

Height-diameter models for mixed-species forests consisting of spruce, fir, and beech

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ABSTRACT

Height-diameter models define the general relationship between the tree height and diameter at each growth stage of the forest stand. This paper presents generalized height-diameter models for mixed-species forest stands consisting of Norway spruce (*Picea abies* Karst.), Silver fir (*Abies alba* L.), and European beech (*Fagus sylvatica* L.) from Slovakia. The models were derived using two growth functions from the exponential family: the two-parameter Michailoff and three-parameter Korf functions. Generalized height-diameter functions must normally be constrained to pass through the mean stand diameter and height, and then the final growth model has only one or two parameters to be estimated. These “free” parameters are then expressed over the quadratic mean diameter, height and stand age and the final mathematical form of the model is obtained. The study material included 50 long-term experimental plots located in the Western Carpathians. The plots were established 40–50 years ago and have been repeatedly measured at 5 to 10-year intervals. The dataset includes 7,950 height measurements of spruce, 21,661 of fir and 5,794 of beech. As many as 9 regression models were derived for each species. Although the “goodness of fit” of all models showed that they were generally well suited for the data, the best results were obtained for silver fir. The coefficient of determination ranged from 0.946 to 0.948, RMSE (m) was in the interval 1.94–1.97 and the bias (m) was –0.031 to 0.063. Although slightly imprecise parameter estimation was established for spruce, the estimations of the regression parameters obtained for beech were quite less precise. The coefficient of determination for beech was 0.854–0.860, RMSE (m) 2.67–2.72, and the bias (m) ranged from –0.144 to –0.056. The majority of models using Korf’s formula produced slightly better estimations than Michailoff’s, and it proved immaterial which estimated parameter was fixed and which parameters were free.

KEY WORDS

height-diameter models, mixed-species forests, *Abies alba*, *Picea abies*, *Fagus sylvatica*

INTRODUCTION

Height growth of trees is their one of the most characteristic biological feature. It is generally accepted that tree height depends not only on age, diameter, species, and species mixture but also on the quality of the site where the trees grow. Height is usually defined by specific models, where the tree height h is the function of its diameter d :

$$h = f(d) \quad (1)$$

While individual height-diameter models were initially defined using simple mathematical functions, such as polynomial function (Näslund 1929; Assmann 1943; Petterson 1955; Kennel 1965), fractional polynomials (Korsuň 1935; Michailoff 1943) or into the exponential functions (Korf 1939; Freese 1964; Sharma and Parton 2007). More complex functions had to be transformed into linear form (Näslund 1936; Prodan 1944). Schmidt (1967) tested 6 functions and the best results were obtained by Korsuň's function. Kennel (1972) tested the same functions in beech stands in Bavaria and Switzerland. Meanwhile, Sterba and Marschall (1976) tested 8 functions using data from the National Forest Inventory in Austria, and they reported that several functions gave quite satisfactory estimations. Zeide (1989, 1993) analyzed many popular growth equations and claimed that Korf's (1939) equation is substantially more accurate than other growth equations. Its standard error of estimate was 2.1–4.8 times less than the errors of the Chapman-Richrds, Weibul, Gompertz, and logistic equations, respectively.

Height-diameter curves have a special shape and position in each growth stage of forest stands, and these emanate from stage-dependent height-diameter models. In this sense, the steeper curve of the older stand is placed above that of the younger one. It has an elongated interval towards larger-diameter trees, but this is shorter for small-diameter trees. However, the height-diameter curve-shift between growth stages is quite irregular during stand development. It is lower for small-diameter trees in older stands and higher for large-diameter trees in younger stands. These properties of height-diameter models can be generalized in one model, where the tree height h is a function of its diameter d and other stand variables such as either the mean diameter d_m and

height h_m , together with the stand age t , are defined in the following equation:

$$h = f(d, d_m, h_m, t) \quad (2)$$

The models using Equation (2) were constructed sequentially. The models were first fitted by the known functions in formula (1) and these parameters were subsequently estimated using selected stand characteristics as independent variables. One parameter was omitted from fitting and later employed in adjusting the final mathematical formula so that the curves passed through the mean stand diameter and height (d_m , h_m). Kennel (1972) used equation (2) to derive a model from 381 height-diameter curves of beech stands for the 3 thinning intensities – light, moderate, and intensive. For this purpose, he utilized the function provided by Petterson (1955). Šmelko et al. (1987) derived height-diameter models for 12 species using the function published by Michailoff (1943). In other countries, the function is known as Schumacher's (1939), and it was developed on the basis of graphical models published by Halaj (1955). Nagel (1991) derived two models for red oak. One was based on Kennel (1972) and the second on the Sloboda et al. (1993) function, based on Michailoff (1943). Similar to Šmelko et al. (1987), he fitted the regression parameters using mean stand diameter and height as independent variables. Using this same formula, Petráš and Mecko (2005) derived their height-diameter models for poplar clones. After testing 3 height-diameter models, Hui and Gadow (1993) proposed their own model which consisted of three allometric functions. Therein, tree heights are modelled as a function of their diameter and top stand height. In addition, Temesgen and Gadow (2004) developed a generalized height-diameter model using a database of permanent sample plots in British Columbia. This is based on a Weibull-type function, where the relationship between diameter and height is influenced by the relative competitive position of the trees and by stand-density variables. Stankova and Diéguez-Aranda (2013) tested ten generalized and six local height-diameter models for pine forests in Bulgaria.

The tree heights in forest stands serve as the basis for evaluation of their growth and production. This is especially essential in mixed-species forests such as spruce-fir-beech, where these three different growth-type species grow in mutual competition. The objec-

tives of this paper is to develop (i) local height-diameter models and forest stand characteristics for the species; (ii) global height-growth models for spruce-fir-beech mixed forests; and (iii) to evaluate height variability, precision and accuracy of the models using a long term experimental data for spruce-fir-beech forests.

MATERIAL AND METHODS

Study site and empirical data

Empirical data used in this study included 252 repeated measurements in 50 long-term experimental plots (LTP) established in the 1960's and 1970's in the Western Carpathians in Slovakia. The plots were established to study the growth and production of homogeneous and mixed-species forest stands, and they are located in the Poľana Massif (48°36'43''N and 19°36'37''E) and the Slovenské Rudohorie Mts. (48°50'20''N and 20°44'06''E) at altitudes ranging from 480 to 970 m. The sites were classified in *Abieto-Fagetum*, *Fageto-Abietum*, *Fagetum abietino-piceosum*, and *Fagetum quercino-abietinum* forest types, after Zlatník (1959, 1976). The species mixture varied in the plots. All three investigated species were present in 22 LTP, while 23 LTP had spruce and fir admixture, 2 LTP had spruce and beech, and the remaining 3 LTP had fir and beech. Fir was most prominent, followed by spruce and then beech. All the study forest stands were one-layered and even-aged. The stand-age at the plots ranged from 32 to 159 years, and stands were repeatedly measured and treated by light thinning; mainly at 5-year intervals. The majority of the plots were measured 4–8 times, and the stand age ranged from 73 to 202 years after a 40-year period. The area of LTP ranged from 0.200 to 1.215 ha. LTP size, when established, was defined so that it included at least 300 trees. All trees in the plots were numbered, marked and had their diameter at breast-height (DBH) and absolute height measured. Diameter was measured using a calliper with precision of 1 mm for the whole study period, however, height was measured by different hypsometers. First it was Blume-Leiss, later Spiegel-Relaskop and since 2002 it was ultrasonic VERTEX. In addition, the trees' social position and both the stem quality and damage were assessed. Although the height of all trees was measured only on the first and last experimental occasions, tree-

height was also measured between these periods for the trees sampled for height-diameter curve construction.

Model construction

The height curve from equation (1) formed the basis for the general height-diameter model in equation (2). After thorough review of published results (Zeide 1989, 1993; Mehtätalo 2004) and the author's own experience, two functions were selected for the model.

Firstly Michailoff's (1943) function formed equation (3):

$$h(d) = 1.3 + a \cdot e^{\left(\frac{-b}{d}\right)} \quad (3)$$

where d denotes the diameter at breast height, and a and b are the regression parameters to be estimated.

This function has several properties. It has an asymptote, with inflexion point and high flexibility. It is quite simple and has the following two relatively stable parameters; a is the intercept which the largest tree heights approximate and b defines the shape of the height-diameter curve.

Secondly, Korf's (1939) function formed equation (4):

$$h(d) = 1.3 + A \cdot \exp\left(\frac{k}{1-n} \cdot d^{(1-n)}\right) \quad (4)$$

This function has three parameters (A , k , and n) and, according to the author's experience, it is much more flexible than the previous one.

To achieve the requirement that each model curve must pass through the mean stand diameter and height (d_m , h_m) (Šmelko et al. 1987; Nagel 1991; Sloboda et al. 1993; Petráš and Mecko 2005), one parameter must be fixed while the others remain free and are later estimated depending on particular stand variables. Two variants were selected for equation (3). The first had parameter a fixed and b free, depending on the mean stand diameter and height (d_q , h_q). This model has the simpler mathematical form (Šmelko et al. 1987):

$$h(d, d_q, h_q) = 1.3 + (h_q - 1.3) \cdot \exp\left[b \cdot \left(\frac{1}{d_q} - \frac{1}{d}\right)\right] \quad (5)$$

where d_q , h_q – mean stand quadratic diameter and height.

The variability of the parameter b was analyzed in relation with mean stand diameter d_q , mean stand height h_q , and stand age t . Product and sum of two power functions and an exponential function were found as the most appropriate functions for this purpose. Three variants were tested for parameter b :

– Michailoff 1

$$b(d_q, h_q) = p1 + p2 \cdot h_q^{p3} \cdot d_q^{p4} \quad (5.1)$$

– Michailoff 2

$$b(d_q, h_q) = p1 + p2 \cdot h_q + p3 \cdot d_q \quad (5.2)$$

– Michailoff 3

$$b(t) = p1 + p2 \cdot e^{\frac{p3}{t}} \quad (5.3)$$

where: t is the stand age.

The second variant of Michailoff's function had parameter a free and b fixed; in the form:

$$h(d, d_q, h_q) = 1.3 + a \cdot \exp\left[\frac{-d_q}{d} \cdot \ln\left(\frac{a}{h_q - 1.3}\right)\right] \quad (6)$$

where \ln is the natural logarithm.

Two variants were tested for parameter a :

– Michailoff 4

$$a(d_q, h_q) = p1 \cdot h_q^{p2} \cdot d_q^{p3} \quad (6.1)$$

– Michailoff 5

$$a(d_q, h_q) = p1 + p2 \cdot h_q^{p3} \cdot d_q^{p4} \quad (6.2)$$

Korf's function (4) has three parameters; one is fixed with an implicit condition while the remaining two are free and depend on individual stand variables. Where parameter A is fixed and parameters k and n are free, the model has the following form:

$$h(d, d_q, h_q) = 1.3 + (h_q - 1.3) \cdot \exp\left[\frac{k}{1-n} \cdot (d^{(1-n)} - d_q^{(1-n)})\right] \quad (7)$$

Similar to the Michailoff's function, the following equations were tested for parameters k and n :

– Korf 1

$$\begin{aligned} k(h_q, d_q) &= p1 \cdot h_q^{p2} \cdot d_q^{p3} \\ n(h_q, d_q) &= p4 \cdot h_q^{p5} \cdot d_q^{p6} \end{aligned} \quad (7.1)$$

– Korf 2

$$\begin{aligned} k(d_q) &= p1 \cdot e^{\frac{p2}{d_q}} \\ n(d_q) &= p3 \cdot e^{\frac{p4}{d_q}} \end{aligned} \quad (7.2)$$

– Korf 3

$$\begin{aligned} k(t) &= p1 \cdot t^{p2} \\ n(t) &= p3 \cdot t^{p4} \end{aligned} \quad (7.3)$$

where parameter k is fixed, and A and n are free, the model has the form:

$$\begin{aligned} h(d, d_q, h_q) &= 1.3 + \\ &+ A \cdot \exp\left[\ln\left(\frac{h_q - 1.3}{A}\right) \cdot \left(\frac{d}{d_q}\right)^{(1-n)}\right] \end{aligned} \quad (8)$$

where \ln is the natural logarithm.

The following equations were tested for parameters A and n :

– Korf 4

$$\begin{aligned} A(d_q, h_q) &= p1 \cdot h_q^{p2} \cdot d_q^{p3} \\ n(d_q, h_q) &= p4 \cdot h_q^{p5} \cdot d_q^{p6} \end{aligned} \quad (8.1)$$

– Korf 5

$$\begin{aligned} A(d_q, h_q) &= p1 + p2 \cdot h_q^{p3} \cdot d_q^{p4} \\ n(d_q, h_q) &= p5 \cdot h_q^{p6} \cdot d_q^{p7} \end{aligned} \quad (8.2)$$

– Korf 6

$$\begin{aligned} A(d_q, h_q) &= p1 + p2 \cdot h_q^{p3} \cdot d_q^{p4} \\ 1 - n(d_q, h_q) &= p5 + p6 \cdot h_q^{p7} \cdot d_q^{p8} \end{aligned} \quad (8.3)$$

The third variant of the Korf function, where the parameter n would be fixed and the other parameters free was not tested, because the parameter n best de-

termines the shape of the curve. It is desired to have the parameter n free. The mathematical form of the generalized height-diameter models according to formula (2) were derived by adding the equations for the free parameters to the general formula. Here, equations (5.1)–(5.3) were added to model (5), (6.1) and (6.2) to formula (6), (7.1)–(7.3) to (7), and (8.1)–(8.3) to (8).

Heights at the long-term experimental plots

The height-diameter models for each repeated measurement and species were derived using Michailoff's function (3). Parameters a and b were tested and fitted using the stand age t as the independent variable. The following allometric functions were selected as the most relevant:

$$a(t) = a_1 \cdot t^{a_2} \quad (9)$$

$$b(t) = b_1 \cdot t^{b_2} \quad (10)$$

The allometric function is simple and has only 2 parameters, compared to the one proposed by Johann (1990). Addition of the functions to formula (3), where tree height h is a continuous function of the diameter d and age t , provides a model in the following form:

$$h(d, t) = 1.3 + a_1 \cdot t^{a_2} \cdot \exp\left(\frac{-b_1 \cdot t^{b_2}}{d}\right) \quad (11)$$

Regression models were derived for each species and each LTP from formula (11), with parameters a_1 , a_2 and b_1 , b_2 estimated using the Gauss-Newton optimization technique in a non-linear least-squares procedure using QC.Expert (Kupka 2008). The precision and accuracy of the models were then evaluated using root mean square error (RMSE), coefficient of determination (R^2), and bias.

The quadratic mean stand diameter d_q for each species and each repeated measurement was calcu-

lated for each LTP. The mean stand height h_q was calculated using equation (11). Hence, the $(5 \times n)$ matrix was created for each species from experimental data. This comprised 5 columns where mutually related tree characteristics (d , h), mean stand variables (d_q , h_q), and stand age t were listed. The row number (n) presents the number of trees with measured heights. There were 21,661 silver fir, 7,950 spruce and 5,794 beech height measurements.

RESULTS

Height-diameter models at the long-term experimental plots

The height-diameter equations (using equation 11) were derived for individual tree species at the LTP. The coefficient of determination (R^2) for all three species (tab. 1) ranged from 0.555 to 0.970 and the root mean square errors (RMSE) were from 1.07 to 3.72 m. Their arithmetic averages had variation; $R^2 = 0.815$ – 0.870 , and RMSE = 1.99 m for fir, 2.02 m for spruce and 2.49 m for beech. The highest variability and RMSE were in LTP's where low numbers of trees were measured.

The reason for such a high error rate was the wide variability in tree-height in individual forest stands, but some errors may also be due to imprecise height measurement. This is exemplified by the height-diameter models of silver fir from repeated measurements on LTP 83 (fig. 1). Here, differences in tree height at approximately the same diameter ranged from 3 to 5 m. The model precision of RMSE was 1.27 m and $R^2 = 0.962$. However, there was obvious bias between the model and measurements at stand ages of 46 and 79 years, and we assume that this was due to imprecise height measurement rather than the function accuracy. Such phenomena are quite frequent in long-term plots

Table 1. "Goodness of fit" of the height-diameter function using modified Michailoff function (equation 11)

Species	Number of LTP	Number of heights per LTP		R^2 -Coefficient of determination		RMSE-Root mean square error (m)	
		min-max	mean	min-max	mean	min-max	mean
Norway spruce	47	16–856	177	0.555–0.970	0.826	1.07–3.72	2.02
Silver fir	48	34–1,098	470	0.701–0.962	0.870	1.22–3.42	1.99
European beech	27	23–1,177	246	0.601–0.937	0.815	1.45–3.31	2.49

(Kennel 1972, Johann 1990). Where imprecise measurements are less frequent and their identification is possible, it is better to exclude them from modelling or correct them before modelling. In addition, if the model curves are not regularly arranged above one another and it is uncertain which one is wrong, it is better to derive them in one generalized model (Curtis 1967; Pretzsch 2009).

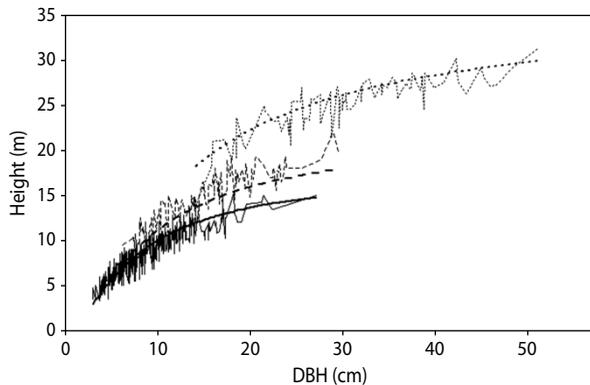


Figure 1. Selected generalized height-diameter models for silver fir based on repeated measurements on LTP 83, at stand ages of 36 years (the first measurement), 46 and 79 years (the last measurement). $R^2 = 0.962$, RMSE = 1.27 m

Species-specific generalized height-diameter models

Although all models were reliable according to "goodness-of-fit" statistics (tab. 2), the most precise estimation was for silver fir. Coefficients of determination ranged from 0.946 to 0.948, RMSE (m) 1.94 to 1.97 and bias (m) -0.031 to 0.063 . The estimation of the parameters for spruce was slightly less precise with an R^2 range of 0.921–0.924, RMSE (m) 2.05–2.12, and bias (m) -0.007 to 0.117 . The precision and accuracy of the estimation for beech was as follows: R^2 from 0.854–0.860, RMSE (m) 2.67–2.72 and bias (m) from -0.228 to -0.056 , which is essentially less precise compared to the previous species. The majority of the models using Korf's function gave slightly better estimates than those from Michailoff's function, irrespective of which parameters were fixed or free. Although the Michailoff (3) and Korf (3) models had free parameters explained only in the relation to stand age, they are inferior to other models because they have an extra variable to be estimated. The residuals ($h-h_{mod}$) greatly varied for each species (fig. 2–4). For the all species the residuals were ± 10 m. However, the majority was up to ± 5 m. The largest residuals were found for trees with DBH between 15 and 40 cm, and

Table 2. Statistical characteristics of the generalized height-diameter models for the three species (gray cell indicates the smallest sum of squares)

Model	Norway spruce				Silver fir				European beech			
	sum of squares	R^2	RMSE (m)	bias (m)	sum of squares	R^2	RMSE (m)	bias (m)	sum of squares	R^2	RMSE (m)	bias (m)
Michaj 1	34,551	0.922	2.085	-0.003	82,769	0.947	1.955	-0.012	41,956	0.857	2.692	-0.103
Michaj 2	34,956	0.921	2.097	-0.006	84,382	0.946	1.974	-0.015	42,043	0.857	2.694	-0.100
Michaj 3	34,130	0.923	2.072	-0.007	81,954	0.948	1.945	-0.031	41,284	0.859	2.670	-0.144
Michaj 4	34,403	0.922	2.081	-0.004	83,015	0.947	1.958	-0.015	42,145	0.856	2.698	-0.102
Michaj 5	34,220	0.922	2.075	-0.003	82,273	0.948	1.949	-0.017	42,010	0.857	2.694	-0.103
Korf 1	33,451	0.924	2.052	0.115	82,924	0.947	1.957	-0.005	41,766	0.858	2.686	-0.063
Korf 2	33,620	0.924	2.057	0.116	82,328	0.947	1.950	0.043	42,758	0.854	2.717	-0.081
Korf 3	33,638	0.924	2.058	0.052	81,940	0.948	1.945	-0.017	41,614	0.858	2.681	-0.228
Korf 4	33,590	0.924	2.056	0.117	82,381	0.947	1.950	0.051	41,676	0.858	2.683	-0.061
Korf 5	35,581	0.924	2.116	0.117	81,191	0.948	1.936	0.057	41,805	0.858	2.687	-0.075
Korf 6	33,494	0.924	2.054	0.109	81,158	0.948	1.936	0.063	41,134	0.860	2.666	-0.056

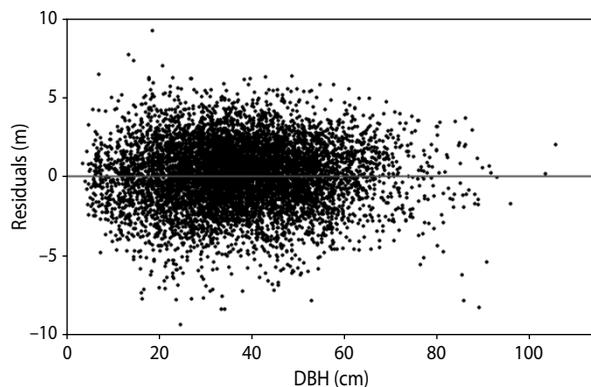


Figure 2. Residuals of Korf model 1 for spruce along tree diameter

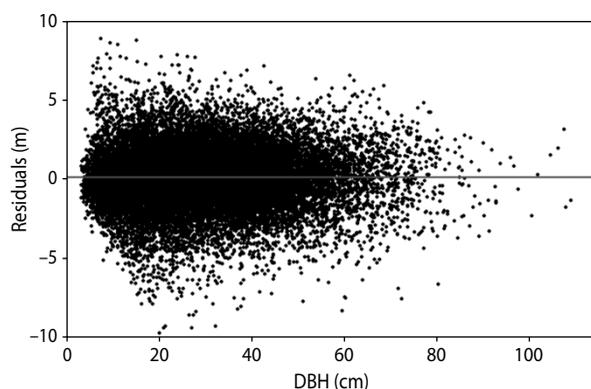


Figure 3. Residuals of Korf model 1 for silver fir along tree diameter

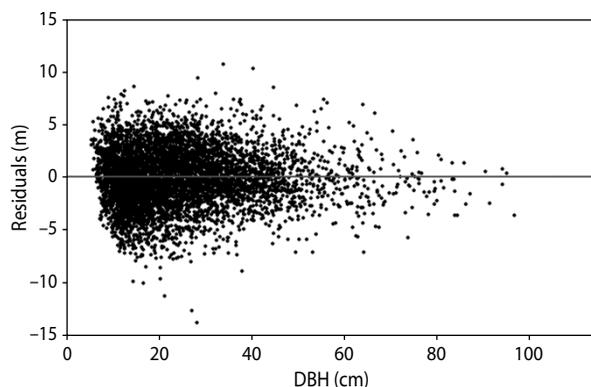


Figure 4. Residuals of Korf model 1 for beech along tree diameter

not for larger trees as expected. In addition, it is important that there is no trend in residuals along DBH, which makes the function applicable. Although the bias was small for all variants, it is important that the

bias is normally distributed throughout the diameter range. Therefore, mean residuals were calculated for the 10 cm diameter classes (tab. 3–5). The residuals were most appropriately distributed for Korf function (1) and Michailoff function (5). Silver fir (tab. 4) had the lowest residuals at ± 0.1 m, most closely approximated to the normal distribution throughout the entire diameter range. Higher bias was found only in trees exceeding 70 cm in diameter, and spruce (tab. 3) and beech (tab. 5) had slightly higher bias than fir trees. Meanwhile, Michailoff's model (5) had similar, but somewhat negatively biased residuals for all species. This was most likely due to less flexibility in the models based on Michailoff's formula.

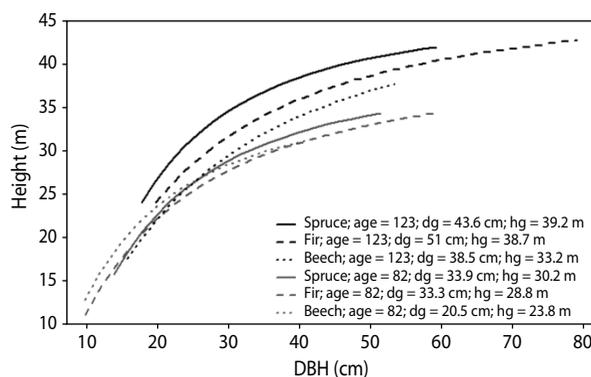


Figure 5. Height-diameter models for spruce, fir, and beech based on Korf function 1 applied to LTP 45 on the first measurement when the stand was 82 years old, and on the last measurement when the stand age was 123 years. (dg and hg are mean stand quadratic diameter and height)

Although all 11 models had relatively similar values for the estimated parameters, according to bias distribution we propose that the optimal model is Korf 1. This was derived from Korf function (4) and contains 3 parameters, A , k and n . Parameter A was fixed using d_q and h_q , while k and n were dependent on the mean stand diameter and height d_q , h_q according to the product of their power (7.1). Since the height-diameter models were constructed for mixed-species forests of spruce, fir and beech with wide diameter and height variability, they were compared using the individual LTP's, rather than simulation stands where all species would have the same mean stand heights and diameters. The height-based dominance of spruce at 82 years of age, and especially 41 years later, is clearly evident for LTP 45 (fig. 5). Although the position and the shape of the models of two

growth stages were symmetrical for spruce and fir, the curves for beech intersected each other. This was most likely due to beech height-growth dynamics, especially

in mixed-species stands. Since the models had quite similar “goodness of fit” characteristics, it may eventuate, that a more optimal model than Korf 1 will be found

Table 3. Bias (m) of the height-diameter models for spruce at the individual diameter classes (gray cell indicates the best model – Korf 1)

N ¹	DBH (cm)	Model Korf 1–6						Model Michailoff 1–5					Šmelko (1987)
		1	2	3	4	5	6	1	2	3	4	5	
172	7.6	-0.10	-0.02	-0.16	0.05	0.07	0.14	0.21	0.54	0.11	0.37	-0.01	0.20
744	15.9	0.06	0.10	-0.02	-0.02	0.00	-0.05	-0.44	-0.51	-0.31	-0.39	-0.33	-0.48
1,607	25.4	0.01	0.02	-0.04	0.02	0.04	0.01	-0.09	-0.15	-0.08	-0.09	-0.03	-0.02
2,246	35.1	0.20	0.20	0.15	0.23	0.23	0.23	0.26	0.26	0.20	0.23	0.22	0.37
1,751	44.8	0.15	0.15	0.08	0.15	0.14	0.13	0.10	0.13	0.07	0.08	0.05	0.15
957	54.3	0.04	0.03	-0.03	0.01	0.00	0.00	-0.18	-0.15	-0.17	-0.18	-0.20	-0.24
327	64.0	0.47	0.44	0.41	0.43	0.40	0.45	-0.02	-0.02	0.10	0.03	0.04	-0.27
102	74.1	-0.10	-0.19	-0.16	-0.13	-0.15	-0.03	-0.91	-0.96	-0.70	-0.76	-0.68	-1.40
37	84.7	-0.70	-0.93	-0.80	-0.73	-0.79	-0.62	-1.96	-2.10	-1.75	-1.67	-1.49	-2.70
5	92.1	-1.25	-1.66	-1.39	-1.32	-1.42	-1.25	-2.95	-3.19	-2.73	-2.49	-2.20	-3.90
2	104.2	1.10	0.66	1.06	1.18	1.10	1.48	-0.89	-1.18	-0.59	-0.31	0.07	-2.14
7,950 ²	37.2	0.12	0.12	0.05	0.12	0.12	0.11	0.00	-0.01	-0.01	0.00	0.00	0.02

Note: ¹ – number of height measurements; ² – last row indicates the sums for columns and overall averages.

Table 4. Bias (m) of the height-diameter models for fir at the individual diameter classes (gray cell indicates the best model – Korf 1)

N ¹	DBH (cm)	Model Korf 1–6						Model Michailoff 1–5					Šmelko (1987)
		1	2	3	4	5	6	1	2	3	4	5	
1,799	7.5	-0.03	0.06	-0.01	0.22	0.02	0.06	0.02	0.09	-0.08	0.18	-0.02	0.57
4,705	15.2	-0.10	-0.02	-0.14	-0.11	-0.01	-0.01	-0.17	-0.21	-0.10	-0.20	-0.10	-0.25
5,382	25.1	0.03	0.10	0.05	0.07	0.14	0.14	0.08	0.06	0.07	0.05	0.06	0.00
4,584	34.9	0.05	0.09	0.04	0.15	0.13	0.14	0.11	0.11	0.03	0.09	0.06	0.11
2,873	44.5	0.02	0.05	-0.02	0.15	0.07	0.08	0.03	0.05	-0.06	0.03	-0.02	0.08
1,444	54.4	-0.06	-0.07	-0.11	-0.03	-0.10	-0.10	-0.17	-0.15	-0.19	-0.15	-0.18	-0.13
583	64.2	0.05	-0.05	0.03	-0.12	-0.12	-0.13	-0.22	-0.23	-0.11	-0.18	-0.14	-0.25
227	74.0	0.10	-0.10	0.12	-0.32	-0.19	-0.22	-0.37	-0.43	-0.11	-0.29	-0.15	-0.50
45	84.1	-0.45	-0.70	-0.35	-1.09	-0.87	-0.89	-1.08	-1.10	-0.63	-0.94	-0.80	-1.30
11	93.9	-0.64	-0.94	-0.65	-1.51	-1.24	-1.25	-1.46	-1.46	-1.06	-1.28	-1.11	-1.75
6	105.7	0.55	0.00	0.61	-0.75	-0.22	-0.29	-0.65	-0.80	0.07	-0.41	0.00	-1.12
2	116.5	1.12	0.55	0.84	0.52	1.01	0.88	-0.11	-0.50	0.08	0.06	0.65	-0.45
21,661 ²	29.8	-0.01	0.04	-0.02	0.05	0.06	0.06	-0.01	-0.02	-0.03	-0.02	-0.02	0.00

Note: ¹ – number of height measurements; ² – last row indicates the sums for columns and overall averages.

Table 5. Bias (m) of the height-diameter models for beech at the individual diameter classes (gray cell indicates the best model – Korf 1)

N ¹	DBH (cm)	Model Korf 1–6						Model Michailoff 1–5					Šmelko (1987)
		1	2	3	4	5	6	1	2	3	4	5	
446	8.5	0.50	0.57	0.52	0.38	0.30	0.56	0.43	0.39	0.71	0.33	0.38	-1.13
2,188	15.0	-0.34	-0.31	-0.54	-0.32	-0.31	-0.36	-0.38	-0.36	-0.47	-0.35	-0.36	-1.57
1,589	24.7	0.10	0.06	-0.12	0.10	0.09	0.10	0.07	0.07	-0.10	0.08	0.08	-0.08
878	34.3	0.16	0.06	-0.03	0.12	0.09	0.15	0.13	0.11	0.07	0.10	0.11	0.78
417	44.3	-0.12	-0.20	-0.24	-0.13	-0.16	-0.07	-0.17	-0.19	-0.07	-0.21	-0.19	1.23
142	54.3	-0.07	-0.12	-0.05	0.00	0.02	0.00	-0.18	-0.14	0.18	-0.18	-0.20	1.82
80	63.8	-0.46	-0.57	-0.49	-0.34	-0.33	-0.38	-0.56	-0.53	-0.13	-0.56	-0.57	1.91
36	74.8	-0.60	-0.60	-0.24	-0.30	-0.27	-0.57	-0.51	-0.46	0.12	-0.42	-0.37	2.04
12	83.3	-0.86	-0.86	-0.42	-0.48	-0.40	-0.91	-0.78	-0.66	0.09	-0.61	-0.65	2.10
6	93.2	-0.87	-1.04	-1.19	-0.39	-0.01	-0.52	-1.83	-1.38	-0.18	-1.54	-1.93	3.18
5,794 ²	24.4	-0.06	-0.08	-0.23	-0.06	-0.08	-0.06	-0.10	-0.10	-0.14	-0.10	-0.10	-0.40

Note: ¹ – number of height measurements; ² – last row indicates the sums for columns and overall averages.

using different datasets. However, if this occurred there is a high probability that such a model would be part of the Korf function-family. Although the models using Michailoff function were relatively satisfactory, they were less flexible for mixed-species stands with wide diameter and height variability, because they have simpler shape and less number of regression parameters to be estimated.

Regression functions of the height-diameter curves express tree height depending on their diameter, mean stand diameter and mean stand height. Although the regression parameters were estimated with high precision and the explained variability was rather high (85–95%), there is still question if there are other factors (for instance social position of trees, stand density, stand mixture, etc.) that would significantly contribute to explaining the variability in tree heights.

Comparison between the height-diameter models

The models derived in this study were not evaluated using an independent dataset. They were, however, compared to the existing models developed by Šmelko et al. (1987) and derived using the graphical height-diameter curves published by Halaj (1955). For model comparison and to avoid possible systematic errors in our model, Šmelko's model was tested by the empirical ma-

terial used in this study. Residuals between measured height and those predicted by the Šmelko model were calculated, together with bias for average tree diameter (tab. 3–5) and RMSE. Biases (m) for fir (tab. 4), spruce (tab. 3), and beech (tab. 5) were 0.004, 0.018 and -0.404 and RMSE (m) were 1.99, 2.11 and 3.06, respectively. It is evident from the average bias that the models had systematic errors for all species. For spruce and fir, the predicted heights were systematically higher than the measured ones for tree diameters exceeding 55 cm. For beech, the predicted heights were higher when diameters were less than 25 cm and they were lower in larger sized trees, where differences ranged from 2–3 m in the largest trees.

Temesgen and Gadow (2004) derived 10 height-diameter models for each of 8 species in British Columbia on permanent sample plots. The models were based on the three-parameter function (Yang et al. 1978). Two parameters involved independent variables such as the basal area of larger trees, the number of trees per hectare and basal area per hectare. The lowest RMSE (m) 1.44 was found for cedar, but other species' models recorded RMSE from 1.95 to 2.44. The spruce model had RMSE of 2.34 m and bias of 0.101 m, and therefore higher values than our results. The original models developed for spruce in British Columbia were not subjected to testing for our dataset as it was done for the model derived by

Šmelko et al. (1987) because it used different independent variables for model generalization.

DISCUSSION

It is generally known that the tree height depends not only on the diameter, age, species and site, but also on the mixture of species in the stand. Most of the studies that have been conducted so far have proposed such models for poor forests, and most of growth models developed for Central European conditions included the relationships in poor forests: BWIN (Nagel 1999, 2009), Moses (Hasenauer 1994; Kindermann and Hasenauer 2005), Prognaus (Hasenauer and Monserud 1996; Monserud and Sterba 1996; Nachtmann 2006), Silva (Pretzsch 1992; Kahn 1995) and SIBYLA (Fabrika 2005). Only recently, research has been focusing on mixed-species forests, and several studies proposed height-diameter models for mixed-species forests (e.g. Temesgen and Gadow 2004). In our study, we developed height-diameter models for spruce-fir-beech forests in the Western Carpathians. However, the three species did not occur on each plot, and the forests were mixed with different combination of the species. Co-occurrence of the all species was found only on a half of the plots. The co-occurrence of fir and spruce was found also on a half of the plots. Despite these shortcomings the dataset includes stands with the age from 30 to 200 years. Another advantage is that trees were repeatedly measured within the 40–45-year period. This help in evaluating outliers or imprecise height measurements of individual trees (Curtis 1967; Pretzsch 2009).

We used own experiences as well as published knowledge for the selection of the mathematical formulas. From among many, only two were identified as most appropriate for the study: two-parameter Michajloff's (1943) and three-parameter Korf's (1939) function. The criterion that the curve passes through the one point defined by mean diameter and height was implemented for the study. The advantage of this approach compared to the approach where the point would be defined by the top diameter and height was a higher precision and a lower bias in tree volume calculation. After applying the criterion, the number of regression parameters was reduced to only one (Michajloff 1943) or two (Korf 1939). If the parameters are expressed depending on some stand

characteristics, it defines a final shape and position of the model curve. Tree diameter, mean stand diameter and height are the parameters that rather precisely determine the tree height. Stand age as the other independent parameter did not improved significantly the performance of the height-diameter models. An influence of other factors, for example top stand diameter and height (Hui and Gadow 1993; Stankova and Diéguez-Aranda 2013) would probably not contribute to increase the precision of the models. The influence of the stand density and stand mixture on tree height can be, however, discussed. Temesgen and Gadow (2004) and Stankova and Diéguez-Aranda (2013) expressed the stand density by the number of trees per hectare. They based their models on the knowledge that the trees growing in more dense stands have usually higher heights. In our case, when the stands are consisted of both broadleaved and coniferous species, intra-species competition is compensated by inter-species competition.

CONCLUSIONS

Tree height is a very important variable characterizing stand structure. It provides an essential basis for tree volume calculation and site quality assessment. Although it has rather wide variability, it can be defined by a particular height-diameter model at a specific stand growth stage. Height-diameter models of one or several stands can be organized in a mathematical model which generalizes the growth of trees and forest stands at particular stages throughout their lifespan. The generalized height-diameter models were derived from a great amount of experimental data, including height measurements on 50 long-term experimental plots established in spruce-fir-beech mixed-species forest stands. Five models based on Michajloff's function and six models based on Korf's function were tested for each species. Each model was adjusted so that the curves were constrained to pass through the mean stand height and diameter. Although tree height varied greatly within stands, all models had rather similar "goodness of fit" characteristics. The lowest RMSE errors of 1.94–1.97 m were found for silver fir, while spruce had 2.05–2.12 m and beech registered 2.67–2.72 m. These findings correspond to biological knowledge of the studied species. The final generalized model explained 95% of height

variability in silver fir, 92% in spruce, and 85–86% in beech. The remaining unexplained variability is presumed to be due to factors not included in the models for various reasons. In these mixed-species forests, this may have been due to the horizontal and vertical distribution of the trees. However, it is difficult to explain this effect of horizontal and vertical arrangement in these three ecologically different species, not only at the time of measurement but also prior to measurement.

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REFERENCES

- Assmann E. 1943. Untersuchungen über die Höhenkurven von Fichtenbeständen. *Allgemeine Forst- und Jagdzeitung*, 119, 77–88.
- Curtis R.O. 1967. Height-diameter and height-diameter-age equations for second-growth Douglas-fir. *Forest Science*, 13, 365–375.
- Fabrika M. 2005. Simulátor biodynamiky lesa SIBY-LA. Koncepcia, konštrukcia a programové riešenie. Habilitačná práca, Technická univerzita vo Zvolene.
- Freese F. 1964. Linear regression methods for forest research. US Forest Service Research Paper, Forest Products Laboratory, Washington.
- Halaj J. 1955. Tabuľky jednotných hmotových kriviek pre určovanie hmoty porastov. Štátne pôdohospodárske nakladateľstvo, Bratislava.
- Hasenauer H. 1994. Ein Einzelbaumwachstumssimulator für ungleichaltrige Fichten- Kiefern- und Buchen-Fichtenmischbestände. Forstliche Schriftenreihe Universität für Bodenkultur Wien 8, Österreichische Gesellschaft für Waldökosystemforschung und experimentelle Baumforschung, Wien.
- Hasenauer H., Monserud R.A. 1996. A crown model for Austrian forests. *Forest Ecology and Management*, 84, 49–60.
- Hui G.Y., Gadow K.V. 1993. Zur Entwicklung von Einheitshöhenkurven am Beispiel der Baumart *Cunninghamia lanceolata*. *Allgemeine Forst- und Jagdzeitung*, 164, 218–220.
- Johann K. 1990. Adjustierung von Bestandeshöhenkurvenscharen nach der Methode des Koeffizientenausgleichs. In: Meth Proposal Working Group “Auswertemethodik bei langfristigen Versuchen“ der Sek Ertragskd Dt Verb Forstl Forschungsanstalten, München.
- Kahn M. 1995. Die Fuzzy Logik basierte Modellierung von Durchforstungseingriffen. *Allgemeine Forst- und Jagdzeitung*, 166 (9/10), 169–176.
- Kennel R. 1965. Untersuchungen über die Leistung von Fichte und Buche im Rein- und Mischbestand. *Allgemeine Forst- und Jagdzeitung*, 136, 149–161, 173–189.
- Kennel R. 1972. Buchendurchforstungsversuche in Bayern von 1870 bis 1970. Forschungsberichte Nr. 7, Forstliche Forschungsanstalt, München.
- Kindermann G.E., Hasenauer H. 2005. Zusammenstellung der Oberhöhenfunktionen für die wichtigsten Baumarten in Österreich. *Austrian Journal of Forest Science*, 122 (4), 163–184.
- Korf V. 1939. Příspěvek k matematické definici vzrůstového zákona hmot lesních porostů. *Lesnická práce*, 18, 339–379.
- Korsuň F. 1935. Život normálního porostu ve vzorcích. *Lesnická práce*, 14, 289–300.
- Kupka K. 2008. QC.Expert, ADSTAT. User’s manual, TryloByte, Ltd., Pardubice.
- Lang A. 1938. Bestandes-Einheitshöhenkurven der Württ. Forsteinrichtungsanstalt. *Allgemeine Forst- und Jagdzeitung*, 114, 168–176.
- Mehätälö L. 2004. A longitudinal height-diameter model for Norway spruce in Finland. *Canadian Journal of Forest Research*, 34 (1), 131–140.
- Michailoff I. 1943. Zahlenmässiges Verfahren für die Ausführung der Bestandeshöhenkurven. *Forstwissenschaftliches Centralblatt – Tharandter Forstliches Jahrbuch*, 6, 273–279.
- Monserud R.A., Sterba, H. 1996. A basal area increment model for individual trees growing in even- and unevenaged forest stands in Austria. *Forest Ecology and Management*, 80, 57–80.
- Nachtmann G. 2006. Height increment models for individual trees in Austria depending on site and competition. *Austrian Journal of Forest Science*, 123, 199–222.

- Nagel J. 1991. Einheitshöhenkurvenmodell für Roteiche. *Allgemeine Forst- und Jagdzeitung* 162 (1), 16–18.
- Nagel J. 1999. Konzeptionelle Überlegungen zum schrittweisen Aufbau eines waldwachstumkundlichen Simulationssystems für Norddeutschland. Schriften aus der forstlichen Fakultät der Universität Göttingen 128, Paul Parey, Frankfurt.
- Nagel J. 2009. Waldwachstumssimulation mit dem Java Software Paket TreeGross. Nordwestdeutsche Forstliche Versuchsanstalt.
- Näslund M. 1929. Antalet provträäd och höjdkurvans noggrannhet. *Meddelanden fran Statens Skogsforskingsinstitut*, 25, 93–170.
- Näslund M. 1936. Die Durchforstungsversuche der forstlichen Versuchsanstalt Schwedens im Kiefernwald. *Meddelanden fran Statens Skogsforskingsinstitut*, 29, 1–169.
- Petráš R., Mecko J. 2005. Ein Einheitshöhenmodell für Pappelklone. *Allgemeine Forst- und Jagdzeitung*, 176 (4), 57–60.
- Petterson H. 1955. Barrskogens volymproduktion. Die Massenproduktion des Nadelwaldes. *Meddelanden fran Statens Skogsforskingsinstitut*, 45, 1–391.
- Pretzsch H. 1992. Konzeption und Konstruktion von Wachstumsmodellen für Rein- und Mischbestände. *Forstliche Forschungsberichte München*, 115, 1–332.
- Pretzsch H. 2009. *Forest Dynamics, Growth and Yield*. Springer-Verlag, Berlin Heidelberg.
- Prodan M. 1944. Untersuchungen über die Höhenkurven von Fichtenbeständen. *Allgemeine Forst- und Jagdzeitung*, 120, 48–53.
- Schmidt A. 1967. Der rechnerische Ausgleich von Bestandeshöhenkurven. *Forstwissenschaftliches Centralblatt*, 86 (6), 370–382.
- Schumacher F.X. 1939. A new growth curve and its application to timber yield studies. *Journal of Forestry*, 37, 819–820.
- Sharma M., Parton J. 2007. Height-diameter equations for boreal tree species in Ontario using a mixed – effects modelling approach. *Forest Ecology and Management*, 249 (3), 187–198.
- Sloboda B., Gaffrey D., Matsumura N. 1993. Regionale und lokale Systeme von Höhenkurven für gleichaltrige Waldbestände. *Allgemeine Forst- und Jagdzeitung*, 164 (12), 225–228.
- Šmelko Š., Pánek F., Zanvit B. 1987. Matematická formulácia systému jednotných výškových kriviek rovnovekých porastov. *Acta Facultatis forestalis Zvolen*, 29, 151–172.
- Stankova T.V., Diéguez-Aranda U. 2013. Height-diameter relationships for Scots pine plantations in Bulgaria: optimal combination of model type and application. *Annals of Forest Research*, 56 (1), 149–163.
- Sterba H., Marschall J., 1976. Einheitshöhenkurven aus und für Stichprobeninventuren. *Allgemeine Forst- und Jagdzeitung*, 87, 349–350.
- Temesgen H., Gadow K. 2004. Generalized height-diameter models – an application for major tree species in complex stands of interior British Columbia. *European Journal of Forest Research*, 123, 45–51.
- Yang R.C., Kozak A., Smith J.H.G. 1978. The potential of Weibull-type functions as flexible growth curves. *Canadian Journal of Forest Research*, 8, 424–431.
- Zeide B. 1989. Accuracy of equations describing diameter growth. *Canadian Journal of Forest Research*, 19, 1283–1286.
- Zeide B. 1993. Analysis of growth equations. *Forest Science*, 39, 594–616.
- Zlatník A. 1959. Přehled slovenských lesů podle skupin lesních typů. Spisy vědecké laboratoře biocenologie a typologie lesa, LF VŠZ v Brně, 3, 1–159.
- Zlatník A. 1976. Přehled skupin typů geobiocenů původně lesních a křovinných v ČSSR. *Zprávy geografického ústavu ČSAV*, 13, 55–64.