DECISION-MAKING IN FLAT AND HIERARCHICAL DECISION PROBLEMS

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Abstract: The article is dedicated to the modelling of the essence of decision-taking processes in flat and hierarchical decision problems. In flat decision problems particular attention is drawn to the effectiveness of strategies in seeking decision variants on solution decomposition trees, taking into account the strength of their predefined contradictions. For hierarchical decision processes, the issue of iterative balancing of global (hierarchical) decisions is expressed, based on the valuation of the significance of flat decisions.

Keywords: hierarchical decision problems, modelling of decision-taking processes, AIDA technique, valuation of decisions, solution tree decomposition procedure, flat decision problems with internal contradictions.

1 Introduction

Taking decisions in a flat decision problem (or in other words: in a single-layer system) involves the defining of the decision problem in the form of a set of homogenous decision areas, and determining for each decision variant one elementary decision from each of these areas. Details on how to proceed are presented on the example of the AIDA technique [14] drawn up by J. Luckman. Flat decision problems refer to the execution of simple tasks involving the choice of one from numerous alternative decisions, fully prepared beforehand. An example of a flat decision problem could be the selecting of an offer in a tender, following assessment of the offers in line with the specification of the relevant conditions in the order (in which case one decision area corresponds to each criterion for assessing the offers submitted).

An hierarchical decision problem arises when elementary decisions in insufficient (hypothetical) form for the direct carrying out of assessments and comparisons occur in at least one decision area of the decision problem being analysed. In such a situation the decision areas with hypothetical elementary decisions should be expanded either directly or indirectly into the form of flat decision problems.

Preparation of a nonempty set of decision variants followed by the indication of one of its components "for execution" constitutes the essence of any decision process. In the case of an hierarchical decision problem, the decision area whose elementary decisions emerge as a result of resolving the decision problem situated in the layer directly preceding the decision area's layer (see Fig. 5), i.e. through the consolidation of one elementary decision from each decision area of the decision problem being resolved, corresponds to the set of decision variants in a mutually explicit manner.

The indication of a decision variant as a decision should be preceded by valuation of its individual elementary decisions and the relations of contradiction occurring between them. The calculative complexity of this process belongs to the NP class – difficult due to the exponential dependence of the number of operations of comparing pairs on the number of sets of elementary decisions participating in creation of the decision space (see chapters 3 and 4).

2 The essence of the decision process

Decisions are taken at various levels of management – operational, tactical and strategic – resulting rather from the organisational necessities of management processes and not the essence of the actual decision-taking process, which in a procedural respect cannot always be formalised and frequently derives directly from work regulations or remains intuitive. The classification of management levels derives historically from needs and applications which, initially, were above all military, followed by those of an economic and administrative nature.

A turbulent economic context forces enterprises to adapt quickly to their environment and react accordingly to transformations taking place. Changes must also frequently affect long-term goals. This is reflected in the necessity to harmonise strategic plans on numerous levels with changes and decision of a medium-term and current scope, on the execution of which the success of strategic goals depends.

Three types of decision are distinguished in decision processes: operational, tactical and strategic. Examples of operational decisions are: employing a new member of staff; purchasing document scanning equipment needed for the management system; and increasing the number of parking spaces. Operational decisions are taken by airport personnel controlling flights and air space, by entire states or even by a continent. However, the consequences of operational decisions taken may be tactical or even strategic in character, depending on the circumstances (the conditions, the threats) in which they are taken.

Tactical decisions, in an operational perspective, apply to the planning and organising of resources and processes essential for carrying out operational tasks, but in a strategic perspective their task is to ensure the conditions for executing an organisation's strategic (long-term) plans.

Tactical decisions apply to the planning and organising of resources and processes essential for the direct execution of operational decisions. The goal of taking decisions at a tactical level is to ensure an enterprise with the effective functioning of material and informational infrastructure. Tactical decisions are reserved for medium-level management or the managerial boards of the organisations in which they are taken. The time horizon for carrying out and assessing the direct consequences of tactical decisions should not exceed one year. Examples of tactical decisions: implementation of an IT Decision Support System (DSS); compiling a prospectus for a planned stock market flotation; and initiating online sales of products and services.

Strategic decisions apply to the planning and allocation in time and space of processes and resources which will be essential for achieving an organisation's strategic goals via appropriate decisions on the tactical level. Strategic decisions are most often reserved for organisations' managerial and supervisory boards. The time horizon for strategic decisions is usually a few years, and is significant for the enterprise or organisation in economic or social-political terms.

The character and time horizon of tactical and strategic decisions is determined by the period after which there should be a return on the expenses incurred along with the appearance of benefits reflecting the operational, tactical and strategic goals – depending on the size of the organisation and the magnitude of the decisions taken.

Among other things, decision processes demand: the systematic accumulation and analysis of information on the goals, resources and processes of the organisation and its context; the development of methods and tools serving the preparation and selection of decisions from among alternative decision variants, as well as the compilation of decision process assessment criteria and the monitoring of their values.

3 Flat and hierarchical decision problems

A decision process may proceed in a single problem layer (flat decision problem) or may take on a multi-layered form (hierarchical decision problem). Flat decision processes refer to the execution of simple tasks, for example involving the processing of a single resource or consolidation of resources available in line with a known procedure (technology). The preparation of alternative decision variants entails defining, for the decision problem posed, corresponding decision areas from which component decisions (elementary decisions) will be drawn in such a way for them to mutually complement each other and display only minimum contradiction in relation to each other in the decision variant being constructed.

The models for flat and hierarchical decision problems derive from the morphological analysis of related decision areas, proposed in 1948 by F. Zwicky [17][1]. Among the numerous known methods of morphological analysis (the morphological box method, the randomisation method using fuzzy sets, Moles' method, and the sequential model of steering events), the AIDA technique (an acronym of the words Analysis of Interconnected Decision Areas) developed by J. Luckman stands out in its high level of effectiveness and simplicity. The AIDA technique may be applied for solving many problems whose morphological models take on a finite, grainy form.

[1] Creator of the grounds for morphological analysis and discoverer of neutron stars, the Swiss astrophysicist F. Zwicky (1898 – 1974) treated morphological research as "glimpsing such a picture of reality in which all the major structural connections between objects, phenomena, ideas and actions would be clearly taken into account…"

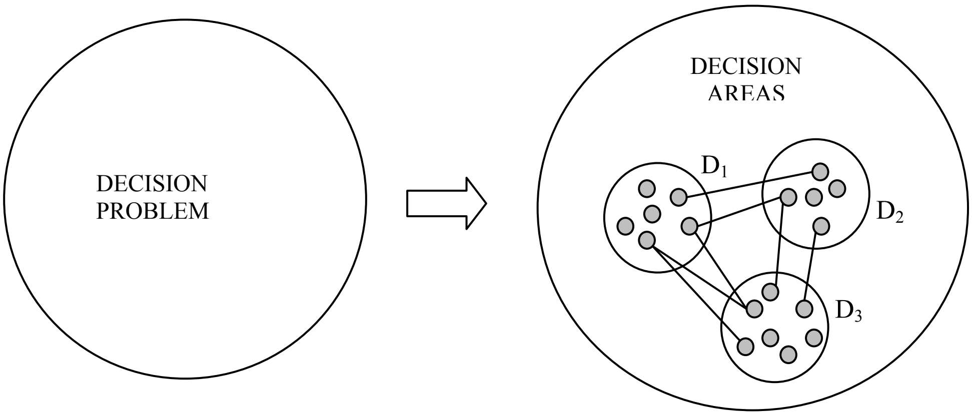

Figure 1. Model of a flat decision problem; D_1, D_2, D_3 – decision areas and their elementary decisions (*source: the authors*)

Distinguishing between a flat and hierarchical (multi-layered, see Figs. 1 and 5) decision problem boils down to resolving whether the decision problem expressed for a specific issue may be settled (resolved) without having to consider more detailed issues and as a consequence expressing separate decision problems for them. Hence the decisions on a strategic and tactical level should be taken as solutions to hierarchical problems, while operational decisions are by definition the solution to flat decision problems. All data at the operational level should be attainable without having to take decisions on more detailed (elementary) levels. With a flat decision problem we are dealing purely with a situation in which the elementary decisions have already been taken, and their results (the products) are available for combining (consolidating) into decision variants.

The essence and expression of a decision problem may be presented in various ways depending on the language convention adopted for the formal record of the problem situation's model. For our needs, in order to obtain a general expression of a decision problem, we will be using the convention used in the AIDA technique (see Fig. 1).

In the language of the AIDA technique a decision problem is recorded in the form of a finite set of decision areas, each of which in turn is a finite set of elementary decisions. The elementary decisions (the points marked in grey in Fig. 1) belonging to the single decision area D_i are alternatives in relation to each other, meaning that a particular elementary decision contained in the decision model excludes the introduction of a different elementary decision from the same decision area. Likewise, elementary decisions from different decision areas declared as in contradiction with each other (the points in Fig. 1 belonging to different decision areas D_i joined by an unbroken straight line) are excluded from the solution (the decision). Thus we have constructed a model of the problem situation and its solution in the form of a decision. A typical decision process comprises a few phases, among which one can distinguish:

- the problem situation model, in which the essence of the decision problem being resolved and the expected solutions for this problem should be recorded in a formalised manner,
- the decision model, specifying all required attributes and values of the features characteristic for the decisions taken,
- the model of solving a problem situation, enabling – based on the problem situation model and decision model – presentation of the process of generating a set of alternative decisions and distinguishing (indicating) within it the solutions most favourably fulfilling the criteria adopted for assessing the decision.

The phases in the decision process enable presentation of the action cycle leading from the essence of the decision problem, through the desired decision model, up to the generation and relative multi-parametric assessment of decisions selected from the problem situation model. The linked decision areas of the decision problem under consideration are analysed using the solution tree of the AIDA technique, presented later in this article (see Fig. 3), this technique deriving from the apparatus of the so-called morphological analysis ([3], [4], [5] and [17]). The model of the decision recorded in the convention of the AIDA technique is a (brief) sequence of elementary decisions, one from each of the decision areas D_i, of the form…

$$\langle d_{j1}, d_{j2}, ..., d_{jm} \rangle \quad (1)$$

…and this sequence belongs to decision space D of the decision problem in question, this space noted down as the Cartesian product of the decision areas D_i :

$$D_1 \times D_2 \times ... \times D_m \quad (2)$$

4 The AIDA technique

The essence of the AIDA technique boils down to:

- defining a finite set of internally alternative decision areas $D_i \in D$, describing the posed problem situation,
- defining finite sets of elementary decisions d_{ji} for all decision areas $D_i \in D$,
- defining pairs of mutually contradictory elements d_{ji}, d_{kl} (elementary decisions) belonging to different decision areas D_i, D_l,
- obtaining solution variants by generating all possible sequences of elementary decisions <d_{j1}, d_{j2}, ..., d_{jm}> and eliminating those sequences from decision space D in which pairs of contradictory elements d_{ji}, d_{kl}, occur,
- putting in order, analysing and ultimately choosing acceptable solution variants (decisions), one of which will be taken.

The initial phase of morphological analysis is definition of the decision areas recorded in the form of so-called formative sets of elementary decisions contained in these areas. We create as many formative sets as there are decision areas. Each of the formative sets should contain at least one elementary decision. The elementary decisions of each decision area have homologous properties, i.e. in specific solution variants they may be replaced by other elementary decisions from the same decision area. The Cartesian product of all decision areas demarcates the morphological analysis space (decision space D).

In certain practical applications the AIDA technique may significantly accelerate the generation of and orderly searching through a significant number of decision variants which should be taken into account, particularly in situations of limited time for taking a decision or due to the significant costs of drawing up decision variants. It is not difficult to notice that even with 5 decision areas, each containing 10 elementary decisions, we are forced in a simple morphological analysis to carry out 4.5×10^6 operations of comparing pairs of elementary decisions in order to exclude erroneous (incomplete) decisions in the sense of their formula (1) – and in the case of 10 decision areas each with 20 elementary decisions, the number of operations for comparing pairs of elementary decisions increases to 1.9×10^{15}.

5 Valuation of decisions

The generation and analysis of decision variants is significantly speeded up thanks to the application of the graph model and orderly decomposition of decision areas. Constructing a graph model of decision space begins with elimination of those decision areas whose formative sets only contain a single element, because – by definition – such an elementary decision will belong to every decision.

The vertices of a graph corresponding to the elements of a single decision area are alternatives in relation to each other (they cannot belong to one decision). Because of the apriori collisions (contradictions) of certain elements belonging to different decision areas, we connect the appropriate graph vertices marked with these elements with edges (unbroken lines).

Figure 2 illustrates the procedure. A dotted line is used to mark elementary decisions forming an example of a correctly constructed decision, in which there are no pairs of elementary decisions constituting alternatives for each other (no contradiction). No pair of elementary decisions belonging to different decision areas can, in a correctly constructed decision, be a pair of elements connected by an unbroken line. An example of another correctly created decision is the sequence: <d_{21}, d_{22}, d_{23}>. In the decision problem example we are looking at, there are 7 such correct decisions, which we will demonstrate when building the solutions tree (see chapter 6).

The decision areas and sets of elementary decisions contained within them have imposed upon them, for practical reasons, restrictions valuing by percentage the significance of decision areas V_i and the weightings v_{ji} of the significance of elementary decisions of all decisions areas, where the sum in each decision area is taken for normalisation as equal to 1:

Example (see Fig. 2)

$V_1 = 20$	$V_2 = 30$	$V_3 = 50$
$v_{11} = 0.75$	$v_{12} = 0.50$	$v_{13} = 0.40$
$v_{21} = 0.25$	$v_{22} = 0.10$	$v_{23} = 0.30$
	$v_{32} = 0.40$	$v_{33} = 0.30$

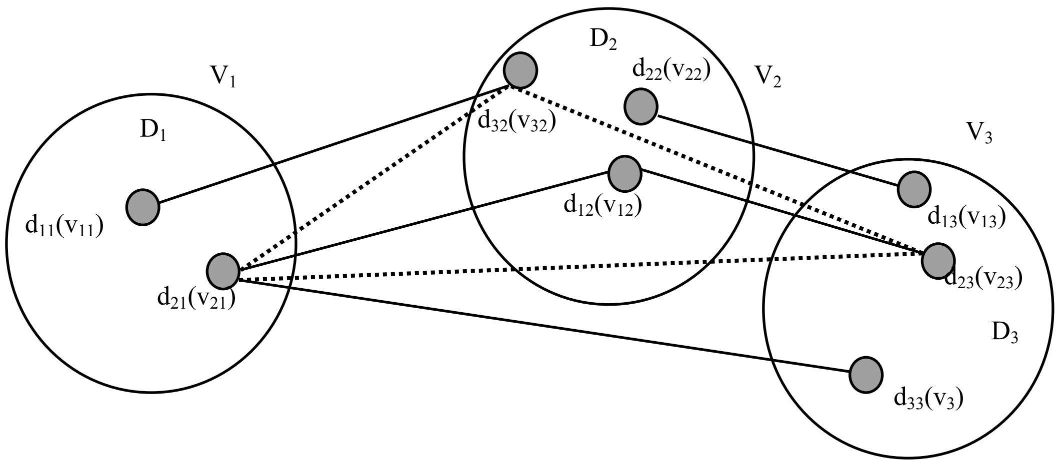

Figure 2. Decision variant model (elementary decisions d_{21}, d_{32}, d_{23} connected with a dotted line) against a decision problem model (unbroken line – total contradiction; absence of a line or a dotted line means no contradiction) (*source: the authors*)

The relative significance weighting of a single decision Q is calculated as the sum of the products of the significance weightings of the decision areas and the significance weightings of the elementary decisions from their corresponding decision areas, using the formula:

$$Q = \sum V_i * v_{ji} \quad (3)$$

where for each index i of the decision area D_i, the value of index j applies to the elementary decision d_{ji} from this area.

Significance weighting Q, on the example of the decision variant <{d_{21}}{d_{32}}{d_{23}}>, has the value:

$$Q = V_1 \text{ x } v_{21} + V_2 \text{ x } v_{32} + V_3 \text{ x } v_{23} = 20 \text{ x } 0.25 + 30 \text{ x } 0.40 + 50 \text{ x } 0.30 = 32$$

In the example under consideration, there are seven possible decisions for which the weighting of the decision's significance may be calculated (see Table 1).

In table 1 the value $Q_{max} = 50$ was obtained as a result of adding up the partial values using the formula:

$$Q_{max} = V_1 \text{ x } (\max v_{j1} = 0.75) + V_2 \text{ x } (\max v_{j2} = 0.50) + V_3 \text{ x } (\max v_{j3} = 0.40) = 20 \text{ x } 0.75 + 30 \text{ x } 0.50 + 50 \text{ x } 0.40 = 50$$

Likewise the value $Q_{min} = 23$ was obtained by adding up the partial values using the formula:

$$Q_{min} = V_1 \text{ x } (\min v_{j1} = 0.25) + V_2 \text{ x } (\min v_{j2} = 0.10) + V_3 \text{ x } (\min v_{j3} = 0.30) = 20 \text{ x } 0.25 + 30 \text{ x } 0.10 + 50 \text{ x } 0.30 = 23$$

The weightings Q_{max} and Q_{min} are used in the procedure for decomposition of the solution tree of decision variants in the process of determining the strategy for searching for these variants. In the example in use, it turned out by chance that decisions 1 and 3 correspond to the calculated values of Q_{max} and Q_{min}. In general, correctly formed decision variants do not have to correspond to the threshold values due to the internal contradictions of elementary decision pairs.

Table 1. Significance weightings of decisions for the decision problem example in Fig. 2 *(source: the authors)*

Decision no.	Decision	Weighting of Decision Significance
1	<{d_{11}}{d_{12}}{d_{13}}>	Q_{max} = Q_1 = 20 x 0.75 + 30 x 0.50 + 50 x 0.40 = 50
2	<{d_{11}}{d_{12}}{d_{33}}>	Q_2 = 20 x 0.75 + 30 x 0.50 + 50 x 0.30 = 45
3	<{d_{21}}{d_{22}}{d_{23}}>	Q_{min} = Q_3 = 20 x 0.25 + 30 x 0.10 + 50 x 0.30 = 23
4	<{d_{21}}{d_{32}}{d_{13}}>	Q_4 = 20 x 0.25 + 30 x 0.40 + 50 x 0.40 = 37
5	<{d_{21}}{d_{32}}{d_{23}}>	Q_5 = 20 x 0.25 + 30 x 0.40 + 50 x 0.30 = 32
6	<{d_{11}}{d_{22}}{d_{23}}>	Q_6 = 20 x 0.75 + 30 x 0.10 + 50 x 0.30 = 33
7	<{d_{11}}{d_{22}}{d_{33}}>	Q_7 = 20 x 0.75 + 30 x 0.10 + 50 x 0.30 = 33

6 Solution tree decomposition procedure

The procedure for generating decision variants is based on decomposition of the graph model of decision space. Decomposition involves the systematic extraction of Internally Stable Groups of Formative Sets (ISGFS), each fulfilling two conditions for being a decision variant (a decision):

- an ISGFS contains one elementary decision from each decision area (formative set),
- an ISGFS does not contain pairs of self-eliminating (alternative) decisions.

For example, for decision areas D_1, D_2, D_3 (see fig. 2), the sets $\{d_{11}, d_{22}, d_{33}\}$, $\{d_{11}, d_{12}, d_{13}\}$ and $\{d_{21}, d_{32}, d_{23}\}$ belonging to the set of acceptable decisions in the decision problem in question will be internally stable. Altogether we can identify 7 ISGFS variants in Fig. 2 (see Table 1).

The generation of decision variants (see Fig. 3) may proceed as follows:

(a) the cardinality of each formative set is defined;

(b) formative sets are put in descending order in regard to their cardinality values;

(c) the formative sets of decisions belonging to all decision areas are split into as many groups of sets as the value of the cardinality of the most numerous formative set;

(d) another vertex is added to the solution tree, along with the edges emerging from it, to which the respective groups of formative sets are assigned.

Various strategies may be applied for choosing a formative set, for example taking into account the significance V_i of a specific decision area in relation to the other areas, or the maximum number of edges connecting a particular set with the others; in the latter case, in the example used, the formative set corresponding to decision area D_2 would also be chosen.

The formative sets of the first vertex in the solution tree being constructed are equal to the sets of elementary decisions in the decision areas of the decision problem being resolved. These sets form the initial group of formative sets (GFS).

When creating successive GFS one has to remember that they should not contain decisions which are alternatives to the elementary decisions from the formative set in relation to which the GFS was split. If, as a result of the splitting, the cardinality of one of the formative sets in a GFS equals 0 – then this particular GFS is eliminated from the splitting process, and marked as an EGFS (Eliminated Group of Formative sets). If the cardinality of all formative sets in a particular GFS attains a value of 1 as a result of successive splits, then this group is a variant of an internally stable set of elementary decisions and is marked as ISGFS.

Formative sets are put in descending order according to their cardinality, in order to indicate the sets most suitable for effective decomposition of the solution tree (with the minimum number of splits).

Operations (a) to (d) are repeated until only groups marked as EGFS or ISGFS remain within the GFS of the solution tree undergoing decomposition. Groups marked as ISGFS constitute a set of all possible decision variants such that no variant contains a pair of alternative decisions.

In the procedure for decomposing the solution tree a significant role is played by the cardinality of the formative set according to which in a specific vertex the decision problem described by the collection of formative sets ascribed to this vertex is to be decomposed. The operation of decomposing one of the formative sets is accompanied by the operation of the unfolding of the current vertex into a set of vertices equal in number to this set's cardinality (see Fig. 3 and Table 3). Intuition (heuristics) suggests that the most numerous formative set, from the collection of formative sets assigned to the vertex in question, should be subjected to decomposition. A strategy preferring decomposition according to the formative set (decision area) whose elementary decisions display the most numerous conflicts (contradictions) with the elementary decisions of the remaining decision areas may be equally as justified (in the example we are using, this is decision area D_2).

The execution of operations (a) to (d) is illustrated by Fig. 3 and Table 2. The ISGFS decision variants on the solution tree are marked in grey. EGFS (position 11 in Table 3) signifies an eliminated decision due to it lacking elementary decision d_{21}.

The procedure for decomposing the solution tree explicitly defines the proceedings which should be carried out in order to obtain the decision variants (tuples of elementary decisions).

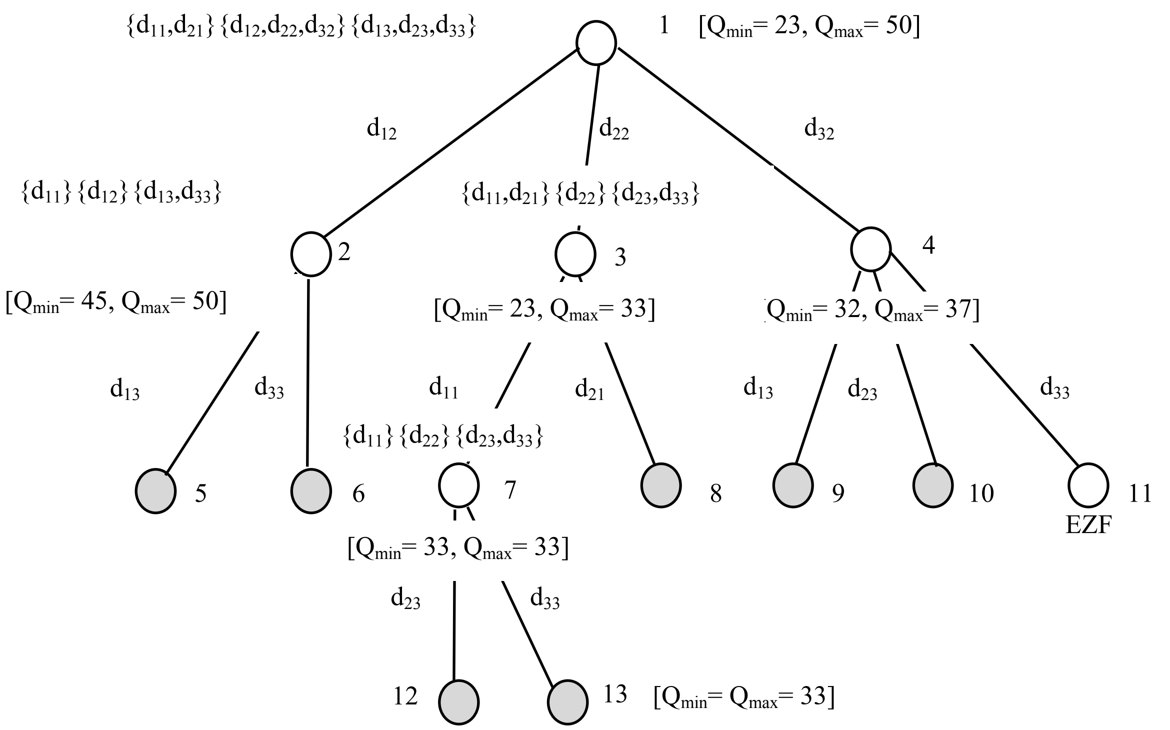

Figure 3. Solution Tree
(*source: the authors*)

Table 3. Vertices, formative sets, and ISGFS solutions of the decomposition tree for the decision problem; the symbol → indicates tree vertices occurring after the current vertex
(source: the authors)

Vertex no.	Formative sets	Q_{min}	Q_{max}	ISGFS solutions	For decomposing
1→2,3,4	$\{d_{11},d_{21}\}\{d_{12},d_{22},d_{32}\}\{d_{13},d_{23},d_{33}\}$	23	50		$\{d_{12},d_{22},d_{32}\}$
2→5,6	$\{d_{11}\}\{d_{12}\}\{d_{13},d_{33}\}$	45	50		$\{d_{13},d_{33}\}$
3→7,8	$\{d_{11},d_{21}\}\{d_{22}\}\{d_{23},d_{33}\}$	23	33		$\{d_{11},d_{21}\}$
4→9,10.11	$\{d_{21}\}\{d_{32}\}\{d_{13},d_{23}\}$	32	37		$\{d_{13},d_{23}\}$
5	$\{d_{11}\}\{d_{12}\}\{d_{13}\}$	50	50	$<d_{11},d_{12},d_{13}>$	ISGFS
6	$\{d_{11}\}\{d_{12}\}\{d_{33}\}$	45	45	$<d_{11},d_{12},d_{33}>$	ISGFS
7→12,13	$\{d_{11}\}\{d_{22}\}\{d_{23},d_{33}\}$	33	33		$\{d_{23},d_{33}\}$
8	$\{d_{21}\}\{d_{22}\}\{d_{23}\}$	23	23	$<d_{21},d_{22},d_{23}>$	ISGFS
9	$\{d_{21}\}\{d_{32}\}\{d_{13}\}$	37	37	$<d_{21},d_{32},d_{13}>$	ISGFS
10	$\{d_{21}\}\{d_{32}\}\{d_{23}\}$	32	32	$<d_{21},d_{32},d_{23}>$	ISGFS
11	$\{\phi\}\{d_{32}\}\{d_{33}\}$	-	-	$< \phi, d_{32},d_{23}>$	EGFS
12	$\{d_{11}\}\{d_{22}\}\{d_{23}\}$	33	33	$<d_{11},d_{22},d_{23}>$	ISGFS
13	$\{d_{11}\}\{d_{22}\}\{d_{33}\}$	33	33	$<d_{11},d_{22},d_{33}>$	ISGFS

During the decomposition of successive decision tree vertices, different strategies for decomposing the formative sets may be applied. They may take different criteria into account (e.g. the cardinality of formative sets, the significance of decision areas, or the value of internal stresses between elementary decisions of different decision areas).

In the process of decomposing a solution tree's vertices, in each vertex unfolded (subjected to decomposition) it is important to assess the anticipated maximum

Q_{max} and minimum Q_{min} significance weightings of the decisions which might be obtained as a result of this vertex's decomposition. In successive vertices subjected to decomposition, on the tree shown in Fig. 3, the current GFS was taken into account (see Table 3) for calculating the Q_{max} and Q_{min}.

The valuation of decisions when decomposing a solution tree's vertices leads to the determining for each newly-created vertex its potential minimum and maximum significance weightings. The term "potential" expresses respectively the lower and upper thresholds of the expected significance weighting of the decision, which due to possible contradictions # (see chapter 7) will not be possible to achieve (such a case does not occur in the example being used). For example, in Fig. 3, vertex no. 3 has a calculated span [Q_{min} = 23, Q_{max} = 33] which decisions (vertices) nos. 8 [Q = 23], 12 [Q = 33] and 13 [Q = 33] correspond to. Early determining of the thresholds Q of potentially anticipated decisions leads to a significant narrowing of the search for decisions fulfilling a defined scope. However, one must bear in mind that due to eliminated elementary decisions also having a part in the calculations it may turn out that the chosen search direction leads us to an empty decision set.

7 Flat decision problems with internal contradictions

Elementary decisions belonging to different decision areas (see fig. 4) may be in a relation of contradiction $\#(d_{kp}, d_{jq})$ with one another, which in the figure is marked with an unbroken line connecting a specific pair of elementary decisions (where # = 1 means total contradiction, which does not allow for the simultaneous occurrence of both elementary decisions in a single decision variant) or dashed line (where 1 > # > 0 means a partial contradiction, which may occur in a decision if appropriately implemented, which is most often related to an increase in the costs of executing such a decision). A value of # = 0 means there is no contradiction, or in other words the cost-free simultaneous presence of both elementary decisions in the decision variant is possible. Contradictions $\#(d_{kp}, d_{jq})$, characteristic of relations occurring between pairs of elementary decisions, can also cause positive effects occurring as a result of positive synergy in the decision areas p, q.

The simultaneous contradiction occurring between m elementary decisions (m-contradiction) belonging to different decision areas is calculated as the sum of the averaged contradictions occurring between each pair of elementary decisions forming a particular m-contradiction, in keeping with the definition:

$$\#<d_{j1}, d_{j2}, ..., d_{jm}> = \sum \#<d_{jp}, d_{jq}> \quad (4)$$

$$\#<d_{jp}, d_{jq}> = [\#(d_{jp}, d_{jq}) / (1 - \#(d_{jp}, d_{jq}))] \text{ x } [(v_{jp} \text{ x } V_p + v_{jq} \text{ x } V_q) / (V_p + V_q)] \quad (5)$$

where:
p = 1..(q-1); q = 2..m,
$\#(d_{jp}, d_{jq})$ - the value of the contradiction (stress) between elementary decisions d_{jp}, d_{jq} measured on a scale [0..1).

The example below, together with Fig. 4, constitutes an illustration of the calculations for valuation of decisions taking internal stresses into account.

Calculations for the internal contradiction for decision no. 5 $\#<d_{21}, d_{32}, d_{23}>$ proceed as follows:

$$\#<d_{21}, d_{32}, d_{23}> = \#<d_{21}, d_{32}> + \#<d_{21}, d_{23}> + \#<d_{32}, d_{23}>$$

$$\#(d_{21}, d_{32}) = 0.20$$
$$\#<d_{21}, d_{32}> = [\#(d_{21}, d_{32}) / (1 - \#(d_{21}, d_{32}))] \text{ x } [(v_{21} \text{ x } V_1 + v_{32} \text{ x } V_2) / (V_1 + V_2)]$$
$$= [0.20 / (1 – 0.20)] \text{ x } [(0.25 \text{ x } 20 + 0.40 \text{ x } 30) (20 + 30)]$$
$$= [0.25] \text{ x } [(17) / (50)]= 0.25 \text{ x } 0.34 = 0.085$$

$$\#(d_{21}, d_{23}) = 0.10$$
$$\#<d_{21}, d_{23}> = [\#(d_{21}, d_{23}) / (1 - \#(d_{21}, d_{23}))] \text{ x } [(v_{21} \text{ x } V_1 + v_{23} \text{ x } V_3) / (V_1 + V_3)] = [0.10 / (1 – 0.10)] \text{ x } [(0.25 \text{ x } 20 + 0.30 \text{ x } 50) / (20 + 50)]$$
$$= [0.11] \text{ x } [(20) / (70)]= 0.11 \text{ x } 0.29 = 0.032$$

$$\#(d_{32}, d_{23}) = 0.15$$
$$\#<d_{32}, d_{23}> = \#(d_{32}, d_{23}) / (1 - \#(d_{32}, d_{23})) \text{ x } [(v_{32} \text{ x } V_2 + v_{23} \text{ x } V_3) / (V_2 + V_3)] = [0.15 / (1 – 0.15)] \text{ x } [(0.40 \text{ x } 30 + 0.30 \text{ x } 50) / (30 + 50)]$$
$$= [0.18] \text{ x } [(27) / (78)] = 0.18 \text{ x } 0.35 = 0.063$$

$$\#<d_{21}, d_{32}, d_{23}> = \#<d_{21}, d_{32}> + \#<d_{21}, d_{23}> + \#<d_{32}, d_{23}> = 0.085 + 0.032 + 0.063 = 0.180$$

The value of Q for contradiction # > 0 is calculated using the formula:

$$Q^{\#} = Q \text{ x } (1 + \# \{ d_{j1}, d_{j2},... d_{jm} \}) \quad (6)$$

In the case of decision no. 5, the value of $Q^{\#}$ calculated when taking into account the internal contradictions of the elementary decisions is:

$$Q_5^{\#} = 32 \text{ x } (1 + 0.180) = 37.76$$

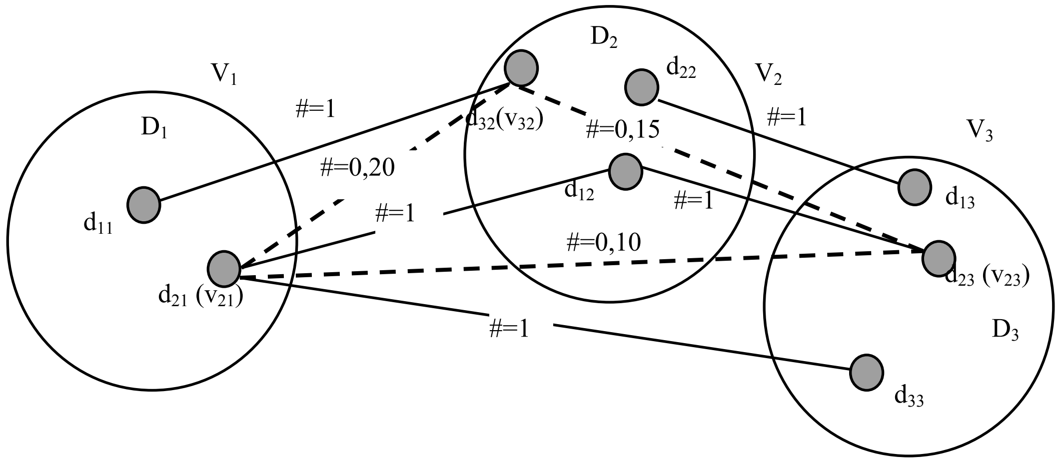

Figure 4. Decision variant model (connections between elementary decisions d_{21}, d_{32}, d_{23} highlighted with a dashed line) against the decision problem model (unbroken line # = 1 – total contradiction; dashed line contradictions within the range [0.10..0.20]; no line # = 0 – no contradiction) (*source: the authors*)

Table 4. Comparison of decision significance weightings for the decision problem example in Fig. 4 taking internal contradictions into account; the table does not give the results $Q^{\#}$ for decisions 3 and 4; grey is used to mark elementary decisions which are in contradictions #; the symbol ↑ marks an expected growth in the decision's significance weighting *(source: the authors)*

Decision no.	Decision	Q	$Q^{\#}$
1	<$\{d_{11}\}$ $\{d_{12}\}$ $\{d_{13}\}$>	50	50,00
2	<$\{d_{11}\}$ $\{d_{12}\}$ $\{d_{33}\}$>	45	45,00
3	<$\{d_{21}\}$ $\{d_{22}\}$ $\{d_{23}\}$>	23	23↑
4	<$\{d_{21}\}$ $\{d_{32}\}$ $\{d_{13}\}$>	37	37↑
5*	<$\{d_{21}\}$ $\{d_{32}\}$ $\{d_{23}\}$>	32	37,76
6	<$\{d_{11}\}$ $\{d_{22}\}$ $\{d_{23}\}$>	33	33,00
7	<$\{d_{11}\}$ $\{d_{22}\}$ $\{d_{33}\}$>	33	33,00

A comparison of values Q and $Q^{\#}$ calculated for the individual decisions makes it possible to choose the most favourable option (see Table 4). An increase in the value of $Q^{\#}$ was noted for decisions 3, 4 and 5, because they contain internal contradictions #.

8 Hierarchical Decision Problems

In general the essence of a decision process is the taking and execution of decisions in an hierarchical system, at the base of which are operational decisions resolved in flat decision problems (their decision areas and elementary decisions are known). At the tactical and strategic levels intermediate tasks are tackled and resolved.

Strategic, tactical and operational decision problems, and the tasks constituting their solution, belong to defined organisational structures and the competences of their members. In other words, hierarchical decision problems are "fastened" to organisational structures in the form of a "competence grid", which causes collisions in the decisions of their accompanying executive processes and the jamming of essential resources (projects). In this situation it is important to have available methodology enabling the coordinated addressing (steering) of decision problems and their resultant tasks for carrying out within the right cells and at the appropriate levels (rungs) of the organisational structure.

The decision-taking process in an hierarchical system is a process conditioned by context: in the superior layer, at the level of the organisation's board of management, decisions depend on business strategy currently being applied, verified by the supervisory board, and on the general state of execution of tasks in intermediate layers and in the operational layer. The taking of decisions in the intermediate layers is determined by the results in the lower layers and the planning decisions of the higher layers.

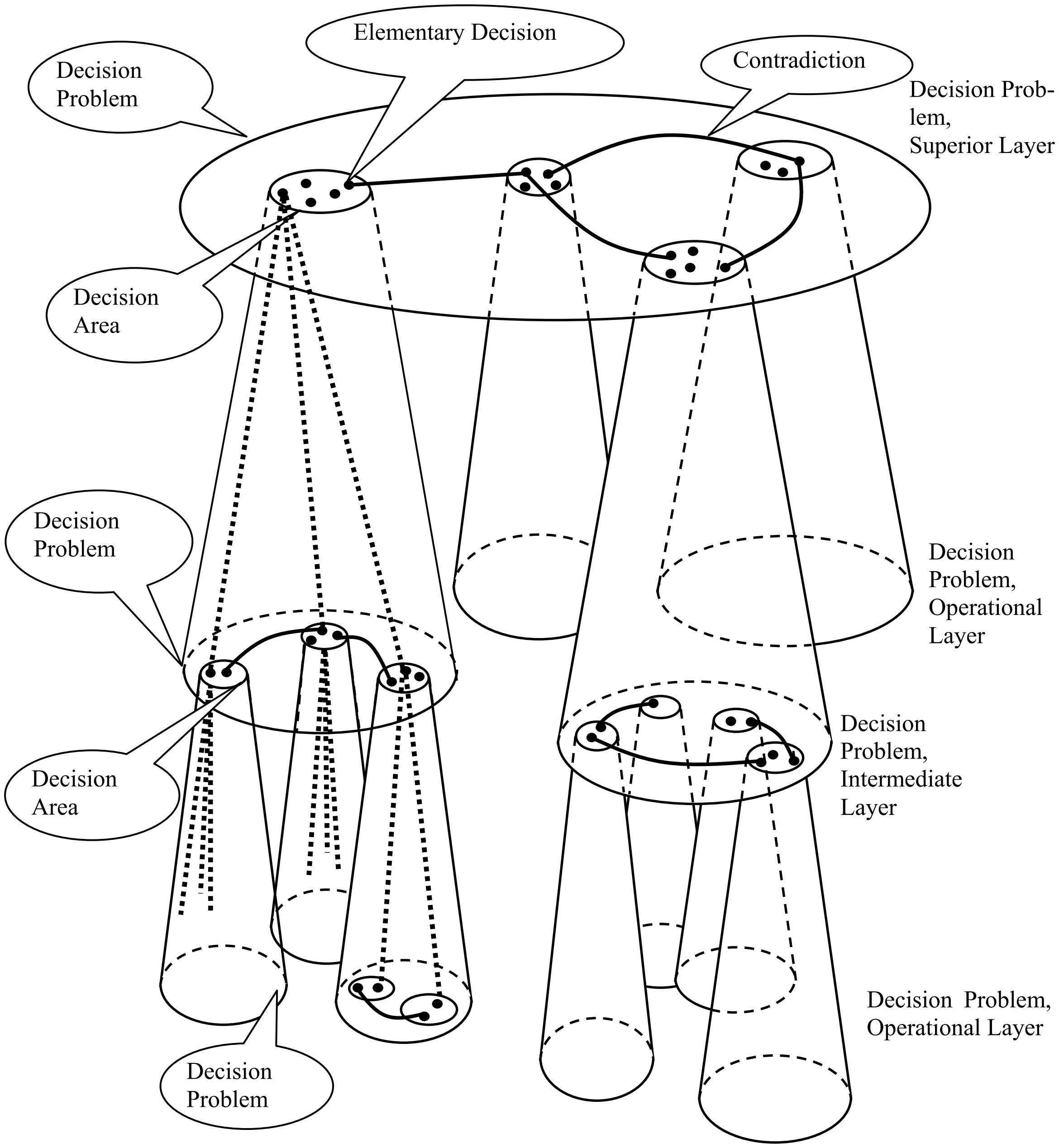

Figure. 5. Decision Problem Model in a Hierarchical System
(*source: the authors*)

In the operational layer the decision-taking is limited by the determinism of its decision areas. A significant parameter of the decision processes is time and the related necessity to keep pace with the coordination of decisions.

The cardinal rule in the construction of hierarchical decision systems (HDS) is to ascribe to each decision area in a higher level a decision problem linked directly to it, the solutions (decisions) of which become the elementary decisions of the decision area in question.

In general, decisions in HDS are taken at a few levels (layers) simultaneously, based on the situation which has arisen in the neighbouring layers. The process of designing and taking decisions depends in such a case on the character of the organisation – although decisions in higher layers are always shaped on the basis of the state of execution of tasks in the lower layers – and symmetrically the tasks in the lower layers are formulated on the basis of the plans and their resultant decisions in the upper layers. We can therefore observe two opposite streams: a decision stream ("top to bottom") and an information stream, about the state of task execution (“bottom to top”). Tasks corresponding to the elementary decisions are carried out and decision-taking processes (processes of associating elementary decisions belonging to different decision areas) take place on each of these layers. Fig. 5 illustrates the mutual positioning of decision problems on the superior (strategic), intermediate (tactical) and operational layers. It is easy to notice that the decision area (containing elementary decisions) of a higher layer becomes the decision problem of the layer positioned directly below.

In a hypothetical 3-level hierarchical system the generation of decision variants is a two-phase process, repeated iteratively, which involves:

- in the first phase:
  - definition of the decision problem and its decision areas, as well as the significance weightings of these areas, on the level of the strategic layer (see Fig. 5),
  - on the level of the tactical layer: the decision problem and its decision areas, as well as the significance weightings of these areas, are defined for each decision area from the strategic layer,
  - on the level of the operational layer: the decision problem and its decision areas, as well as the significance weightings of these areas, are defined for each decision area from the tactical layer,
- in the second phase:
  - on the level of the operational layer: for each decision problem and its decision areas, elementary decisions are defined and pairs of contradictory decisions belonging to different decision areas of the same decision problem are marked, and the weightings of these areas’ elementary decisions are determined; the weightings of the elementary decisions of the operational layer are determined irrespective of the significance weightings of the superior layers, and this is an apriori process,
  - on the level of the tactical layer: for each decision problem and its decision areas, elementary decisions are defined with the help of the decomposition procedures of decision trees corresponding to the decision problems of the operational layer, and pairs of contradictory decisions belonging to different decision areas of the same decision problem are marked in the tactical layer (as in the case of a flat decision problem) and the weightings of these areas’ elementary decisions are determined; the weightings of the elementary decisions of the tactical layer are determined as a result of calculation of the value of the operational level’s decision corresponding to a specific elementary decision in the tactical level; this procedure occurs irrespectively of the significance weightings of the superior layers and is apriori in character in relation to the strategic layer,
  - on the level of the strategic layer – as for the tactical layer – for each decision problem and its decision areas, elementary decisions are defined with the help of the decomposition procedures of decision trees (see fig. 5) corresponding to the decision problems of the intermediate layer, and pairs of mutually contradictory decisions belonging to different decision areas of the same decision problem (as in the case of a flat decision problem) are marked in the layer above, and the weightings of these areas’ elementary decisions are determined; the weightings of the elementary decisions of the strategic layer are determined as a result of calculation of the value of the tactical level’s decision corresponding to a specific elementary decision in the strategic level,
  - above the level of the strategic layer: the decision area comprising the elementary decisions obtained through the decomposition procedure for the decision tree corresponding to the decision problem of the strategic layer is defined, and the elementary decisions of this area are determined; the decision is taken by indicating one of the elementary decisions obtained as a result of decomposition of the decision tree for the strategic layer; taking a decision at the strategic level implies the explicit indication of decisions at all other levels and in all of their decision problems.

Fig. 5 shows with dotted lines how an elementary decision from a superior level (e.g. strategic) is connected to the decision problems and elementary decisions from an intermediate (tactical) or the operational layer.

If the results obtained are for certain reasons dissatisfactory, there may be change in the share of the significance weightings of the decision areas in certain decision problems.

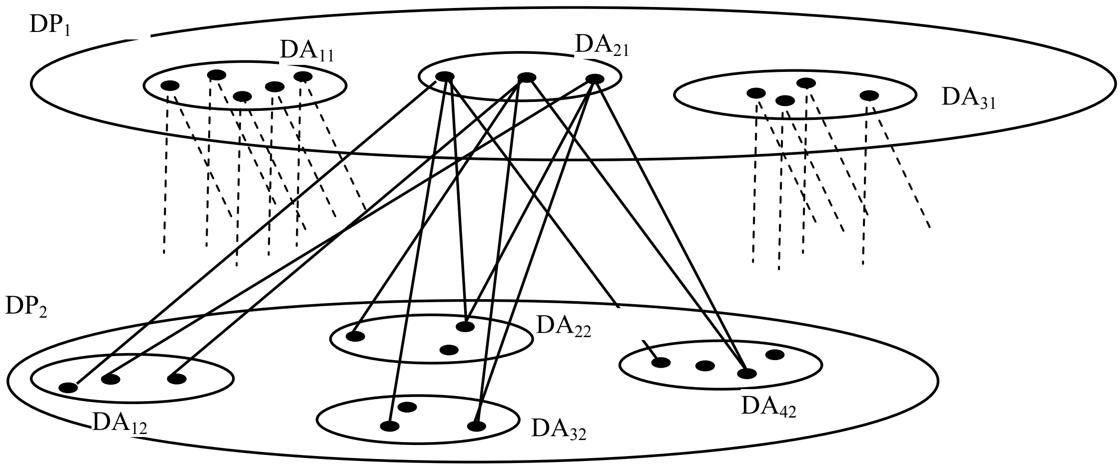

Figure 6. Example of two decision problems in an hierarchical system: DP_1 – superior decision problem, DP_2 – subordinate decision problem (*source: the authors*)

One can also successfully look for decisions whose assessment is situated within limits set in advance, as occurs in the case of flat decision problems.

In the case of settling (resolving) hierarchical decision problems, we are dealing with two types of decision: related to projects and to management, and taken in specific layers of the decision problem model (see Fig. 4 and Fig. 5).

A management decision – within the process guidelines – determines in a specific decision problem alternative variants of processes for the consolidation and/or functioning of structural elements of the object being designed, elements which the design team recommends as acceptable problem solutions. The set of these acceptable solutions becomes a set of elementary decisions for the decision area belonging to the immediately superior decision problem of the design proceedings underway. The same situation occurs in the case of project decisions.

A project decision – within the technical guidelines – determines in a specific decision problem alternative variants of elements which the design team recommend as acceptable solutions for this problem. The set of these acceptable solutions becomes the set of elementary decisions for the decision area belonging to the immediately superior decision problem of the design proceedings underway.

The hierarchical decision problem presented in Fig. 6 features the following properties:

- there are at least two such decision problems (see DP_1 and DP_2), in the first of which there is at least one decision area (see DA_{21}) ensuing as a result of the solving of the second decision problem,
- different scopes of competence among the decision-takers responsible for these problems correspond to different decision problems, in such a manner that the elementary decisions of decision areas belonging to a single decision problem (e.g. decision areas DA_{12}, DA_{22}, DA_{32} and DA_{42} of decision problem DP_2) do not concur, and the decision areas of different decision problems linked directly to one another do not concur; however, the existence of common decision areas for different decision problems not directly linked to one another is permitted (e.g. DA_{22} and DA_{23}, see Fig. 7),
- four decision areas of the subordinate layer's decision problem (in the example used these four areas are: DA_{12}, DA_{22} and $DA_{13} = DA_{23}$) correspond to all the elementary decisions belonging to one decision area of the superior layer's decision problem (e.g. to two elementary decisions from decision area DA_{21} of decision problem DP_1).

Hierarchical decision problems are widespread, and we deal with them in:

- the planning and management of projects, and the designing of complex organisational-technical and business undertakings, such as exchanges and banks,
- the designing of technical objects – such as machinery and complicated technical equipment, such as aircraft, spacecraft and ships,
- the designing of complex property investments such as intelligent buildings and other construction investments, bridges, roads and motorways, and retail chains,
- military and aerospace applications.

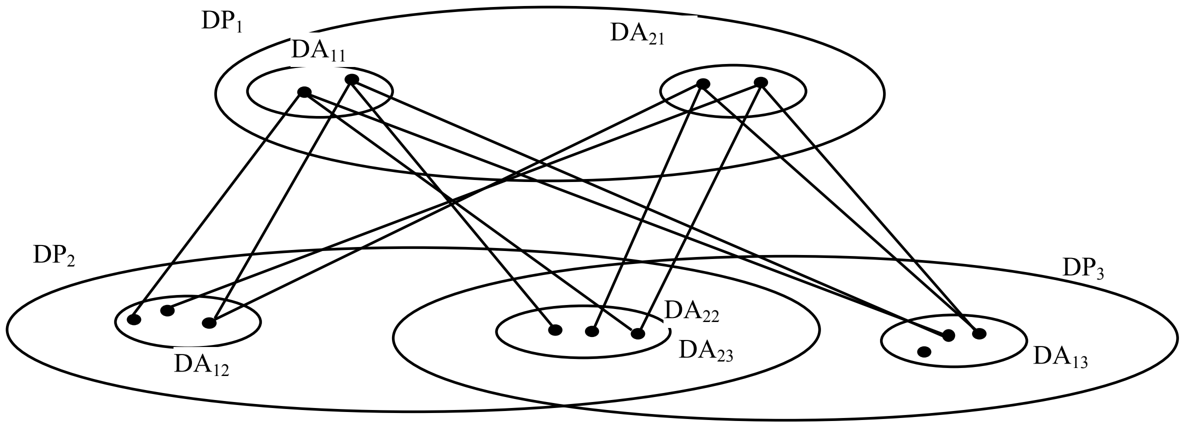

Figure. 7. An example of three decision problems in an hierarchical system: DP_1 – superior decision problem, DP_2, DP_3 – subordinate decision problems with a common decision area marked with two identifiers: DA_{22} and DA_{23}
(*source: the authors*)

9 Practical applications

In a world of rapidly developing informational technologies, the process of taking decisions cannot remain purely intuitive. Artificial intelligence, expert systems, data warehouses and evolutionary programming – these are only examples of the more important activities being carried out with the objective of reinforcing the intellectual and procedural capabilities of mankind. Laying bare the essence of the decision process is not easy – although models of elementary decision acts or more complex decision processes carried out using Decision Support Systems (DSS) seem to be within reach.

We most often understand DSS as methodological and computer solutions which prove useful in those decision situations and problems for which there are no explicit procedures leading to a solution which is correct and effective in all respects.

It was almost half a century ago that researcher at Stanford University developed the first ever expert system, DENDRAL, in 1965. The basic task of the system was to determine the molecular structure of organic compounds based on the analysis of their electromagnetic spectrums. The 1990s saw beginning of the rapid development of the data warehousing technology and OnLine Analytical Processing (OLAP). As a result of advanced methods of artificial intelligence (evolutionary algorithms and neuroidal networks) and the stormy growth in information technologies, semiotic data models are considered above all to be the leading models in decision processes.

As an advanced method of structural and functional research, the AIDA technique may serve as an example of an advanced use of semiotic analyses in at least two different aspects of decision making:

- generating permissible elements in decision space structure D,
- generating the trajectories of a system's functional states in decision space D.The first case, using the AIDA technique, involves the process of decomposing decision areas down to decision variants, from among which the ultimate choice is made. In the second, the AIDA technique serves the "rapid" generation of decision variants interpreted as discrete states of a dynamic system.

The processes of generating the elements and trajectories in space D should be assessed quantitatively and qualitatively. A promising application of the AIDA technique is its usage in situations of large and complex decision spaces, i.e. everywhere where the moment of taking the decision should be preceded by a simulative phase of generating all or "almost all" decision variants.

In certain practical applications one should also count on the AIDA technique – due to the limited time for taking the decision or significant costs of elaborating the decision variants – proving effective in the steered preparation of a significant number of decision variants, among which there should be the locally optimal sub-sets of the variants searched for (this applies in particular to steering tasks in conditions where the variability of a system's parameters is highly dynamic).

Searching for methods of iterative balancing of global (hierarchical) decisions, based on the valuation of flat decisions' significance taking into account the strength of their locally predefined contradictions, invariable remains a long-term direction for research for the example of hierarchical decision problems.

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