## MODELING OF POLISH ENTERPRISES INSOLVENCY PROCESSES WITH THE USE OF GORBATOV CHARACTERIZATION PRINCIPLE – RESEARCH RESULTS

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*Abstract:* Economical activities of enterprises should be based on such managerial decisions that assure quick and effective adjustment of the company to the changes that appear in the market. Enterprises, which are not able to use their opportunities and avoid threats, are bound to face the thread of insolvency. Effects of the insolvency are felt not only by the enterprise, but also by its creditors. Therefore, it is necessary to elaborate a warning system that will beforehand allow diagnosing the condition of the enterprise and setting necessary directions for the company to avoid insolvency. The article presents research results on the use of characterization theory in the creation of insolvency threat evaluation model based on Polish enterprises.

*Key words:* characterization theory, corporate bankruptcy, bankruptcy prediction, corporate crisis management, model of bankruptcy prediction.

### 1 Introduction

Enterprise, which has lost its ability to survive, stands in the face of crisis. It is a common phenomenon in modern economical environment and it affects enterprises on different levels. In case of temporary difficulties, which are overcome, the company will be able to survive and develop its activities. This situation is more dangerous when the management does not see the threat and manage the company towards intensification of the crisis. This results in the thread of insolvency that can lead to bankruptcy.

Results of insolvency (especially the financial problems) are felt not only by the enterprise itself, but also by [1, 11]:

- contractors (suppliers, customers) insolvency directly influences the financial problems and results in bankruptcy of co-operating companies (especially in case of high dependence of subjects on delivered components or sales of finished products),
- creditors (financial institutions, partners) bankruptcy results in problems with execution of debt.
- state bankruptcy of a major economic subject (with considerable market share) can destabilize the functioning of the sector or even the whole economy (e.g. state receives additional cost connected with the increase of unemployment, such as social cost of unemployment, social benefits etc.),

 employees – incoming job reductions can lead to insufficient means of livelihood.

Threat of bankruptcy is also connected with direct and indirect cost of insolvency. Direct cost includes employee and company representative wages, as well as cost of time of managers who manage the company during the bankruptcy process. Indirect cost includes cost of lost sales, revenue and loosing the possibility to receive loans. In terms of direct and indirect insolvency cost one can distinguish [17]:

- administrative costs (direct) resulting from the necessity to perform restructuring process, which is a cost generating mechanism requiring engagement of employees, banks and company representatives,
- cost of lost investment opportunities (indirect) enterprise with financial trouble can have problems with obtaining the capital for new ventures (even more profitable than its basic activities),
- cost of creditor/shareholder conflict (indirect) in the moment of financial crisis creditors are afraid that the company will loose its solvency and invested capital will be lost. On the other hand the shareholders are interested in the increase of market value of the enterprise. That is why managers may be forced to making short-term decisions that please the creditors but ignore the shareholders.

Analysis of the financial and economical condition of the enterprise can be a basis for the model of insolvency threat evaluation. The main objective of such model is to provide an early enough (e.g. allowing taking necessary preventive measures) diagnosis of potential threats (dangers) that can lead to the bankruptcy of the enterprise.

Customers of the insolvency threat evaluation model are the following [8, 11]:

- management board responsible for the constant supervision and evaluation of the financial condition of the enterprise,
- auditors, responsible for e.g. evaluation of financial reports and determination of potential threats for the functioning of the business unit,
- banks, which can evaluate the insolvency threat in the process of granting the loan to the enterprise,
- rating institutions, such as Standard & Poor, Moody's, which can use the models to build enterprise ratings,
- bank guarantee funds, in order to select the banks endangered with insolvency,
- customers or suppliers, to check the financial credibility of the contractors,
- potential investors investing money in stock market or bonds,
- nations and local authorities, to select potential bankruptcies in the national sector,
- judges dealing with insolvency and bankruptcy (increase of correctness of sentences),
- other units e.g. debt selling companies, insurance companies, debt collectors.

### 2 Enterprise insolvency processes – stages

### 2.1 Main causes enterprises bankruptcy

Bankruptcy is not a sudden event that appears overnight – it is the end stage of a process defined as enterprise crisis.

Crisis starts when there is a major difference (endangering the functioning of the company) between the changes in the economical environment and the strategy of market activities as well as the organization of management processes of the enterprise. Such viewpoint is presented in the works of P. Drucker [4], D. Sull [13], E. Urbanowska-Sojkin [15] that determine the "difference" as:

- outdated organization business theory [4],
- active inertia pitfall [13],
- lack of balance between the goals and resources of the enterprise [15].

P. Drucker emphasized that the premises of the crisis situation arise when the business theory in the organization is out of date. According to Drucker the theory of organization consists of three parts [4]:

- assumptions on the organizational external environment: society and its structure, the market, the customers and technology,
- assumptions on the organization's mission, purpose and raison d'etere,
- assumptions on the core competences, the skills and abilities required to accomplish the mission of the organization.

" (...) Assumptions about the external environment determine the source of the organization's benefits. Assumptions about the mission determine the favorable outcome for the organization: in other words, they indicate how it perceives its role in changing the society and economy. Finally, assumptions on the core competences determine the directions necessary to be perfected by the organization, in order to maintain the position of the market leader (...)" [4, p. 34]. When a difference between the goals and the current situation of the enterprise emerges, it usually leads to the crisis in the enterprise.

Similar viewpoint can be found in the works of E. Urbanowska-Sojkin [15], which states that lack of balance between the goals and the resources in the enterprise leads to a crisis. Erroneous perception of the environment and one's position in it results in disturbances of goal formulation and external ability to reach them as well as resources necessary to realize the goals.

Works of D. Sull [13] present the concept of active inertia pitfall (see Figure 1), which describe the situation where once defined success formula is repeated continuously, despite the changing environmental conditions. What is interesting is the fact that in such circumstances the managing board usually intensifies the activities that lead to the success in the past. What is more, managers lack the perspective that would allow a synonymous and neutral evaluation of the success formula.



Figure 1. Pitfall of active inertia (source: [13, p. 94])

Usually the set strategic outline, resources, processes and values of the enterprise are assumed as obvious and the need for change is not noticed. Success formula is understood as the activities that allow the enterprise to come into being and gain a significant market position.

It also is the " (...) unique set of strategic outline (perception of the competitive environment), resources (necessary to gain competitive advantage) processes (ways of operating), relations (permanent relations with external stakeholders and contacts between functional departments), values (inspiring factors, synonymous and modeling the identity of the organization)" [13, p. 90].

Initial success draws customers, investors, imitators. It also assures the managing board that selected strategy is successful and should be reinforced. Such attitude "puts the enterprise managers off their guard" and reassures that current success formula is correct. " (...) Individual elements of the success formula become less flexible: strategic outline changes into a blindfold, resources become a burden, processes are driven by routine, relations change into chains and values into numb doctrines (...)" [13, p. 93].

Changing environment enforces changes in the enterprise. Managing board sees the changes and takes actions. However, the non-flexible structure directs them to the same old tracks. The greater the difference between the environment and the success formula, the more intense actions are undertaken – unfortunately they do not bring expected results. This can lead to the crisis in the enterprise and end in its bankruptcy.

#### 2.2 Stages of insolvency process

Threat of insolvency is a consequence of prolonging crisis within the company. Organization faces bank-ruptcy when all undertaken actions, to deal with the crisis, have failed. Literature presents a four-stage concept of the crisis [10, 18 and 19]:

• Stage 1

Usually concerns situations where the first symptoms of the crisis, generated by internal and external sources, are visible. They are, however, ignored by the management. This results in gradual preservation of the functional inefficiency of the organization.



Figure 2. Developed, multiphase model of the course of crisis situation in an enterprise

(source: [18, p. 44])

#### • Stage 2

Mistakes within the enterprise lead to better threat symptoms visibility than in stage one. Despite this fact management still ignores the threat or takes ineffective /erroneous actions. This results in even more serious functional inefficiency of the organization, which leads to even more serious mistakes and increase of irregularities. This way the enterprise enters stage three, in which significant activity disturbances are revealed.

#### • Stage 3

This is the culmination of the crisis situation development. This moment requires introduction of radical changes, due to symptoms that transformed into the state that threatens future existence of the enterprise. The symptoms are e.g. loss of market position, financial insolvency, dangerous level of loan, significant increase in fixed cost etc. Introduction of changes is based mainly of curative actions.

#### • Stage 4

Final stage that ends with bankruptcy and liquidation of the enterprise.

Works of A. Zelek [18, p. 44] present the concept of developed, multiphase model of the course of crisis situation in an enterprise (see Figure 2). According to A. Zelek, efficient actions taken in the "intervention" stage can save the enterprise form bankruptcy.

On the other hand D. Czajka [3, p. 506] distinguishes three stages of enterprise crisis, where the third stage leads to bankruptcy:

- Initial stage: distribution and production dynamics problem occurs, receivables execution slows down, contradictory decisions accumulate or there is a total lack of decisions. Stock increases or maintains a high level together with decrease or stoppage of sales. Liabilities increase and current assets decrease, first symptoms of financial insolvency occur.
- Intermediate stage: production problems occur with higher frequency, material shortage due to lack of financial resources starts, production process stops to be profitable, quality of products decreases. Receivables are not paid in time and current bank loans limits are used in full. Liabilities are not paid in time and the interest level rises. Some suppliers require to be paid up-front or increased level of prepayments. Receiving a loan is more difficult due to the opinion that the enterprise is not reliable. Wages payment is not regular, highly qualified em-

ployees start to leave the company and go to the competition.

- Final stage: production is stopped, receivables payment slows down, and material assets are being sold out. Enterprise stops paying its liabilities, there are no more financial resources. Creditors apply for the company to receive the bankrupt status.
- 3 Characterization principle use of insolvency threat level evaluation in the construction of the model

# **3.1** Theoretical basis for the characterization principle

"(...) V.A. Gorbatov characterization principle is a part of modern methodology for the systems theory. Main gnoseologic postulates of the characterization principle are:

- characteristics of the solutions rather than the solutions themselves should be sought,
- solution characteristics should relate do the created class representatives (invariants) of equivalent solutions,
- equivalent solutions class is created as a result of input data interpretation of the solved task group of the problem area in the representative solution characteristics categories (...)". [7, p. 190].

"(...) Usually there is less equivalent class solutions than the solutions themselves and the analysis of solution characteristics can be performed without their direct (objective) generation. Characterization theory consists of formal elaboration and methodological verification in the selected characterization theory objective area, main idea of which is based on the mutual interpretability of functioning model of certain object with the model of its structure. Mutual interpretability of models is reached through:

- selection of universal rules of "proper" functioning (expressed in the model of functioning),
- structural (technical) interpretation of the functioning model [7, p. 191].

Universal rules of "proper functioning" are expressed with graph figures determined as [5, 7 and 9]:

• obligatory graph figures – abstract constructions, which, as homeomorphisms, should occur in the model - otherwise it can be incorrect,

- forbidden graph figures easily identifiable objects, which isolation or dispersion (in the functioning model) assures the functional correctness of the object,
- neutral used for functioning model simplification transformations, which do not result in forbidden and obligatory figures.

"(...) Object (resource) will function properly only if a mutually synonymous interpretation between its functioning rules (described with the  $\Psi_a$  functioning model) and the structure that realizes it (described with  $\Psi_b$  structure model) is determined and proven (...)" [7, p. 142]. In order to determine and prove synonymous interpretation of these two models the following assumptions are taken:

- resource functions adequately to its structure,
- resource structure is adequate to its desired way of functioning.

"(...) Basic model of the characterization theory can be described as:

$$\langle \Psi_{a}, \Psi_{b}, P_{0}(\Psi_{a}, \Psi_{b}) \rangle$$
 (1)

where:

 $\Psi_a$  - functioning model,

 $\Psi_{b}$  - structure model,

 $P_0(\Psi_a, \Psi_b)$  – atomic predicate characterizing the interpretation possibility of  $\Psi_a$  functioning model in the categories of  $\Psi_b$  structure model (...)" [9, p.142].

"(...) Practical application of characterization principle to solving the determined group of tasks (problems) requires the elaboration of adequate theory that expresses in detailed determination of  $\Psi_a$ ,  $\Psi_b$  models and  $P_0$  predicate (...)" [9, p. 142].

# **3.2** Characterization principle application methodology in research practice

Search of optimal solution for formulated research problem can be performed on the basis of the Gorbatov characterization principle [5], as part of the stages presented in the Figure 3.

Stage 1 is based on a precise determination of the objective area (research reality) and formulation of the research problem.



Figure 3. Characterization principle application methodology in research practice (source: self study)

As part of stage 2 the following are defined:

- logical predicate (formal recording of the researched problem),
- universal rules of proper functioning in the form of figures: forbidden, obligatory, neutral (usually in the form of graphs).

During stage 3 the defined logical predicate has its functioning model  $\Psi_a$  (resource, object) elaborated, for which its structural interpretation, in the form of structure model  $\Psi_b$  (stage 4), is sought.

Structure model  $\Psi_b$  is a solution for the formulated research problem. The basic condition is to create adequate characterization theory, in the form of basic model of this theory, for the selected subjective area.

# 3.3 Research problem, assumptions, hypothesis, research goal

The following research problem was formulated: two groups of companies are presented: enterprises, which went bankrupt and enterprises, which held their position in the market. Each enterprise makes or made financial statements and reports (balance sheet, cash flow statement). Research method that fulfills the following requirements is sought:

- on the basis of evaluation and comparison of two groups of enterprises will allow to determine the characteristics (described with indicators) of the financial and economical condition of the companies that bankrupted and that held their position in the market,
- on the basis of the financial and economical characteristics of bankrupt enterprises will allow to diagnose the insolvency threat level of any other enterprise,
- on the basis of insolvency threat level diagnosis and comparison with characteristics of enterprises that survived the crisis will allow to determine preventive activities for this process.

The following assumptions were made for such formulated research problem:

 financial reports (balance sheet, cash flow statement, income statement) are a synthetic record of the enterprise's activities,

- results of decisions and mistakes made during management processes are reflected in the financial statements,
- financial indicators, based on financial statements, describe the economical and financial condition of the enterprise,
- insolvency is a process that can be diagnosed on the basis of economical and financial condition changes.

With relation to the research problem and selected assumptions the following research hypotheses were formulated:

- H<sub>1</sub>: set of functional and structural models, describing changes in the economical and financial condition of bankrupt and survivor companies, can be determined with the use of characterization principle,
- H<sub>2</sub>: comparative analysis, which aims at identification of differences in the functioning of two enterprise groups, can be performed on the basis of set of functional and structural models, describing changes in the economical and financial condition of bankrupt and survivor companies,
- H<sub>3</sub>: set of functional and structural models, describing changes in the economical and financial condition of bankrupt and survivor companies, can be used to evaluate the insolvency threat level for any enterprise. Insolvency threat level can be determined on the basis of evaluation of comparison of any enterprise with the set of functional and structural models of selected group of enterprises.

In context of research problem, set assumptions and formulated hypotheses, the following research goal was determined: elaboration of observation models set and insolvency symptoms analysis, described in the form of Gorbatov characterization principle.

### 3.4 Characterization principle application methodology in the solving of the research problem

Characterization principle application methodology in the solving of the research problem formed above (see point 3.3) is based on the elaboration of the theory that, form the economical and financial viewpoint, determines the following:

- functioning models (Ψ<sub>a</sub>) of survivor and bankrupt enterprises,
- structure models  $(\Psi_b)$  of survivor and bankrupt enterprises,
- atomic predicate  $P_0(\Psi_a, \Psi_b)$ , that determines the possibility of functionality model interpretation in the categories of structure model.

Scientific experiments were performed on the basis of financial and economical condition evaluation of survivor and bankrupt enterprises, precisely the changes of financial and economical condition of enterprises over the period of three years. Economical and financial condition of the enterprises was described with a set of indicators used in financial reports analysis.

Figure 4 presents the four main stages of performed research as well as expected results.

Changes of financial and economical condition of enterprises were recorder in the following form:

- Set of logic predicates (stage 1), expected result of which were 3 logic predicates for each enterprise group. One logic predicate corresponded to one financial year. Functions record the changes in the economical and financial status over the researched period (increases and decreases of selected economical and financial indicators). According to the formal recording of the research problem (see Appendix A) the following cases of companies in logic predicate were formulated in the form of disjunction.
- Set of logic predicates graph models (stage 2), expected result of which were 3 logic predicates graph models for each enterprise group. Every logic predicate had one corresponding graph model. Forbidden figures were eliminated from the logic predicate graph models what resulted in functioning graph models.
- Set of functioning graph models (stage 3), expected result of which were 3 functioning graph models for each enterprise group (they represent the common characteristics of the enterprises from the economical and financial viewpoint). Every functioning graph model had one corresponding functioning model.





• Set of structure graph models (stage 4), expected result of which were 3 structure graph models for each enterprise group (these models are the sought solution of set research problem). Every structure graph model had one corresponding structure model.

Graph match index, which allows to determine in what part any enterprise matches the graph functioning (or structure) model of bankrupt enterprises and graph functioning (or structure) model of survivor enterprises, was elaborated. Evaluation of insolvency threat or survival level for the enterprise in upcoming period of 3 years was possible on the basis of comparison of the reached graph match index.

#### 4 Research experiments results

#### 4.1 Research sample

Financial records of 52 manufacturing enterprises that bankrupted in the period of 2000-2004 and 52 enterprises that survived in the period of 2000-2004 were collected for the conduction of this research experiment. The selection of bankrupt and survivor enterprises was selected with the following criteria:

- branch of operation of bankrupt enterprise,
- period (financial year) of the financial statement.

For example if the financial records of a bankrupt clothing company, for the years 2000, 2001, 2002, were gathered than records of a survivor company form the same industry and from the same period of time were gathered. Such selection of the research sample is to eliminate differences between particular company groups (bankrupt, survivor) that can occur due to:

- character of the branch that given company operates in,
- character of the time period (external conditions of the economical environment) that given company operated in.

Elimination of these differences leads to the possibility to compare the experiment results of particular enterprises:

- between enterprises within the same economic branch (bankrupt, survivor),
- between enterprises in different economic branches (bankrupt, survivor).

Research sample was divided in the following manner:

- "learning sample" (42 bankrupt enterprises, 42 survivor enterprises) on its basis the following were constructed for particular company groups: sets of logic predicates, sets of logic predicates graph models, sets of functioning graph models, sets of structure graph models,
- "test sample" (10 bankrupt enterprises, 10 survivor enterprises) – on its basis tests of functioning and structure graph model sets were tested with relation to their "usefulness" in the insolvency threat situation evaluation; the assumption for the measure of functioning and structure graph model sets "usefulness" in the insolvency threat situation evaluation is their "ability" to classify the enterprise as "bankrupt" or "survivor" with the number of erroneous classification not exceeding 40% of the cases (it was assumed that in the test sample in every group for 10 cases up to 4 cases can be wrongly classified).

For every enterprise financial indicators, which indicate changes in the economical and financial condition of the bankrupt and survivor enterprises were calculated on the basis of collected financial statements.

# 4.2 Variables of logic predicate - indicators describing changes in the economical and financial condition

Changes in the economical and financial condition of two enterprises' groups are described with 17 indicators. Indicator set was elaborated on the basis of related literature [2, 6, 12, 14 and 16]. Preparation of economical and financial condition evaluation was possible, on the basis of the indicators, in the aspect of:

- profitability ratios (indicators: return on assets, return on net assets, net profit margin, return on equity),
- efficiency ratios (indicators: fixed assets value, reversed fixed assets turnover ratio, asset turnover ratio, receivables turnover ratio, stock turnover ratio),
- liquidity ratios connected with evaluation of working capital (indicators: current ratio, quick ratio, working capital to sales ratio, working capital in financing of movable assets),
- debt ratios and ability to cover liabilities (indicators: debt ratio, debt to equity ratio, reversed current liquidity ratio, ability to cover debt form cash flow).

Due to the fact that:

- profitability indicators: return on assets, net assets, return on net sales, return on equity,
- working capital indicators: in sales revenues, in financing of moving assets,
- ability to cover debt form cash flow indicator could have positive or negative values – finally (including all remaining indicators) 24 logic predicate variables (F<sub>1</sub>, ... F<sub>24</sub>) were elaborated.

In other words, apart from the change of the indicator in time, selected indicators were analyzed whether their value is positive or negative. For example negative value of working capital indicators means serious problems with financing of current activities (lack of working capital). What is more, if its value decreased in time it corresponded to increase of problems in the enterprise. Additionally if the enterprise had losses and lost its ability to cover debt form cash flow (negative value), it meant that its presence in the market is seriously threatened. Final set of indicators, describing financial and economical condition, and corresponding predicate variables were collected in the Table 1a and Table 1b.

Table 1a. Financial indicators and corresponding predicate variables used in the research (source: own elaboration)

| Description   | Indicator<br>Wsk <sub>i</sub> | Predicate F <sub>i</sub> | Value of the logic predicate F <sub>i</sub>  |  |
|---|-------------------------------|--------------------------|--|--|
| return on assets (positive, negative value)                                   | $Wsk_1$                       | $F_1$                    | $F_1 = 1$ (positive indicator value Wsk <sub>1</sub> )<br>$F_1 = 0$ (negative indicator value Wsk <sub>1</sub> )<br>variable signature $-F_1$                |  |
| return on assets (decrease, increase<br>of value in time)                     | $Wsk_1$                       | $F_2$                    | $F_2 = 1$ (increase of indicator value Wsk <sub>1</sub> )<br>$F_2 = 0$ (decrease of indicator value Wsk <sub>1</sub> )<br>variable signature $-F_2$          |  |
| return on net assets (positive,<br>negative value)                            | $Wsk_2$                       | F <sub>3</sub>           | $F_3 = 1$ (positive indicator value Wsk <sub>2</sub> )<br>$F_3 = 0$ (negative indicator value Wsk <sub>2</sub> )<br>variable signature $-F_3$                |  |
| return on net assets (decrease,<br>increase of value in time)                 | $Wsk_2$                       | $F_4$                    | $F_4 = 1$ (increase of indicator value Wsk <sub>2</sub> )<br>$F_4 = 0$ (decrease of indicator value Wsk <sub>2</sub> )<br>variable signature $-F_4$          |  |
| net profit margin (positive, negative value)                                  | $Wsk_3$                       | $F_5$                    | $F_5 = 1$ (positive indicator value Wsk <sub>3</sub> )<br>$F_5 = 0$ (negative indicator value Wsk <sub>3</sub> )<br>variable signature $-F_5$                |  |
| net profit margin (decrease, increase<br>of value in time)                    | $Wsk_3$                       | $F_6$                    | $F_6 = 1$ (increase of indicator value Wsk <sub>3</sub> )<br>$F_6 = 0$ (decrease of indicator value Wsk <sub>3</sub> )<br>variable signature $-F_6$          |  |
| return on equity (positive, negative value)                                   | $\mathrm{Wsk}_4$              | $F_7$                    | $F_7 = 1$ (positive indicator value Wsk <sub>4</sub> )<br>$F_7 = 0$ (negative indicator value Wsk <sub>4</sub> )<br>variable signature $-F_7$                |  |
| return on equity (positive, negative value)                                   | $\mathrm{Wsk}_4$              | $F_8$                    | $F_8 = 1$ (increase of indicator value Wsk <sub>4</sub> )<br>$F_8 = 0$ (decrease of indicator value Wsk <sub>4</sub> )<br>variable signature $-F_8$          |  |
| fixed assets value (decrease, increase<br>of value in time)                   | Wsk5                          | F9                       | $F_9 = 1$ (increase of indicator value Wsk <sub>5</sub> )<br>$F_9 = 0$ (decrease of indicator value Wsk <sub>5</sub> )<br>variable signature $-F_9$          |  |
| reversed fixed assets turnover ratio<br>(decrease, increase of value in time) | Wsk <sub>6</sub>              | F <sub>10</sub>          | $F_{10} = 1$ (increase of indicator value Wsk <sub>6</sub> )<br>$F_{10} = 0$ (decrease of indicator value Wsk <sub>6</sub> )<br>variable signature $-F_{10}$ |  |
|   |                               |                          | Continued in table $1b \rightarrow$  |  |

| Table 1b Financi  | al indicators and corresponding predicate variables used in the research (cor   | nt)      |
|-------------------|---|----------|
| ruble ro. r maner | the indicators and corresponding prediction variables used in the research (cor | <i>)</i> |
|                   | (source: self study)  |          |

| Description  | Indicator<br>Wsk <sub>i</sub> | Predicate F <sub>i</sub> | Value of the logic predicate F <sub>i</sub>   |
|--|-------------------------------|--------------------------|---|
| asset turnover ratio (decrease,<br>increase of value in time)                              | Wsk <sub>7</sub>              | F <sub>11</sub>          | $F_{11} = 1$ (increase of indicator value Wsk <sub>5</sub> )<br>$F_{11} = 0$ (decrease of indicator value Wsk <sub>7</sub> ,<br>variable signature $-F_{11}$    |
| current liquidity ratio (decrease,<br>increase of value in time)                           | Wsk <sub>8</sub>              | F <sub>12</sub>          | $F_{12} = 1$ (increase of indicator value Wsk <sub>8</sub> )<br>$F_{12} = 0$ (decrease of indicator value Wsk <sub>8</sub> ,<br>variable signature $-F_{12}$    |
| quick liquidity ratio (decrease,<br>increase of value in time)                             | Wsk9                          | F <sub>13</sub>          | $F_{13} = 1$ (increase of indicator value Wsk <sub>9</sub> )<br>$F_{13} = 0$ (decrease of indicator value Wsk <sub>9</sub> )<br>variable signature $-F_{13}$    |
| working capital to sales ratio<br>(positive, negative value)                               | Wsk <sub>10</sub>             | F <sub>14</sub>          | $F_{14} = 1$ (positive indicator value Wsk <sub>10</sub> )<br>$F_{14} = 0$ (negative indicator value Wsk <sub>10</sub> )<br>variable signature $-F_{14}$        |
| working capital to sales ratio<br>(decrease, increase of value in time)                    | Wsk <sub>10</sub>             | F <sub>15</sub>          | $F_{15} = 1$ (increase of indicator value Wsk <sub>10</sub> )<br>$F_{15} = 0$ (decrease of indicator value Wsk <sub>10</sub> )<br>variable signature $-F_{15}$  |
| working capital in financing<br>of movable assets (positive, negative<br>value)            | Wsk11                         | F <sub>16</sub>          | $F_{16} = 1$ (positive indicator value Wsk <sub>11</sub> )<br>$F_{16} = 0$ (negative indicator value Wsk <sub>11</sub> )<br>variable signature $-F_{16}$        |
| working capital in financing<br>of movable assets (decrease, increase<br>of value in time) | Wsk <sub>11</sub>             | F <sub>17</sub>          | $F_{17} = 1$ (increase of indicator value Wsk <sub>11</sub> )<br>$F_{17} = 0$ (decrease of indicator value Wsk <sub>11</sub> )<br>variable signature $-F_{17}$  |
| stock turnover ratio (decrease,<br>increase of value in time)                              | Wsk <sub>12</sub>             | F <sub>18</sub>          | $F_{18} = 1$ (increase of indicator value Wsk <sub>12</sub> )<br>$F_{18} = 0$ (decrease of indicator value Wsk <sub>12</sub> )<br>variable signature $-F_{18}$  |
| receivables turnover ratio (decrease,<br>increase of value in time)                        | Wsk <sub>13</sub>             | F <sub>19</sub>          | $F_{19} = 1$ (increase of indicator value Wsk <sub>13</sub> )<br>$F_{19} = 0$ (decrease of indicator value Wsk <sub>13</sub> ),<br>variable signature $-F_{19}$ |
| debt ratio (decrease, increase of value in time)   | Wsk <sub>14</sub>             | F <sub>20</sub>          | $F_{20} = 1$ (increase of indicator value Wsk <sub>13</sub> )<br>$F_{20} = 0$ (decrease of indicator value Wsk <sub>14</sub> ),<br>variable signature $-F_{20}$ |
| debt to equity ratio (decrease, increase of value in time)                                 | Wsk <sub>15</sub>             | F <sub>21</sub>          | $F_{21} = 1$ (increase of indicator value Wsk <sub>15</sub> )<br>$F_{21} = 0$ (decrease of indicator value Wsk <sub>15</sub> )<br>variable signature $-F_{21}$  |
| reversed current liquidity ratio<br>(decrease, increase of value in time)                  | Wsk <sub>16</sub>             | F <sub>22</sub>          | $F_{22} = 1$ (increase of indicator value Wsk <sub>16</sub> )<br>$F_{22} = 0$ (decrease of indicator value Wsk <sub>16</sub> )<br>variable signature $-F_{22}$  |
| ability to cover debt form cash flow<br>(positive, negative value)                         | Wsk <sub>17</sub>             | F <sub>23</sub>          | $F_{23} = 1$ (positive indicator value Wsk <sub>17</sub> )<br>$F_{23} = 0$ (negative indicator value Wsk <sub>17</sub> )<br>variable signature $-F_{23}$        |
| ability to cover debt form cash flow<br>(decrease, increase of value in time)              | Wsk <sub>17</sub>             | F <sub>24</sub>          | $F_{24} = 1$ (increase of indicator value Wsk <sub>17</sub> )<br>$F_{24} = 0$ (decrease of indicator value Wsk <sub>17</sub> )<br>variable signature $-F_{24}$  |

Evaluation of changes in the economical and financial condition was performed on the basis of the change direction of  $Wsk_1$ , ... $Wsk_{17}$  indicators (decrease, increase – regardless of the value of the change) or their score categorized: positive, negative.

Analysis was performed for every enterprise  $(1 \div 42)$  in every group (bankrupt, survivor) according to the following rules:

- value of the (t<sub>2</sub>) indicator in the second financial year (e.g. 2001) was compared with the value of the (t<sub>3</sub>) indicator in the third financial year (e.g. 2000):
  - if the value of the indicator in the second year was higher than in the third year the evaluation was that its value increased in time,
  - if the value of the indicator in the second year was lower than in the third year the evaluation was that its value decreased in time,
  - if the value of the indicator in the second year was the same as in the third year the evaluation was that the value did not change,
- in case of Wsk<sub>1</sub>, Wsk<sub>2</sub>, Wsk<sub>3</sub>, Wsk<sub>4</sub>, Wsk<sub>10</sub>, Wsk<sub>11</sub>, Wsk<sub>17</sub> indicators additional evaluation was made in the third year and the indicator value was checked whether it is positive or negative (named indicators can be negative if the company in the third financial year: made a loss - Wsk<sub>1</sub>, Wsk<sub>2</sub>, Wsk<sub>3</sub>, Wsk<sub>4</sub>; had a deficiency of working capital - Wsk<sub>10</sub>, Wsk<sub>11</sub>; net loss was higher than the depreciation charge -Wsk<sub>17</sub>),
- value of the indicator in the first financial year (t<sub>1</sub>) (e.g. 2002) was compared with the value of the indicator in the second financial year (t<sub>2</sub>) (e.g. 2001):
  - if the value of the indicator in the first year was higher than in the second year the evaluation was that its value increased in time,
  - if the value of the indicator in the first year was lower than in the second year the evaluation was that its value decreased in time,
  - if the value of the indicator in the first year was the same as in the second year the evaluation was that the value did not change,
- in case of Wsk<sub>1</sub>, Wsk<sub>2</sub>, Wsk<sub>3</sub>, Wsk<sub>4</sub>, Wsk<sub>10</sub>, Wsk<sub>11</sub>, Wsk<sub>17</sub> indicators additional evaluation was made in the second year and the indicator value was checked whether it is positive or negative (named indicators can be negative if the company in the second financial year: made a loss - Wsk<sub>1</sub>, Wsk<sub>2</sub>, Wsk<sub>3</sub>, Wsk<sub>4</sub>; had a deficiency of working capital -Wsk<sub>10</sub>, Wsk<sub>11</sub>; net loss was higher than the depreciation charge - Wsk<sub>17</sub>),

- value of the indicator in the first financial year (t<sub>1</sub>)
   (e.g. 2002) was compared with the value of the indicator in the third financial year (t<sub>3</sub>) (e.g. 2002):
  - if the value of the indicator in the first year was higher than in the third year the evaluation was that its value increased in time,
  - if the value of the indicator in the first year was lower than in the third year the evaluation was that its value decreased in time,
  - if the value of the indicator in the first year was the same as in the third year the evaluation was that the value did not change,
- in case of Wsk<sub>1</sub>, Wsk<sub>2</sub>, Wsk<sub>3</sub>, Wsk<sub>4</sub>, Wsk<sub>10</sub>, Wsk<sub>11</sub>, Wsk<sub>17</sub> indicators additional evaluation was made in the first year and the indicator value was checked whether it is positive or negative (named indicators can be negative if the company in the first financial year: made a loss - Wsk<sub>1</sub>, Wsk<sub>2</sub>, Wsk<sub>3</sub>, Wsk<sub>4</sub>; had a deficiency of working capital - Wsk<sub>10</sub>, Wsk<sub>11</sub>; net loss was higher than the depreciation charge -Wsk<sub>17</sub>).

Therefore the value of a single indicator can have increasing, decreasing or constant (no change) direction. During the analysis an observation was made that the value of particular indicators, in every enterprise (regardless form the group) and in every analyzed time period, was strictly increasing or decreasing. There was no case of a constant indicator in time, that is why the number of logic predicates could be limited to 24 (additional 17 variables would be necessary if the case of constant variable was included).

# 4.3 Construction, interpretation and analysis of logic predicates sets

Logic predicate values table was elaborated on the basis of the analysis described in point 4.2, performed for every enterprise in particular enterprise groups (bankrupt, survivor).

On the basis of the formal research problem solving (see Attachment A) recording, the following was elaborated:

- set of 3 logic predicates describing the group of 42 bankrupt enterprises:
  - $ZB_{t_2-t_3}$  describes changes in the economical and financial condition of enterprises in the time period  $t_2$  in comparison with  $t_3$ ,

- ZB<sub>t1</sub>-t2 describes changes in the economical and financial condition of enterprises in the time period t1 in comparison with t2,
- ZB<sub>t1-t3</sub> describes changes in the economical and financial condition of enterprises in the time period t1 in comparison with t3,
- set of 3 logic predicates describing the group of 42 survivor enterprises:
  - $ZP_{t_2-t_3}$  describes changes in the economical and financial condition of enterprises in the time period  $t_2$  in comparison with  $t_3$ ,
  - ZPt1-t2- describes changes in the economical and financial condition of enterprises in the time period t1 in comparison with t2,
  - ZP<sub>t1-t3</sub>- describes changes in the economical and financial condition of enterprises in the time period t1 in comparison with t3.

Elaborated sets of logic predicates assured full projection of the changes in economical and financial state that occurred in the group of bankrupt and survivor enterprises over the period of 3 years. Successive cases of enterprises were recorded in the form of disjunction.

Exemplary (fragment) logic predicates table was presented in the table 2. Rows include information about successive cases of enterprises columns collect the predicates.

#### Table 2. Exemplary variable logic values for predicates of bankrupt enterprises in the $t_2 - t_3$ period (source: self study)

| (source. self study)    |       |       |                |  |                 |  |  |
|-------------------------|-------|-------|----------------|--|-----------------|--|--|
| Bankrupt<br>enterprises | $F_1$ | $F_2$ | F <sub>3</sub> |  | F <sub>24</sub> |  |  |
| 1                       | 0     | 0     | 0              |  | 0               |  |  |
| 2                       | 1     | 0     | 1              |  | 0               |  |  |
| 3                       | 0     | 1     | 0              |  | 1               |  |  |
|                         |       |       |                |  |                 |  |  |
| 42                      | 1     | 0     | 1              |  | 0               |  |  |

Exemplary form of logic predicate (the part related to data collected in table 2) was presented below.

$$\begin{split} &Z_{t_2-t_3}(F_1,-F_1,F_2,-F_2,F_3,-F_3,...,F_{24},-F_{24}) = \\ &= -F_1 \wedge -F_2 \wedge -F_3 \wedge \ldots \wedge -F_{24} \vee \\ &\vee F_1 \wedge -F_2 \wedge F_3 \wedge \ldots \wedge -F_{24} \vee \\ &\vee -F_1 \wedge F_2 \wedge -F_3 \wedge \ldots \wedge F_{24} \vee \ldots \vee \\ &\vee F_1 \wedge -F_2 \wedge F_3 \wedge \ldots \wedge -F_{24} \end{split}$$

Set of logic predicates should be interpreted in the following way – changes in the economical and financial condition for any enterprise in any selected period indicate that the enterprise:

- no. 1 indicator value was (predicate F<sub>1</sub>) positive (negative) "I",
- no. 1 indicator value (predicate F<sub>2</sub>) decreased (increased) "I",
- no. 2 indicator value was (predicate F<sub>3</sub>) positive (negative) "I",
- no. 2 indicator value (predicate F<sub>4</sub>) decreased (increased) "I",
- ... no. 24 indicator value (predicate F<sub>24</sub>) decreased (increased).

In detail, the set of logic predicates indicates that in given enterprise group:

- in 2nd year in comparison with 3rd year there were changes in the economical and financial condition such as in case of enterprise no 1 (first part of the logic predicate), OR in case of enterprise no 2 (second part of the logic predicate), OR ..., OR in case of enterprise no 42 (forty-second part of the logic predicate),
- in 1st year in comparison with 2nd year there were changes in the economical and financial condition such as in case of enterprise no 1 (first part of the logic predicate), OR in case of enterprise no 2 (second part of the logic predicate), OR ..., OR in case of enterprise no 42 (forty-second part of the logic predicate),
- in 1st year in comparison with 3rd year there were changes in the economical and financial condition such as in case of enterprise no 1 (first part of the logic predicate), OR in case of enterprise no 2 (second part of the logic predicate), OR ..., OR in case of enterprise no 42 (forty-second part of the logic predicate).

Analysis of the elaborated set of logic predicates for the groups of bankrupt and survivor enterprises allowed the formulation of the following conclusions:

 despite of the differences in the functioning of bankrupt and survivor enterprises, in the same period and over the period of 3rd years (parts of the logic predicate vary when it comes to the value of the logic predicate), one can observe some "common characteristics" (e.g. in the group of bankrupt enterprises the logical value of F<sub>1</sub> was usually equal to 0 – recording "- F<sub>1</sub>" indicated that the value of the return on assets ratio was negative what corresponds to financial losses),

- despite of the differences in the functioning of bankrupt and survivor enterprises, one can observe that in the same period of time some cases (parts of the logic predicate) of survivor (bankrupt) enterprises are similar to the bankrupt (survivor) enterprises; this means that there are minor differences in the logic values of predicates in particular parts of the logic predicate; this fact can lead to the following conclusions:
  - survivor (bankrupt) enterprises underwent similar changes in the economical and financial condition – same as in case of bankrupt (survivor) enterprises,
  - totality of economical conditions in macro and micro-environmental scale for the enterprises were similar; however, other factors decided about the final "success" (survival) or "failure" (bankruptcy) of the enterprise,
- one can assume that the "common characteristics" of bankrupt (survivor) enterprises described with logic predicates will be reflected in the structure graph model,
- set of logic predicates is a form of knowledge recording about the changes in the economical and financial condition that occurred in a group of bank-rupt and survivor enterprises.

# 4.4 Construction, analysis and interpretation of functioning graph model sets

Corresponding functioning graph model, with eliminated forbidden figures, was elaborated for all logic predicates. This resulted in the creation of:

- set of 3 functioning graph models that represented (described) the manner of functioning (from the viewpoint of changes in the economical and financial condition) of the group of 42 bankrupt enterprises,
- set of 3 functioning graph models that represented (described) the manner of functioning (from the viewpoint of changes in the economical and finan-

cial condition) of the group of 42 survivor enterprises.

Functioning graph models fully realized the set logic predicate. This means that particular paths in every graph corresponded to the successive parts of the logic predicate. Figure 5 presents an example of functioning graph model of survivor enterprises elaborated on the basis of corresponding logic predicate.

Analysis of functioning graph model sets allowed to draw the following conclusions:

- each graph (tree-shaped) has as many levels as the number of predicate variables, that is 24 levels corresponding to successive predicate variables (F<sub>1</sub>, ... F<sub>24</sub>), which had various logical values in particular paths of the graph,
- every path in a graph has the same length, which means that it connects all 24 vertexes; this corresponds to a single part of ea logic predicate and is compliant with the set economical and financial condition, which is analyzed on the basis of constructed set of indicators,
- every path in a graph corresponds to a single enterprise case; this allows observation and evaluation of differences (characteristics expressed with economical and financial indicators) that occurred between particular enterprises; such differences can indicate "individual" reasons that led to survival or bankruptcy of a company,
- it can be expected that the common characteristics of enterprises will be "connected" in the structure graph models (these characteristics will differ in various enterprises' groups).

Functioning graph model fully corresponds to the logic predicate. That is why the interpretation of particular paths in successive functioning graph model sets should be performed similarly to the one in case of particular parts in the set of logic predicates.

Evaluation of the most frequently used path in the graph (the one taken by the majority of enterprises) was made as part of the functioning graph models analysis. Table 3 collects the most frequent paths in the functioning graph models of bankrupt enterprises.



Figure 5. Example of survivor enterprises functioning graph model that realizes the logic predicate  $ZP_{t_2-t_3}$ (source: self study generated by prototype of IT system)

Analysis of the most frequent paths of bankrupt enterprises allowed formulating the following conclusions:

- "most frequent" path in functioning graph model was identical in every analyzed period; this means that logic values of particular predicate variables are equal,
- enterprises deepened their state of bankruptcy in the successive time periods – increasing number of enterprises is on the most frequent path, what is indicated by the increasing sums values of particular paths,
- enterprise that "found itself" on the frequent path usually characterized with the following changes in economical and financial condition (bankruptcy indicators):
  - in the 3rd , 2nd and 1st year before bankruptcy the company had negative value of the following indicators: return on assets (predicate variable: F<sub>1</sub>), return on net assets (predicate variable: -F<sub>3</sub>), net profit margin (predicate variable: -F<sub>5</sub>), return on equity (predicate variable: -F<sub>7</sub>); this indicates that the enterprise had operating and net losses;

this loss increased what is highlighted by the decreases of the indicators: return on assets (predicate variable:  $-F_2$ ), return on net assets (predicate variable:  $-F_4$ ), net profit margin (predicate variable:  $-F_6$ ), return on equity (predicate variable:  $-F_8$ ) "I",

- fixed assets value decreased (predicate variable: -F<sub>9</sub>) in the following periods, this can mean that the enterprises liquidated its machine park or had increase in the depreciation charge (e.g. through investments in new manufacturing machines), however, losses were so great that they could not assure positive cash flow, calculated as the sum of net profit and depreciation, even if the depreciation charge would increase in time (value of the predicate: F<sub>23</sub> equaled 0 – indicator described with F<sub>23</sub> and F<sub>24</sub> predicates informs about the ability to cover debt form cash flow) "I",
- value of the following indicators decreased in successive time periods: reversed fixed assets turnover ratio (predicate variable:  $-F_{10}$ ) and asset turnover ratio (predicate variable:  $-F_{11}$ ); total analysis of the information allowed to conclude that the pace of revenue decrease and the pace value of fixed assets were similar; what is more, assets were less frequently "renewed" (their turnover dropped) "I",
- value of the following indicators decreased in successive time periods: current liquidity ratio (predicate variable: -F<sub>12</sub>) and quick liquidity ratio (predicate variable: -F<sub>13</sub>); enterprise was loosing solvency "I",
- in the 3rd , 2nd and 1st year before bankruptcy the company had deficiency of working capital (predicate variables:  $-F_{14}$  and  $-F_{16}$ ), that increased in the following periods (predicate variables:  $-F_{15}$  and  $-F_{17}$ ) "I",

| Table 3. Most frequent paths in | the functioning graph | n models of bankrupt | enterprises |
|---------------------------------|-----------------------|----------------------|-------------|
|                                 | (source: self study)  |                      |             |

| pree             | dicate model      | pr               | edicate model          | ate model predicate r |                   |
|------------------|-------------------|------------------|------------------------|-----------------------|-------------------|
|                  | $ZB_{t2-t3}$      |                  | $ZB_{t1-t2}$           |                       | $ZB_{t1-t3}$      |
| Path             | No of enterprises | Path             | Path No of enterprises |                       | No of enterprises |
| -F1              | 27                | -F <sub>1</sub>  | 26                     | -F <sub>1</sub>       | 33                |
| -F <sub>2</sub>  | 16                | -F <sub>2</sub>  | 16                     | -F <sub>2</sub>       | 25                |
| -F <sub>3</sub>  | 15                | -F <sub>3</sub>  | 14                     | -F <sub>3</sub>       | 24                |
| -F4              | 13                | -F4              | 14                     | -F4                   | 23                |
| -F <sub>5</sub>  | 13                | -F <sub>5</sub>  | 14                     | -F <sub>5</sub>       | 23                |
| -F <sub>6</sub>  | 13                | -F <sub>6</sub>  | 14                     | -F <sub>6</sub>       | 23                |
| -F <sub>7</sub>  | 13                | -F <sub>7</sub>  | 14                     | -F <sub>7</sub>       | 23                |
| -F <sub>8</sub>  | 12                | -F <sub>8</sub>  | 14                     | -F <sub>8</sub>       | 22                |
| -F9              | 11                | -F9              | 11                     | -F9                   | 16                |
| -F <sub>10</sub> | 9                 | -F <sub>10</sub> | 10                     | -F <sub>10</sub>      | 13                |
| -F <sub>11</sub> | 6                 | -F <sub>11</sub> | 7                      | -F <sub>11</sub>      | 7                 |
| -F <sub>12</sub> | 6                 | -F <sub>12</sub> | 7                      | -F <sub>12</sub>      | 7                 |
| -F <sub>13</sub> | 6                 | -F <sub>13</sub> | 7                      | -F <sub>13</sub>      | 6                 |
| -F <sub>14</sub> | 5                 | -F <sub>14</sub> | 5                      | -F <sub>14</sub>      | 5                 |
| -F <sub>15</sub> | 5                 | -F <sub>15</sub> | 5                      | -F <sub>15</sub>      | 5                 |
| -F <sub>16</sub> | 5                 | -F <sub>16</sub> | 5                      | -F <sub>16</sub>      | 5                 |
| -F <sub>17</sub> | 5                 | -F <sub>17</sub> | 5                      | -F <sub>17</sub>      | 5                 |
| F <sub>18</sub>  | 5                 | F <sub>18</sub>  | 5                      | F <sub>18</sub>       | 5                 |
| F <sub>19</sub>  | 5                 | F <sub>19</sub>  | 5                      | F <sub>19</sub>       | 5                 |
| -F <sub>20</sub> | 5                 | -F <sub>20</sub> | 5                      | -F <sub>20</sub>      | 5                 |
| -F <sub>21</sub> | 5                 | -F <sub>21</sub> | 5                      | -F <sub>21</sub>      | 5                 |
| -F <sub>22</sub> | 5                 | -F <sub>22</sub> | 5                      | -F <sub>22</sub>      | 5                 |
| -F <sub>23</sub> | 5                 | -F <sub>23</sub> | 5                      | -F <sub>23</sub>      | 5                 |
| -F <sub>24</sub> | 5                 | -F <sub>24</sub> | 5                      | -F <sub>24</sub>      | 5                 |

- value of the following indicators increased in successive time periods: stock turnover (predicate variable: F<sub>18</sub>) and receivables turnover (predicate variable: F<sub>19</sub>), what means that the enterprise was "manufacturing to stock" and had problems with executing of receivables (connected with drop in revenue – see point c) "I",
- value of the following indicators decreased in successive time periods: debt ratio (predicate variable:  $-F_{20}$ ), debt to equity ratio (predicate variable:  $-F_{21}$ ), reversed current liquidity ratio (predicate variable:  $-F_{22}$ ), what corresponds with the necessity to pay up all short and long-term loans and (or) increasing value of moving assets (what is indicated by the increasing value of stock turnover and receivables turnover ratios – stock and receivables are part of the moving assets) "I",
- in the 3rd , 2nd and 1st year before bankruptcy the company had negative value of the following indicator: ability to cover debt form cash flow (predicate variable:  $-F_{23}$ ), what meant that losses of the enterprise were considerable and they were not able to balance the depreciation charges "I",
- value of the following indicator decreased in successive time periods: ability to cover debt form cash flow (predicate variable:  $-F_{24}$ ), what corresponds to the disability to pay the liabilities.

Performed interpretation concerns only the "most frequent" paths in the successive functioning graph models of bankrupt enterprises. This interpretation has its limitations, due to the fact that every case of enterprise is different and not always the selected path led to bankruptcy. Similar analysis was performed on the group of survivor enterprises.

Whereas the comparative analysis of the functioning model sets of bankrupt and survivor companies (especially the most frequent paths) allowed formulation of the following conclusions:

- there are significant differences in the manner of operation of bankrupt and survivor enterprises (differences are visible in the values of successive logic predicates),
- there are slight differences in the most frequent paths among survivor enterprises; this means that

various changes in economical and financial conditions that did not have significant influence on their manner of operation, can occur in this group (the enterprise survived),

similarities in problems, faced both by survivor and bankrupt enterprises, are visible; this means that the totality of functioning conditions for these companies was identical, but only the ones that underwent certain changes in economical and financial condition were able to survive (e.g. increase of stock turnover ratio value is visible in survivor companies but it is connected with the decrease in receivables turnover ratio - despite stock sales problems the survivor companies had better playability of receivables; increase of debt but also possibilities to pay it off can be observed in cases where the stock and receivables turnover ratio increased - logic values of predicate variables F<sub>23</sub> equal 1 and correspond to the positive value of the ability to pay debt from cash flow ratio).

# 4.5 Construction, analysis and interpretation of structure graph model sets

Corresponding structure graph model was elaborated for every elaborated functioning model. Structure graph models are interpreted together with functioning models, which indicates full realization of logic predicate. The following were created as a result of performed experiments:

- set of structure 3-graph models that connect common characteristics of enterprises from the group of 42 bankrupt enterprises,
- set of structure 3-graph models that connect common characteristics of enterprises from the group of 42 survivor enterprises.

Structure graph models are interpreted together with functioning models, which indicates full realization of logic predicate. Mutuality of the interpretation was reached through the use of atomic (transforming) predicate, which is identical for ever functioning model. This indicates that particular paths in every graph correspond to the paths of successive enterprises from the functioning model.

Figure 6 presents an example of a structure graph model of bankrupt enterprises, elaborated on the basis of corresponding functioning model.



Figure 6. Example of a structure graph model elaborated on the basis of corresponding functioning model for the logic predicate  $ZB_{t_2-t_3}$  describing the bankrupt enterprises in  $t_2$  in comparison with  $t_3$  time periods (source: self study generated by prototype of IT system)

Interpretation of particular paths in successive structure graph models should be performed similarly to the functioning graph models – path in the graph describing the changes in the economical and financial condition of the enterprises indicates that it reached:

- positive (negative) value for no. 1 indicator (predicate variable F<sub>1</sub>) "I",
- value for no. 2 indicator (predicate variable F<sub>2</sub>) decreased (increased) "I",
- value for no 3. indicator (predicate variable F<sub>3</sub>) was positive (negative) "I",
- value for no 4. indicator (predicate variable F<sub>4</sub>) decreased (increased) ... ,,I" ... ,

• value for no. 24 indicator (predicate variable F<sub>24</sub>) decreased (increased).

Similarly to the functioning graph models the following can be observed in the structure graph models:

- each graph has as many levels as the number of predicate variables, that is 24 levels corresponding to successive predicate variables (F<sub>1</sub>, ... F<sub>24</sub>), which had various logical values in particular paths of the graph,
- every path in a graph has the same length, which means that it connects all 24 vertexes; this corresponds to a single part of ea logic predicate and is compliant with the set economical and financial

condition, which is analyzed on the basis of constructed set of indicators,

- every path in a graph corresponds to a single enterprise case; this allows observation and evaluation of differences (characteristics expressed with economical and financial indicators) that occurred between particular enterprises; such differences can indicate "individual" reasons that led to survival or bankruptcy of a company.
- the most significant (common) characteristics of the enterprises are located in the upper and lower levels of the structure graph model; this indicates that these characteristics can be the greatest determinants of a survival or a bankruptcy of an enterprise; in other words, the order of appearance of predicates and their logical values is different from the one in functioning graph model (mainly because the main characteristics "merged" – grouped in a form of a path).

Graph match index was elaborated for the need of structure graph models analysis. Index informs about the degree of match (in percentage) of particular path (case of enterprise) to the most frequently used path. In other words, match degree of single enterprise characteristics (changes in economical and financial condition) in relation to the characteristics of the enterprise group.

Average value of the index reached by the selected enterprise, for the set of all structure graph models (within the selected enterprise group), should be interpreted as:

- insolvency threat in three upcoming years (structure models of bankrupt enterprises),
- survival chance in three upcoming years (structure models of survivor enterprises).

Increasing or decreasing index value, between successive graph models, informs about the changes in insolvency threat or survival chance of the enterprise in the upcoming years.

# 4.6 Bankrupt and survivor structure graph models tests in the classification of enterprises

Tests of structure graph models of particular enterprises were performed as part of research experiments. Tests were performed to answer the following questions:

- in what degree the elaborated structure models properly classify (differentiate) the enterprises that originate form different enterprise groups (evaluation on the basis of research sample of 42 cases of bankrupt and survivor enterprises)?
- in what degree the elaborated structure models properly classify the enterprises that do originate from the research sample (evaluation on the basis of 10 enterprises not included in the research sample)?

Tests based on the graph match index were performed to reach an answer for these questions:

- for every case from 42 bankrupt enterprises the graph match index to the structure graph models of survivor companies was calculated (for a selected period of 2nd year in comparison with 3rd year, 1st year in comparison with 2nd year, 1st year in comparison with 3rd year),
- for every case from 42 survivor enterprises the graph match index to the structure graph models of bankrupt companies was calculated (for a selected period of 2nd year in comparison with 3th year, 1st year in comparison with 2nd year, 1st year in comparison with 3rd year),
- for every case from 10 bankrupt enterprises from the test sample the graph match index to the structure graph models of survivor companies was calculated (for a selected period of 2nd year in comparison with 3rd year, 1st year in comparison with 2nd year, 1st year in comparison with 3rd year),
- for every case from 10 survivor enterprises from the test sample the graph match index to the structure graph models of bankrupt companies was calculated (for a selected period of 2nd year in comparison with 3rd year, 1st year in comparison with 2nd year, 1st year in comparison with 3rd year).

In case when the enterprise went bankrupt (or survived) had an average index value in the structure model of survivor (or bankrupt) companies it was classified as "survivor" (or "bankrupt"). The number of wrongly classified companies in relation to 42 enterprises sets the general efficiency of functioning graph model sets as part of particular enterprise group. Table 4 collects the results of performed tests.

|  | Structure               | e models       | Structure models        |                |  |  |  |
|--|-------------------------|----------------|-------------------------|----------------|--|--|--|
|  | of bankrupt enterprises |                | of survivor enterprises |                |  |  |  |
|  | Proper                  | Classification | Proper                  | Classification |  |  |  |
|  | classification          | error          | classification          | error          |  |  |  |
| Research sample (42 cases of bankrupt          | 69%                     | 31%            | 78%                     | 22%            |  |  |  |
| enterprises, 42 cases of survivor enterprises) | (29 enterpr.)           | (13 enterpr.)  | (33 enterpr.)           | (9 enterpr.)   |  |  |  |
| Test sample (10 cases of bankrupt enterprises, | 70%                     | 30%            | 90%                     | 10%            |  |  |  |
| 10 cases of survivor enterprises)              | (7 enterpr.)            | (3 enterpr.)   | (9 enterpr.)            | (1 enterpr.)   |  |  |  |

Table 4. Efficiency of structure graph models in classification of enterprises (research sample, test sample) (source: self study)

The following can be assumed on the basis of tests performed on the research sample:

- overall efficiency of bankrupt enterprises' structure graph models set (classified as bankrupt, survivor on the basis of economical and financial status condition changes) equals 69%; in other words, when the enterprise reaches higher average of graph match index, for bankrupt enterprises structure graph model sets, it can correspond with the manner of functioning of this enterprise group with 69% certainty,
- overall efficiency of survivor enterprises' structure graph models set (classified as bankrupt, survivor on the basis of economical and financial status condition changes) equals 78%; in other words, when the enterprise reaches higher average of graph match index, for survivor enterprises structure graph model sets, it can correspond with the manner of functioning of this enterprise group with 78% certainty.

Test analysis, performed on 10 bankrupt and 10 survivor enterprises that did not originate from the research sample, allowed drawing the following conclusions:

- Efficiency of enterprise classification, based on structure graph model sets, is similar both for the "research sample" and "test sample" and equals approximately 70% (with 69% for "research sample"). Result of this test should be evaluated positively. Whereas, in case of structure models for survivor enterprises the classification efficiency equaled approximately 90%. This can indicate that the sample of 10 survivor companies was strongly correlated with the "research sample".
- Reached result can contain certain error due to relatively small number of enterprises (thus a small diversity of the enterprises) – probably different results would be reached if the "test sample" would be as numerous as the "research sample" – 42 cases of enterprises.

- Average match level, to the structure graph models' set for bankrupt (survivor) enterprises, exceeded 50% in all properly classified enterprises originating from "test sample". Reached result should be evaluated positively. It indicates that changes in economical and financial condition of these companies in a great degree corresponded with bankrupt (survivor) enterprises.
- Graph match index, as an indicator of insolvency threat and survival chance, can be evaluated as sufficient to perform an analysis of any enterprise on the basis of structure graph models' set. This indicates that classification to a particular enterprise group (structure graph model) is, to a great degree, independent of the research sample, on the basis of which the graph models were constructed ("test sample" results greatly emphasize this fact).

### 4.7 IT system prototype

IT system prototype was constructed for this particular research. System allows the detailed analysis of:

- every enterprise that is the part of the "research sample" in the context of:
  - set path in the functioning and structure graph model,
  - match degree to the functioning and structure graph model,
  - comparison of particular enterprises among the group they belong to (e.g. bankrupt enterprises) an between groups (bankrupt, survivor) on the basis of set path and (or) graph match index,
- any enterprise, which can be introduced to the prototype in the following manner:
  - creation of logic predicate based on constructed set of indicators,

- comparison of match degree to the structure graph model of bankrupt and survivor enterprises,
- comparison of the path set for the analyzed enterprise with the "most frequently" used path in the structure graph models' set of bankrupt and survivor enterprises.

Elaborated prototype version can be a tool that supports enterprise management in case of insolvency threat. However, its current functional limitations can be a major issue in practical application. It might be necessary to construct the final version of the IT system.

Figure 7a and Figure 7b present main screenshots form the prototype application.



Figure 7a. Screen of the application prototype with functions adding an enterprise and adding predicate variables to the system
(source: self study - generated by the prototype IT system)



Figure 7b. Screen of the application prototype with generated functioning graph model (source: self study - generated by the prototype IT system)

### 5 Summary

Presented experiment results allow to conclude that structure and functioning graph models' sets, elaborated on the basis of Gorbatov principle, can be used in enterprise management to:

- evaluate the insolvency threat level (or survival chance) in the context of changes in economical and financial condition,
- determine activities, which need to be executed to minimize the insolvency threat (or increase the chance to survive).

Manner of functioning of the model in the management process is presented in the Figure 8:

• identification <1> stage records changes in economical and financial condition of the enterprise (characterization principle), which is described with financial indicators,

- diagnosis <2> stage aims at the evaluation if the economical and financial condition of the enterprise and comparison (confrontation) with the set of functional and structural models elaborated according to the characterization theory; set of structure and functioning models is a knowledge registration form of mechanisms that:
  - enable the enterprises to survive in the market,
  - led the enterprise to bankruptcy,
- conclusions from this stage can be treated as remarks for the future <3> for the management of the enterprise, which needs to implement corrective activities that will prevent insolvency.



Figure 8. Insolvency threat level evaluation model functioning in the management process (*source: self study*)

Contrary to the currently used insolvency threat evaluation models (multidimensional discriminant analysis, logic analysis, artificial neural networks, genetic algorithms), elaborated models allow to:

- evaluate the condition of an enterprise threatened with insolvency on the basis of a more numerous,
- set of economical and financial indicators, including their mutual relations,
- analysis of increasing or decreasing insolvency threat in time,
- evaluation of activities that need to be undertaken in the basis of comparison with structure graph models' set of survivor enterprises.

What is more, elaborated set of functioning and structure graph models' includes changes in the economical and financial condition of bankrupt (or survivor) enterprises. In other words, sets of graph models reveal the reasons that caused bankruptcy or survival of the enterprise.

Authors of this paper have not encountered ay research that would present models able to compare groups of bankrupt and survivor enterprises (what was achieved with the help of Gorbatov characterization principle). Results of performed experiments indicate that the comparative analysis of models elaborated for particular groups can provide information about characteristics of insolvency threat and conditions favoring survival in the market.

#### 6 References

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#### Attachment A

Attachment presents the formal recording of a research problem solution that was presented in point 3.3. This recording was formulated on the basis of Gorbatov characterization principle and was the basis for the experiments described in point 4.

# A1 Formulas describing enterprises and their financial and economical status

Let us consider the formula below:

$$W = B \cup P \tag{a1}$$

where:

W - collection of all n analyzed enterprises W<sub>i</sub>, also:

$$\forall W_{i=1}^{n} W_{i} \in W$$

P – collection of m analyzed enterprises  $P_i$  that survived, also:

$$\forall P_{i=1}^{m} P_{i} \in P$$

B - collection of o analyzed enterprises  $B_i$  that bank-rupted, also:

$$\forall_{B_{i=1}^{\circ}} B_i \in B$$

where n = m + o.

Formula (a1) states that considered set of enterprises is a sum of sets of bankrupt and survivor (still functioning) enterprises.

Every enterprise ( $P_i$  - survivor and  $B_i$  - bankrupt) has financial records, which determine its financial and economical status. This status can be expressed with indicators calculated on the basis of certain positions in the financial statement of the enterprise.

Let us consider the following formulas describing the financial statement of the enterprise and its financial indicators:

$$\forall_{\mathsf{PS}_{\mathsf{t},\mathsf{i}=1}^{\mathsf{l}}}\mathsf{PS}_{\mathsf{t},\mathsf{i}} \in \mathsf{PS}_{\mathsf{t}} \tag{a2}$$

$$SP_t(PS_t) = SP_t(Ps_{t,1}, \dots, Ps_{t,l}) = \Lambda_{i=1}^l Ps_t$$
(a3)

where:

SPt – financial statement for the t period,

 $Ps_{t,i}$  – i position in the financial statement for the t period,

 $PS_t$  – collection of positions in financial statement for the t period,

- t considered financial period,
- 1- number of positions in financial statement.

Formulas (a2) and (a3) represent the following sentence: financial statement for the t period consists of l positions form t period that are included in this period.

Let us consider the (WSK) set of economical and financial indicators (Wsk<sub>i</sub>):

$$\forall_{Wsk_{t,i=1}^{p}}Wsk_{t,i} \in WSK_{t}$$
(a4)

where:

p - number of considered indicators Wski,

t – considered financial period.

Formula (a4) states that every considered economical and financial indicator for particular time period t originates only form the indicator set that was appointed for this period.

Formula that describes the financial indicators has the following form:

$$\forall_{\mathsf{W}sk_{\mathsf{i},\mathsf{i}=1}}^{\mathsf{p}}\mathsf{R}\big(\mathsf{W}sk_{\mathsf{i},\mathsf{j}}\big) \tag{a5}$$

where:

R – projection of considered indicator Wsk<sub>t,i</sub> for the position PS<sub>t</sub> of the financial statement SP<sub>t</sub>; the R projection has the following form:

$$R(WSK_t) = = \{(Wsk_t, PS_t) \in R \text{ for a certain } Wsk_t \in WSK_t\}$$
(a6)

Formula (a5) represents the sentence: every considered indicator  $Wsk_1$  for the t period is a projection of specific positions from the financial statement from this period. Projection R can have different forms according to the considered indicator. It means that direct position from the statement or quotient of positions e.g.Ps<sub>1</sub>/Ps<sub>2</sub>, product e.g. Ps<sub>2</sub>·Ps<sub>3</sub>, combination e.g. Ps<sub>2</sub>·Ps<sub>3</sub>/Ps<sub>4</sub>, also with the use of addition e.g. [(Ps<sub>1</sub> + Ps<sub>2</sub>)/ Ps<sub>4</sub>] or subtraction e.g. [(Ps<sub>1</sub> - Ps<sub>2</sub>)/ Ps<sub>4</sub>].

Let us consider the following formula, which describes economical and financial status (S) of the enterprise in selected period t:

$$S_{t}(WSK) = S_{t}(Wsk_{t,1}, ... Wsk_{t,p}) = \Lambda_{i=1}^{p} Wsk_{t,j}$$
(a7)

Formula (a7) represents the sentence: economical and financial status of an enterprise in considered period t is described with p indicators  $Wsk_{t,i}$  calculated for this period.

Basing on the formulas (a6) and (a7) one can state that the following formula is true:

$$S_{t}(WSK_{t}) = \bigwedge_{i=1}^{p} R(Wsk_{t,i})$$
(a8)

Formula (a8) represents the sentence: economical and financial status (S) in period t is described with a prod-

uct of all financial indicators (Wsk<sub>t,i</sub>) belonging to the WSKt, set based on the (Ps<sub>t,i</sub>)positions of the financial statement (SP<sub>t</sub>) from this period.

Every considered case of enterprise belonging to the W set can be analyzed with indicators (Wsk<sub>t,i</sub>) that describe its economical and financial status in selected period t (formula 7). Therefore the following formula is true:

$$\forall_{W_i} Q(W_i) \tag{a9}$$

where:

Q – projection of and enterprise  $W_i$  belonging to the W set for its economical and financial status  $S_t$ , described with indicators.

 $Q(W_i) = {S_t \in S: (W_i, S_t \in Q \text{ for certain } W_i \in W)}$ 

 $S_t$  – set of economical and financial statuses of enterprises from considered period t.

On the basis of the formula (a9), determining the projection of selected enterprise on its economical and financial status, later in this paper, concept of economical and financial status and enterprise (company) will be used interchangeable.

### A2 Definition and formal recording of logic predicate

It is assumed that in logic predicate  $(Z_t)$ , for the t period, the predicate variable  $(F_{t,i})$  for this period will be the recording of the  $(Wsk_{t,i})$  indicator changes direction in the period t. Predicate variable can assume two logical values: truth (1) or false (0). For the formulated research problem:

- predicate variable F<sub>t,i</sub> in the period t assumes the value 0, when the Wsk<sub>t,i</sub> indicator value in the period t+1 decreased in comparison with period t,
- predicate variable F<sub>t,i</sub> in the period t assumes the value 1, when the Wsk<sub>t,i</sub> indicator value in the period t+1 increased or did not change in comparison with period t

or:

- predicate variable F<sub>t,i</sub> in the period t assumes the value 0, when the Wsk<sub>t,i</sub> indicator value in the period t was negative,
- predicate variable  $F_{t,i}$  in the period t assumes the value 1, when the Wsk<sub>t,i</sub> indicator value in the period t was positive or equaled 0

what can be recorded in the following way:

$$\begin{aligned} \forall_{F_{t,i}} O\bigl(F_{t,i}\bigr) & (a10) \\ F_{t,i} &= \{0,1\} \end{aligned}$$

where:

 $F_{t,i}$  – predicate variable in the period t assuming one of the values (1) or (0),

O – projection of  $Wsk_{t,i}$  indicator change on the predicate variable  $F_{t,i}$ .

$$\begin{split} O\big(F_{t,i}\big) &= \big\{ Wsk_{t,i} \in WSK_t : \big(F_{t,i}, Wsk_{t,i}\big) \\ &\in 0 \text{ for a certain } F_{t,i} \in F_t \big\} \end{split}$$

 $F_t$  – set of considered predicate variables  $F_{t,i}$ .

Assuming that formulas (a7), (a8) and (a9) describe the economical and financial status ( $S_t$ ) of any enterprise ( $W_i$ ) in the period t, one can see that the number of predicate variables is equal to the amount of p considered indicators ( $Wsk_{t,i}$ ).

Including formula (a10), formula (a7) assumes the following form:

$$S_t(F_t) = S_t(F_{t,1}, \dots F_{t,p}) = \Lambda_{i=1}^p F_{t,i}$$
 (a11)

Formula (a11) is the representation of a sentence: economical and financial status of an enterprise in considered period t is described with p predicate variable  $F_{t,i}$  for this period.

On the basis of formula (a11) one can see that theoretically there is  $k = 2^p$  (p – number of considered financial indicators Wsk<sub>t,i</sub>) zero-one possibilities of occurrence of all predicate variables in a logical sentence. For example, if we consider economical and financial status (S<sub>t</sub>) of an enterprise described with p-element set (WSK) of financial indicators (Wsk<sub>i</sub>) than e.g. for the set of 21 financial indicators (predicate variables) we have 2 097 152 theoretical possible states that can describe the enterprise<sup>1</sup>.

Identifying the enterprise  $(W_i)$ , with its status  $(S_t)$  in the period t that describes it, one can see that in the (W) enterprise set there are as many disjunctive economical and financial sates as the number of considered enterprises. That is why the following formula is true:

$$Z_t(S_t) = \bigvee_{i=1}^n W_{t,i}$$
(a12)

On the basis of formula (a12) the following recording of logic predicate was assumed for the bankrupt (B<sub>i</sub>) enterprises was assumed:

$$ZB_t(S_t) = \bigvee_{i=1}^0 B_{t,i}$$
(a13)

where:

o – number of considered bankrupt enterprises' cases  $B_i$ ,

 $ZB_t$  – logic predicate describing bankrupt enterprises in the period t.

Identifying bankrupt enterprise  $(B_i)$  with the economical and financial status that describes it we have:

$$ZB_t(S_t) = \bigvee_{j=1}^{o} \left( \bigwedge_{i=1}^{p} F_{t,i} \right)$$
(a14)

where:

o - number of bankrupt enterprises (B<sub>i</sub>),

p – number of predicate variables ( $F_{t,i}$ ).

Predicate  $(ZB_t)$  represents the set of considered z economical and financial states of bankrupt enterprises  $(B_1, B_2, ..., B_o)$ . Because of differences in functioning of enterprises, every economical and financial state (single enterprise case) is presented in the form of disjunction.

The following logic predicate recording was assumed on the basis of formula (12) for the survivor enterprises  $(P_i)$ :

$$ZP_t(S_t) = \bigvee_{i=1}^m P_{t,i}$$
(a15)

where:

m – number of considered survivor enterprise cases P<sub>i</sub>,

 $ZP_t$  – logic predicate describing survivor enterprises in the period t.

Identifying survivor enterprise  $(P_i)$  with economical and financial status, which describes it, we have:

$$ZP_t(S_t) = \bigvee_{j=1}^m \left( \bigwedge_{i=1}^p F_{t,i} \right)$$
(a16)

where:

m – number of survivor enterprises (P<sub>i</sub>),

p – number of predicate variables ( $F_{t,i}$ ).

Predicate  $(ZP_t)$  represents set of "economical and financial states" of survivor enterprises  $(P_1, P_2, ..., P_m)$ . Every single economical and financial state (single enterprise case) has the form of disjunction, due to differences in the functioning of enterprises.

On the basis of Figure 14 and Figure 16 one can assume that every part of a logic predicate has the length of (p) – includes the same number of predicate variables ( $F_{t,i}$ ). Length of the alternative part results directly form the assumed knowledge representation model - economical and financial state of enterprises is analyzed on the basis of considered set of indicators (set of 17 indicators for which 24 predicate variables were elaborated). Number of predicates for selected group of enterprises is equal to the number of analyzed time periods.

<sup>&</sup>lt;sup>1</sup> The use of Gorbatov characterization principle allows the search of a solution based on the analysis of its characteristics without the need to consider as many possible cases.

For every group of enterprises a set of three logic predicates will be constructed, considering the data form the period of three years.

### A3 Forbidden figures – definition and interpretation

Functioning graph model is created, due to fission of forbidden figures in predicate graph model. Elimination of forbidden figures form the predicate graph model allows such connection of vertexes that:

- do not fit any of analyzed enterprises there is no relevant part in the logic predicate, which can realize the graph connections,
- do not suit the economical and financial status definition (formula (a7)) every enterprise is considered according to a strictly determined set of financial indicators. Therefore, analysis based on a sample of fewer indicators than considered set, is not allowed (it does not provide the full information about the economical and financial status of the enterprise).

Concept of path, which determines the connections between graph vertexes, was defined for the functioning graph model. Path (v) was defined with the following formula:

$$v(M) = \Lambda_{j=1}^{s} M_{j}$$
(a17)

$$\forall_{\mathbf{M}_{i=1}^{\mathbf{r}}}\mathbf{M}_{j} \in \mathbf{M} \tag{a18}$$

where:

 $s = \{1, ..., r\}$  – determines length of the path,

 $M-set \mbox{ of predicate variables } M_j \mbox{ present in a logic predicate.}$ 

Formulas (a17) and (a18) indicate that the path is created due to connection of graph vertexes, which belong to the M set. However, two vertexes to belonging to the same path need to be different, what is described with the formula below:

$$\forall_{M_i M_i} \in v(M) \quad M_i \neq M_j \tag{a19}$$

Also in case of bankrupt companies (B):

$$\forall_{\left(\mathsf{M}_{i},\mathsf{M}_{j}\in\mathsf{v}(\mathsf{M})\right)}\mathsf{T}\left(\mathsf{M}_{i},\mathsf{M}_{j}\right)\wedge\mathsf{U}\left(\mathsf{M}_{i},\mathsf{M}_{j},\mathsf{B}\right)$$
(a20)

where:

T – relation sequence, determining the  $M_j$  element (graph vertex) as the consequent of the  $M_i$  (graph vertex) element in the path v(M),

 $U-M_i,\,M_j$  appurtenance relation to any alternative part of the B collection (bankrupt enterprises).

However, in case of survivor companies (P):

$$\forall_{\left(\mathbf{M}_{i},\mathbf{M}_{j}\in\mathbf{v}(\mathbf{M})\right)}\mathbf{T}\left(\mathbf{M}_{i},\mathbf{M}_{j}\right)\wedge\mathbf{U}\left(\mathbf{M}_{i},\mathbf{M}_{j},\mathbf{P}\right)$$
(a21)

where:

T - relation sequence, determining the  $M_j$  element (graph vertex) as the consequent of the  $M_i$  (graph vertex) element in the path v(M),

U -  $M_i$ ,  $M_j$  appurtenance relation to any alternative part of the P collection (survivor enterprises).

Formulas (a19) - (a21) represent the sentence: for each pair of path successive vertexes there must be an alternative part, which includes these vertexes. Path in a functioning graph model is a directed path and the graph cannot be cyclical.

With the use of the (a17) - (a21) formulas forbidden figures are determined as any paths v(M), which do not fulfill the following formulas:

$$\forall_{M_{i=1}^{s} \in v(M)} \exists_{B_{j} \in B} M_{i} \in B_{j}$$
(a22)

$$\forall_{M_{i=1}^{S} \in v(M)} \exists_{P_{j} \in P} M_{i} \in P_{j}$$
(a23)

$$s = p \tag{a24}$$

 $\exists_{M_i \in v(M)} \{ \bigwedge_{j=1}^t [M_i \in v_j(M)] \} \land O[M_i, M, v(M)] \quad (a25)$ where:

B - set of bankrupt enterprises,

P – set of survivor enterprises,

v(M) – path in a graph,

- M<sub>i</sub> i path vertex in a graph,
- s number of vertexes (M),

p – number of predicates (considered financial indicators),

t – number of considered paths,

O - "relation of common predecessors" determining whether the set of preceding vertexes (starting form the considered vertex  $M_i$ ) in considered paths is identical.

Formulas (a22) - (a25) indicate that:

- every path vertex must belong to at least one alternative part (from the collection of all alternative parts) in logic predicate for any group of enterprises
   formula (a22) and (a23),
- length of the path has to correspond with the length of the alternative part (formula (a24),
- condition of common vertexes has to be met particular vertex is common for considered paths if it belongs to every of them and all preceding in these paths are identical (formula (a25)).

#### A4 Functioning graph model

Functioning model is a graph representation of considered enterprises' "manner of operation", bankrupt and survivor, from the viewpoint of changes in their economical and financial condition. This model is created when forbidden figures are eliminated from the graph model of logic predicate. Elimination of these figures results in the functioning model assuring the proper realization of the logic predicate. Proper functioning model is created for every group of enterprises and every, constructed for it, logic predicate. Construction of functioning model is started form its definition, which in case of bankrupt enterprises is given as the combination:

$$\Psi_{a}(B) = \langle M, B \rangle \tag{a26}$$

In case of survivor enterprises as:

$$\Psi_{a}(B) = \langle M, P \rangle$$
 (a27)  
where:

M – set of predicate variables, with inclusion of their values in the logic predicate:

$$\begin{split} \mathbf{M} &= (\mathbf{F}_{1}, \overline{\mathbf{F}_{1}}, \mathbf{F}_{2}, \overline{\mathbf{F}_{2}}, \mathbf{F}_{3}, \overline{\mathbf{F}_{3}}, \mathbf{F}_{4}, \overline{\mathbf{F}_{4}}, \mathbf{F}_{5}, \overline{\mathbf{F}_{5}}, \mathbf{F}_{6}, \overline{\mathbf{F}_{6}}, \\ \mathbf{F}_{7}, \overline{\mathbf{F}_{7}}, \mathbf{F}_{8}, \overline{\mathbf{F}_{8}}, \mathbf{F}_{9}, \overline{\mathbf{F}_{9}}, \mathbf{F}_{10}, \overline{\mathbf{F}_{10}}, \mathbf{F}_{11}, \overline{\mathbf{F}_{11}}, \mathbf{F}_{12}, \overline{\mathbf{F}_{12}}, \\ \mathbf{F}_{13}, \overline{\mathbf{F}_{13}}, \mathbf{F}_{14}, \overline{\mathbf{F}_{14}}, \mathbf{F}_{15}, \overline{\mathbf{F}_{15}}, \mathbf{F}_{16}, \overline{\mathbf{F}_{16}}, \mathbf{F}_{17}, \overline{\mathbf{F}_{17}}, \mathbf{F}_{18}, \overline{\mathbf{F}_{18}}, \\ \mathbf{F}_{19}, \overline{\mathbf{F}_{19}}, \mathbf{F}_{20}, \overline{\mathbf{F}_{20}}, \mathbf{F}_{21}, \overline{\mathbf{F}_{21}}, \mathbf{F}_{22}, \overline{\mathbf{F}_{22}}, \mathbf{F}_{23}, \overline{\mathbf{F}_{23}}, \mathbf{F}_{24}, \overline{\mathbf{F}_{24}} \end{split}$$

B – set of relations determined with p – element alternative parts for the group of bankrupt enterprises,

P – set of relations determined with p - element alternative parts for the group of survivor enterprises.

One can see that the number (card (M) = r) of possible logic predicates is included in the range:

$$p \le r \le 2^p \tag{a28}$$

### A5 Construction procedure of structure graph model – definition of atomic predicate

Possibility to transform the functioning model to structure model is described with atomic predicate:

$$P_0(\psi_a,\psi_b)$$

where:

 $\Psi_a$  – functioning model of any group of enterprises,

 $\Psi_b$  - structure model of any group of enterprises.

Use of atomic predicate allows determining (on the basis of the functioning model  $\Psi_a$ ) the structure model

 $\Psi_{b.}$  Structure and functioning models are mutually interpretable<sup>2</sup>. Observation and analysis of structure models allows determining common characteristics of enterprise groups form the economical and financial viewpoint. Structure models allow:

- determination of characteristics that indicate bankruptcy (structure models of bankrupt enterprises) or contributed to the survival in the market (structure models of survivor enterprises),
- determination of insolvency threat level for any enterprise (on the basis of match degree of economical and financial condition of an enterprise to the structure model of bankrupt and survivor enterprises),
- determination of necessary activities preventing bankruptcy (on the basis of comparison of structure models of bankrupt enterprises with structure models of survivor enterprises).

Atomic predicate  $P_0$  takes a form of a procedure, which transforms functioning model  $\Psi_a$  into the structure model  $\Psi_b$ . Successive steps of the procedure are:

• Step 1

Set of vertexes is created for any predicate variable  $F_i$  (regardless of its logical value) belonging to different paths v(M) in graph functioning model. All possible two-element subsets, as a combination of vertex pairs ( $F_i$ ,  $F_j$ ), are created on the basis of this set. Every pair is subjected to analysis in step 2.

#### • Step 2

For every pair of vertexes  $(F_i, F_j)$  the compatibility condition, of their logical values and the logical values of their successors in relation to their logical values, is checked. If they are compatible than:

- predecessor of F<sub>i</sub> vertex is connected via a path with vertex F<sub>i</sub>,
- vertex F<sub>i</sub>, with successors is disconnected from its predecessor.
- Step 3

(a29)

Existing connections between vertexes and their predecessors are transformed into a path directed form predecessor to the vertex, after the consideration of all vertex pairs.<sup>3</sup>

W

<sup>&</sup>lt;sup>2</sup> Functioning model and corresponding structure model are created for every logic predicate - 3 functioning and 3 structure models are constructed in case of three considered time periods.

<sup>&</sup>lt;sup>3</sup> Presented procedure leads to the creation of structure graph models without forbidden figures, through connection of common characteristics.