d_1) = 0.9799$, $N(d_2) = 0.9720$
so that the call option is even more cheaper, namely it costs $c_{90} = 28.87$ PLN.

Fact 3

The change of parameter q from 2% to 3% and next to 4% implies the corresponding declines of c_{90} from 30.03 PLN to 29.45 PLN and next to 28.87 PLN $(\sigma = 20\%)$ by the same amount of 0.58 PLN in each of these two cases.

Now, let's see how different values of parameter q affect the value of our put option with strike price of 115 PLN when $\sigma = 20\%$.

Assuming $q = 2\%$ we have
$d_1 = 0.3893$ $d_2 = 0.2479$ and
$N(d_1) = 0.6515$, $N(d_2) = 0.5979$
so that $p_{115} = 4.26$ PLN.

When dividend
$q = 3\%$, then $d_1 = 0.3540$, $d_2 = 0.2126$
and consequently
$N(d_1) = 0.6383$, $N(d_2) = 0.5842$
so that $p_{115} = 4.47$ PLN.

The put option costs more for the same reason as previously. Finally, when
$q = 4\%$ then $d_1 = 0.3186$, $d_2 = 0.1772$ and
$N(d_1) = 0.6250$, $N(d_2) = 0.5703$
imply that $p_{115} = 4.69$ PLN.

Fact 4

The change of parameter q from 2% to 3% and next to 4% implies the corresponding increases of p_{115} from 4.26 PLN to 4.47 PLN and next to 4.69 PLN by approximately the same amount of 0.21 PLN.

7.1 Cost calculation for portfolio $\hat{x}_2$ when $\sigma = 20\%$

For q = 2%, the price vector S = $\begin{bmatrix} 120 \\ 98.5 \\ 30.03 \\ 4.26 \end{bmatrix}$ and consequently the cost of $\hat{x}_2$ is negative and equal to $< S^T; \hat{x}_2 > =$ –0.15 PLN. With dividend q = 3% the price vector S = $\begin{bmatrix} 120 \\ 98.5 \\ 29.45 \\ 4.47 \end{bmatrix}$ so that the price of $\hat{x}_2$ goes up by 0.45 PLN, to 0.30 PLN.

Finally, when q = 4% and S = $\begin{bmatrix} 120 \\ 98.5 \\ 28.87 \\ 4.69 \end{bmatrix}$ the cost of portfolio $\hat{x}_2$ rises by additional 0.46 PLN to 0.76 PLN.

Corollary 3

In case when volatility σ of KGHM's share prices is equal to 20%, the cost of acquiring the best approximate portfolio $\hat{x}_2$ is close to zero, depending on dividend yield (q) paid annually by KGHM. When dividend q raises from 2% to 4%, the price to be paid for acquiring $\hat{x}_2$ increases from –0.15 PLN to 0.76 PLN.

The pay-offs generated by $\hat{x}_2$ and payments resulting from holding one share of KGHM in February 2016 are the same as in the case when $\sigma = 35\%$.

Bibliography

[1] Cerny, A., 2009. *Mathematical Techniques in Finance.* Princeton University Press.

[2] Zaremba, L., 2015. Inżynieria finansowa na rynkach zupełnych (*Financial Engineering in the Complete Markets*). *Kwartalnik Naukowy Uniwersytetu Vistula (Vistula Scientific Quarterly)*, Vol. 46, No. 4, pp.101-115.