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# MULTICRITERIA EVALUATION OF MANAGERIAL COMPETENCES: AN APPLICATION OF THE DOMINANCE PRINCIPLE AND THE ROUGH SET THEORY

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**Abstract.** This study demonstrates an analysis of executives based on their management competences. The aim of the study is to identify those executives that show a greater strategic alignment with the guidelines set by an organization. The analysis is performed using self-evaluation and corporate evaluation by considering each management competence as a criterion. It makes use of the Dominance principle (i.e., the Dominance-based Rough Set Approach, denoted here as DRSA). The use of DRSA permits the inference of decision rules and identifies the revisions that violate that principle. The main advantage for an organization from using this procedure (i.e., DRSA and RST) is offering the organizational decision makers a comprehensive analysis of their executives. As a major result, training programs aligned to the expectations and needs of the organization can be developed and implemented.

**Keywords:** Multiple criteria decision analysis, Executive competences, Rough set theory, Dominance principle.

## 1. Introduction

The evaluation of executives in an organization, from a multicriteria perspective, is based on an analysis of their management or executive competences by means of self-evaluations, in some cases, and also by independent assessment. An accurate evaluation of executives with the aim of preparing a training programme aligned with the organizational strategy can constitute a competitive advantage in the face of competitors.

This study demonstrates the application of the concepts of Rough Set Theory (RST) extended by the Dominance principle – the Dominance-based Rough Set Approach (DRSA) – as multicriteria decision aiding in the evaluation of executives, considering their

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management competences. The choice of RST and DRSA was chosen because it does not require any preliminary information about the data at hand, such as its probability distribution. Other theories might be used [8] – e.g., the Fuzzy Set Theory, proposed by Lotfi Asker Zadeh in 1965 [18], as an extension of conventional (boolean) logic that introduces the concept of non-absolute truth and serves as a tool for dealing with uncertainties in natural language [5]. RST and Fuzzy Set Theory are independent approaches to the treatment of imperfect (incomplete) and uncertain (vague, indeterminate) knowledge [15]. Both theories provide a mathematical framework to capture uncertainties associated with the data [10], [16]. RST has shown to be of fundamental importance to artificial intelligence and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, decision analysis, expert systems, decision support systems, inductive reasoning and pattern recognition [15].

In this case, information is obtained by means of self-evaluation and an independent evaluation (an appraisal committee, for example). Afterwards, the probable executive candidates for the position of model executive are identified from the set of executives of an organization. For this purpose, their evaluations are considered, even if they are not of the highest degree. This permits the preparation of executive training programmes by means of the "socialisation" of knowledge (the transmission of "tacit for tacit" knowledge), from the model executives to the other executives. Facilitating the exchange of experiences among executives not only permits their retraining, but also better knowledge management within an organization. Large corporations both in Brazil and around the world already use their more experienced executives for "coaching" and "mentoring" of executives at the beginning of their careers.

Initially, this work presents RST with its principal concepts and, the extension of these by DRSA. Afterwards, it demonstrates the application of RST together with DRSA, as multicriteria decision support for simulations with 10 and 25 executives, considering 8 management competences as condition criteria and an independent assessment as the decision criteria. As a decision aiding tool, software developed in *Visual Basic for Applications* (VBA/Excel) was used, based on the DOMLEM algorithm, for type "1" rules. At the end of the article, the conclusions and possibilities for future studies are presented.

## 2. Rough Set Theory

RST originated with the seminal work of Zdzislaw Pawlak, in 1982, and approaches the treatment of imprecise data, using "upper and lower approximations" of a set of data [12]. The "relation of indiscernibility" is conceived as that which identifies objects with the same property. In other words, those objects which have the same properties are "indiscernible" (they are treated as identical or similar). An information system can be defined as a tuple $S = (U, Q, V, f)$, where $U$ is a finite set of objects, $Q$ is a finite set of attributes, $V = \bigcup_{q \in Q} V_q$, where $V_q$ is the domain of the attribute $q$ and, $f\colon U \chi Q \rightarrow V$ is a total function so that, $f(x, q) \in V_q$ for each $q \in Q$, $x \in U$, known as the "information function" [14]. In addition, given an information system, $S = (U, Q, V, f)$, and $P \subseteq Q$, and $x,y \in U$, it is said that x and y are "indiscernible" by the set of attributes $P$ in $S$, if $f(x,q) = f(y,q)$ for all $q \in P$. In this way, all $P \subseteq Q$ generate a binary relation in $U$, known as the "indiscernibility relation", denoted by IND(*P*). Given that, $P \subseteq Q$ and $Y \subseteq U$, the lower approximation ($\underline{P}\,Y$) and the upper approximation ($\overline{P}\,Y$) are defined as:

$$\underline{P}\,Y = \cup \{X \in U/P: X \subseteq Y\} \text{ and } \overline{P}\,Y = \cup \{X \in U/P: X \cap Y \neq \emptyset\} \quad (1)$$

The difference between $\overline{P}\,Y$ and $\underline{P}\,Y$, denominates "the boundary region" of *Y*:

$$BN_P(Y) = \overline{P}\,Y - \underline{P}\,Y \quad (2)$$

The concept of a measurement of accuracy is also added:

$$\alpha_P(Y) = \text{card } \underline{P} \,/\, \text{card } \overline{P}, \quad (3)$$

which captures the degree of completeness of the knowledge of the set *Y*. As an illustration of the application of the previous concepts (1) [13], a table (Table 1) is used composed of six shops and four attributes:

**Table 1. Example with six shops and four initial attributes**

| Shop | E | Q | L | P |
|---|---|---|---|---|
| **1** | High | Good | No | Profit |
| **2** | Average | Good | No | Loss |
| **3** | Average | Good | No | Profit |
| **4** | None | Average | No | Loss |
| **5** | Average | Average | Yes | Loss |
| **6** | High | Average | Yes | Profit |

In Table 1 the following attributes are identified: E – autonomy of sales staff; Q – merchandise quality; L – location with intense movement; P – result (profit or loss). Each shop is characterized by the attributes E, Q, L and P. In this way, all the shops are "discernible" by the use of the contents (values) of these attributes. However, shops 2 and 3 are "indiscernible" in relation to attributes E, Q and L, taking into consideration that they have the same values. Each subset of attributes determines a "partition" ("classification") of all the objects in "classes", which have the same description in terms of those attributes. For example, attributes Q and L aggregate all the shops in the classes {1,2,3}, {4} and {5,6}, as shown in Table 2.

**Table 2. Example with six shops and two attributes**

| Shop | Q | L |
|---|---|---|
| **1** | Good | No |
| **2** | Good | No |
| **3** | Good | No |
| **4** | Average | No |
| **5** | Average | Yes |
| **6** | Average | Yes |

Consider the following problem: what are the characteristics of the shops which made a profit (or made a loss) in terms of the attributes E, Q and L? In other words, the focus is on describing the set (concept) {1,3,6} (or {2,4,5}), shown in Table 1. It can easily be observed that this question cannot be answered in a single way, as shops 2 and 3 have the same characteristics in regards to attributes E, Q and L, but shop 2 made a loss, while shop 3 made a profit. Based on Table 1, it can be said that: shops 1and 6 made a profit, shops 4 and 5 made a loss and shops 2 and 3 cannot be classified (in terms of profit or loss). Table 3 shows the result of this analysis.

**Table 3. Example with six shops and four final attributes**

| Shop | E | Q | L | P | Result |
|---|---|---|---|---|---|
| **1** | High | Good | No | Profit | **PROFIT** |
| **2** | Average | Good | No | Loss | **?** |
| **3** | Average | Good | No | Profit | **?** |
| **4** | None | Average | No | Loss | **LOSS** |
| **5** | Average | Average | Yes | Loss | **LOSS** |
| **6** | High | Average | Yes | Profit | **PROFIT** |

Considering the attributes E, Q and L, it can be deduced that: shops 1 and 6 <u>certainly</u> made a profit, that is, <u>certainly</u> belong to the set {1,3,6}; shops 1, 2, 3 and 6 <u>possibly</u> made a profit, that is, <u>possibly</u> belong to the set {1,3,6}. The sets {1,6} and {1,2,3,6} represent, respectively, the lower and upper approximations of the set {1,3,6}. The set {2,3} represents the difference between the upper and lower approximations and characterises the "boundary region" of the set {1,3,6}. In RST there are two important concepts: the "reduct" and the "core" of an information system. The reduct is its essential part, that is, the set of attributes which supply the same quality of classification as the original set of attributes (it permits the same decisions to be made if there are all the attributes of the condition). The core represents the most important subset of this knowledge (the collection of the most important attributes) [12], [14]. If **R** represents a family of relations and $R \in \mathbf{R}$ [12], it is said that $R$ is "dispensable" in **R** if IND(**R**) = IND(**R** – {$R$}); otherwise, $R$ is "indispensible" in **R**. The family **R** is "independent" if each $R \in \mathbf{R}$ is indispensible in **R**; if the contrary, **R** is "dependent". In this way, the following propositions were defined:

a) If **R** is independent and $\mathbf{P} \subseteq \mathbf{R}$, then **P** is also independent.

b) CORE(**P**) = $\bigcap$ RED(**P**), where RED(**P**) is the family of all the "reducts" of **P**.

As an example of the application of RST in Human Resources, one can use the decision to be made to determine the quantitative of executives based on replicated and inconsistent data [4].

## 3. Dominance Principle

The key aspect of multicriteria decision making is the consideration of objects described by multiple criteria which represent conflicting points of view. Criteria are attributes with domains with an order of preference – e.g., in the choice of a car it can be considered that price and fuel consumption are characteristics which should serve as criteria for its purchase, as usually, a low price is considered better than a high price and average fuel consumption is more desirable than high fuel consumption. Generally, for other attributes such as colour and country of origin, domains in which there is no order of preference, these are not considered as decision criteria - they are regular attributes. Thus, multicriteria decision problems cannot be analysed by the RST approach given that the analysis is only on regular attributes. In addition to this, it is not possible to identify inconsistencies which violate the Dominance principle: "objects which have a better evaluation or which have at least the same evaluation (class decision) cannot be associated with a worse decision class, having considered all the decision criteria". RST ignores not only the preference order in the value sets of attributes but the monotonic relationship between evaluations of objects on such attributes ("criteria") and the preference ordered value of decision (classification decision or degree of preference) [7], [17]. This question is treated by the extension of RST: DRSA is applied [17]. By this principle, indiscernibility relations are substituted by relations of Dominance in the approximations of the decision classes. By DRSA, due to the order of preference among the decision classes, the sets become approximations and are known as "upward" and "downward" unions of decision classes. In this way, for a tuple $S = (U, Q, V, f)$, the set $Q$ is, in general, divided into condition attributes (set $C$) and decision attributes (set $D$). Based on the fact that all the condition attributes ($q \in C$) are decision criteria, $S_q$ represents a relation outside the classification in $U$ with respect to criterion $q$ so that, $xS_qy$ represents "x is at least as good as y in relation to criterion $q$". Assuming that the set of decision attributes $D$ establishes a partition of $U$ in a finite number of classes – $\boldsymbol{Cl} = \{Cl_t, t \in T\}$, $T = \{1, ..., n\}$, is a set of these classes so that each $x \in U$ belongs to one and only one $Cl_t \in \boldsymbol{Cl}$. It is supposed that these classes are ordered, that is, for all $r,s \in T$, so that $r > s$, the objects of $Cl_r$ are preferable to the objects of the class $Cl_s$. In this way, the objects can be approximated by upward and downward unions of decision classes respectively: $Cl_t^{\geq} = \bigcup_{s \geq t} Cl_s$, $Cl_t^{\leq} = \bigcup_{s \leq t} Cl_s$, $t$=1, ...,n.

In this way, the indiscernibility relation is substituted by a Dominance relation. It is said that $x$ dominates $y$ in relation to $P \subseteq C$, denoted by $xD_Py$, if $xS_qy$ for all $q \in P$. The Dominance relation is reflexive and transitive. Given that, $P \subseteq C$ and $x \in U$, the "knowledge granules" used in the approximations in DRSA are:

- a set of dominant objects x, called set P-dominant: $D_P^+(x) = \{y \in \bigcup : yD_Px\}$,
- a set of objects dominated by x, called set P-dominated: $D_P^-(x) = \{y \in \bigcup : xD_Py\}$.

Using the sets $D_P^+(x)$, the approximations P-lower and P-upper of $Cl_t^{\geq}$ are: $\underline{P}(Cl_t^{\geq}) = \{x \in \bigcup : D_P^+(x) \subseteq Cl_t^{\geq}\}$, $\overline{P}(Cl_t^{\geq}) = \bigcup_{x \in Cl_t^{\geq}} D_P^+(x)$, for $t$=1,...,n.

In the same way, the approximations P-lower and P-upper of $Cl_t^{\leq}$ are: $\underline{P}(Cl_t^{\leq}) = \{x \in \bigcup : D_P^-(x) \subseteq Cl_t^{\leq}\}$, $\overline{P}(Cl_t^{\leq}) = \bigcup_{x \in Cl_t^{\leq}} D_P^-(x)$, for $t$=1,...,n.

The P-boundaries sets of $Cl_t^{\geq}$ and $Cl_t^{\leq}$ are: $Bn_P(Cl_t^{\geq}) = \overline{P}(Cl_t^{\geq}) - \underline{P}(Cl_t^{\geq})$, $Bn_P(Cl_t^{\leq}) = \overline{P}(Cl_t^{\leq}) - \underline{P}(Cl_t^{\leq})$, for $t$=1,...,n. These approximations of the "upward" and "downward" unions of classes can serve to induce decision rules "*if* ... *then* ...". For each "upward" or "downward" union of class, $Cl_t^{\geq}$ or $Cl_t^{\leq}$, $s,t \in T$, the rules inferred from the hypothesis that the objects belonging to the lower approximations $\underline{P}(Cl_t^{\geq})$ or $\underline{P}(Cl_t^{\leq})$ are positive and all the others are negative, suggest the attribution of an object to "at least a class $Cl_t$" or to "at most a class $Cl_s$", respectively. These rules are known as "certain decision rules" ($D_{\leq}$ or $D_{\geq}$) because they attribute the objects to unions of decision classes without any ambiguity. In contrast, if the objects belong to upper approximations, the rules are known as "possible decision rules"; in this way, the objects could belong to "at least a class $Cl_t$" or to "at most a class $Cl_s$". And, if the objects belong to the intersection $\overline{P}(Cl_s^{\leq}) \bigcap \overline{P}(Cl_t^{\geq})$ ($s<t$), the rules induced are known as "approximate rules", that is, the objects are between the classes $Cl_s$ and $Cl_t$.

Thus, if for each criterion $q \in C$, $V_q \subseteq \mathbf{R}$ ($V_q$ is quantitative) and that for each $x,y \in U$, $f(x,q) \geq f(y,q)$ implies $xS_qy$ ($V_q$ has a preference ranking), the decision rules can be considered according to five types:

1- certain decision rules-$D_{\geq}$:

*if* $f(x,q_1) \geq r_{q1}$ *and* $f(x,q_2) \geq r_{q2}$ *and ...* $f(x,q_p) \geq r_{qp}$, *then* $x \in Cl_t^{\geq}$

2- possible decision rules-$D_{\geq}$:

*if* $f(x,q_1) \geq r_{q1}$ *and* $f(x,q_2) \geq r_{q2}$ *and ...* $f(x,q_p) \geq r_{qp}$, *then x could belong to* $Cl_t^{\geq}$

3- certain decision rules-$D_{\leq}$:

*if* $f(x,q_1) \leq r_{q1}$ *and* $f(x,q_2) \leq r_{q2}$ *and ...* $f(x,q_p) \leq r_{qp}$, *then* $x \in Cl_t^{\leq}$

4- possible decision rules-$D_{\leq}$:

*if $f(x,q_1) \leq r_{q1}$ and $f(x,q_2) \leq r_{q2}$ and ... $f(x,q_p) \leq r_{qp}$, then x could belong to $Cl_t^{\leq}$*

where $P = \{q_1, ..., q_p\} \subseteq C$, $(r_{q1}, ..., r_{qp}) \in V_{q1} \times V_{q2} \times ... \times V_{qp}$ and $t \in T$;

5- approximate rules- $D_{\leq\geq}$:

*if $f(x,q_1) \geq r_{q1}$ and $f(x,q_2) \geq r_{q2}$ and ... $f(x,q_k) \geq r_{qk}$ and $f(x,q_{k+1}) \leq r_{qk+1}$ and $f(x,q_p) \leq r_{qp}$, then $x \in Cl_s \cup Cl_{s+1} \cup ... \cup Cl_t$.*

The rules of types “1” and “3” represent “certain knowledge” extracted from a table of data (or information system); the rules of types “2” and “4” represent “possible knowledge” and, the rule of type “5”, “ambiguous knowledge”.

In order to apply the previous concepts, Table 4, taken from [6], displays data with three condition criteria, $C = \{q_1, q_2, q_3\}$. Those are to be maximized according to the preferences of the decision maker. Table 4 shows a decision attribute *d* that classifies the objects into decision classes, $Cl_1$, $Cl_2$ and $Cl_3$, of preference in increasing numerical order.

**Table 4. Table with 3 condition criteria and 3 decision classes**

| Object | *q1* | *q2* | *q3* | *D* |
|---|---|---|---|---|
| **001** | 1.5 | 3 | 12 | $Cl_2$ |
| **002** | 1.7 | 5 | 9.5 | $Cl_2$ |
| **003** | 0.5 | 2 | 2.5 | $Cl_1$ |
| **004** | 0.7 | 0.5 | 1.5 | $Cl_1$ |
| **005** | 3 | 4.3 | 9 | $Cl_3$ |
| **006** | 1 | 2 | 4.5 | $Cl_2$ |
| **007** | 1 | 1.2 | 8 | $Cl_1$ |
| **008** | 2.3 | 3.3 | 9 | $Cl_3$ |
| **009** | 1 | 3 | 5 | $Cl_1$ |
| **010** | 1.7 | 2.8 | 3.5 | $Cl_2$ |
| **011** | 2.5 | 4 | 11 | $Cl_2$ |
| **012** | 0.5 | 3 | 6 | $Cl_2$ |
| **013** | 1.2 | 1 | 7 | $Cl_2$ |
| **014** | 2 | 2.4 | 6 | $Cl_1$ |
| **015** | 1.9 | 4.3 | 14 | $Cl_2$ |
| **016** | 2.3 | 4 | 13 | $Cl_3$ |
| **017** | 2.7 | 5.5 | 15 | $Cl_3$ |

The unions of classes are as follows:

$Cl_1^{\leq} = \{3,4,7,9,14\}$,

$Cl_2^{\leq} = \{1,2,3,4,6,7,9,10,11,12,13,14,15\}$,

$Cl_2^{\geq} = \{1,2,5,6,8,10,11,12,13,15,16,17\}$,

$Cl_3^{\geq} = \{5,8,16,17\}$.

There are 5 objects which violate the Dominance principle: 6, 8, 9, 11 and 14. For example, object "9" dominates object "6" because it is better in all the condition criteria ($q_1$, $q_2$ and $q_3$). However, it has been attributed to a decision class $Cl_1$ which is worse than $Cl_2$. The lower and upper approximations of each decision class were extracted as follows:

$\underline{C}(Cl_1^{\leq}) = \{3, 4, 7\}$

$\overline{C}(Cl_1^{\leq}) = \{3, 4, 6, 7, 9, 14\}$

$\underline{C}(Cl_2^{\leq}) = \{1, 2, 3, 4, 6, 7, 9, 10, 12, 13, 14, 15\}$

$\overline{C}(Cl_2^{\leq}) = \{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$\underline{C}(Cl_2^{\geq}) = \{1, 2, 5, 8, 10, 11, 12, 13, 15, 16, 17\}$

$\overline{C}(Cl_2^{\geq}) = \{1, 2, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$

$\underline{C}(Cl_3^{\geq}) = \{5, 16, 17\}$

$\overline{C}(Cl_3^{\geq}) = \{5, 8,11, 16, 17\}$

Following the sequence of the analysis proposed by the algorithm DOMLEM (Annex) [6], for rules type "1", the decision rules were extracted with the respective objects which satisfy the rule and its evaluation measure - ($[e_i] \cap G/[e_i]$) and ($[e_i] \cap G$), where "$e_i$" represents a rule and "G", the upper approximation under analysis– $\underline{C}(Cl_3^{\geq})$:

$e_1 = (f(x,q_1) \geq 2.3)$, $\{5, 8, 11,16, 17\}$, 0.6, 3;

$e_2 = (f(x,q_1) \geq 2.7)$, $\{5, 17\}$, 1.0, 2;

$e_3 = (f(x,q_2) \geq 4)$, $\{2, 5, 11, 15, 16, 17\}$, 0.5, 3;

$e_4 = (f(x,q_2) \geq 4.3)$, $\{2, 5, 15, 17\}$, 0.5, 2;

$e_5 = (f(x,q_2) \geq 5.5)$, $\{17\}$, 1.0, 1;

$e_6 = (f(x,q_3) \geq 9)$, $\{1, 2, 5, 8, 11, 15, 16, 17\}$, 0.38, 3;

$e_7 = (f(x,q_3) \geq 13)$, $\{15, 16, 17\}$, 0.67, 2;

$e_8 = (f(x,q_3) \geq 15)$, $\{17\}$, 1.0, 1.

The decision rule $e_2$ is the one chosen, given the greater evaluation measure (1.0) and as it has more objects (2) in the intersection "$[e_i] \cap G$", in addition to satisfying the condition "$[e_2] \subseteq B$". Afterwards, these objects are excluded from G and the logic of extraction of the decision rules for the remaining object ("16") is repeated. The rules inferred are:

$e_9 = (f(x,q_1) \geq 2.3)$, $\{8, 11, 16\}$, 0.33, 1;

$e_{10} = (f(x,q_2) \geq 4)$, $\{2, 11, 15, 16\}$, 0.25, 1;

$e_{11} = (f(x,q_3) \geq 13)$, $\{15, 16\}$, 0.5, 1.

The rule $e_{11}$ is the rule with the greatest evaluation measure (0.5), but, as the object "15" does not belong to the approximation under analysis ($\underline{C}$ $(Cl_3^{\geq})$), it becomes necessary to infer "complex" rules ("^"): $e_9$ ^ $e_{11}$ e $e_{10}$ ^ $e_{11}$. In this way, the rule $e_9$ ^ $e_{11}$ is the one chosen as it has the greatest evaluation measure and covers the elements of the lower approximation. In summary, only considering the lower approximation of the decision class $Cl_3$, the rules inferred and respective objects are:

*if* $(f(x,q_1) \geq 2.7)$, *then* $x \in Cl_3^{\geq}$ {5, 17}

*if* $(f(x,q_1) \geq 2.3)$ *and* $(f(x,q_3) \geq 13.0)$, *then* $x \in Cl_3^{\geq}$ {16, 17}

A generalisation for DRSA has been proposed, called VC-DRSA (*Variable consistency-DRSA*) [1], [2], [3], [6], which allows one to define lower approximations to unions of decision classes that take a limited number of negative examples controlled by a predefined "consistency level" $l \in (0, 1]$. In VC-DRSA, given $P \subseteq C$ and consistency level $l$, approximations $\underline{P}$ and $\overline{P}$ of the unions of "upper" classes are as follows:

$$\underline{P}^l(Cl_t^{\geq}) = \{x \in Cl_t^{\geq} : \frac{card(D_P^+(x) \cap Cl_t^{\geq})}{card(D_P^+(x))} \geq l\} \quad (4)$$

$$\overline{P}^l(Cl_t^{\geq}) = Cl_t^{\geq} \cup \{x \in Cl_{t-1}^{\leq} : \frac{card(D_P^-(x) \cap Cl_{t-1}^{\leq})}{card(D_P^-(x))} < l\} \quad (5)$$

In VC-DRSA, each decision rule is characterised by an additional parameter "α" known as the rule's "confidence" (level); it is the ratio of the number of objects that satisfy the rule to the number of objects the rule covers. Consequently, this model allows a rule to cover more objects than DRSA. Some of its basic concepts are as follows: a rule's "strength" is the ratio of the number of objects that satisfy the rule to the total number of objects, its "certainty" is the ratio of the number of objects that satisfy the rule to the number of objects that satisfy the rule's condition criteria, and its "coverage" is the ratio of the number of objects that satisfy the rule to the number of objects that satisfy the rule's decision criteria.

## 4. Multicriteria Decision based on Executive Competences and Corporate Evaluation

The following example shows the application of the Dominance principle in the evaluation of executives in the upper and middle management levels of a large organization. The same set of executive competences is considered in both levels. This evaluation aims to identify those executives who might be considered models for the training of other executives that belong to the same organization. This has been a practice in large corporations both in Brazil and around the world as those organizations try to identify more experienced executives for the coaching and mentoring of executives at the beginning of their careers.

In this context, management (or executive) competence is understood to be the knowledge, skills or attitudes necessary to perform well as a manager in the company [11]. For the problem under analysis, 8 executive competences were proposed for self-evaluation (without precedence):

- Leadership 1 ("inspires other people") - C1;
- Leadership 2 ("capacity to make decisions") - C2;

• Knowledge Management ("carries out Knowledge Management, both tacit and explicit") - C3;

• Cooperation ("contributes to the success of the work") - C4;

• Communication ("communicates quite well with the team") - C5;

• Process Management ("carries out management by working process") - C6;

• Negotiation ("promotes the progress of work") - C7;

• Strategic Alignment ("plans actions in alignment with the company strategy") - C8.

For each competence to be evaluated a response scale was supplied: 1- does not meet, 2 - sometimes meets, 3 - in most cases meets, 4 - fully meets. In addition to self-evaluations, an evaluation ("*A*") carried out by a committee (corporate evaluation) was added: "3" (performance acceptable or above), "2" (acceptable performance; need for training) or "1" (performance below acceptable; need for mentoring or coaching, as well as training). Each executive was then classified according to this evaluation (3, 2 or 1), with "3", the best performance and "1", the worst performance. In this study the corporate evaluations ("*A*") are called evaluation "classes". Classes $Cl_1$, $Cl_2$ and $Cl_3$ were associated with the corporate evaluations "1", "2" and "3" respectively. The management competences were considered "attributes or condition criteria" and the corporate evaluation, an "attribute or decision criterion". In this way, the (multicriteria) decision making problem consists of classifying or reclassifying the executive competences (self-evaluations) and the corporate evaluation. This decision seeks to identify the executives who should be recycled (trained) and those who can serve as models for the training programme. For example: by means of the socialisation of knowledge ("tacit to tacit" transmission of knowledge), the executives considered to have "acceptable or above" performance (corporate evaluation equal to "3" or $Cl_3$) can transmit their experience to the other executives. It is a decision problem "type α" ($P_\alpha$), which is: to identify the executive(s) with the best performance in their self-evaluations and corporate evaluation, even if these are not at the highest evaluation grade. In addition to this, it also seeks to infer the decision rules which permit support for this classification (or reclassification).

In what follows, simulations show an evaluation based on two management levels from the same company: board of directors, composed of 10 "executive directors" and "heads of department" of a particular business area, considering up to 25 executives. They represent the upper (except, chairman and vice-chairman) and middle management levels, respectively, in large organizations. This example shows how to identify those executives who might be considered models for executive training (i.e., "coaching" and "mentoring"). Thus, the Figure 1 illustrates the simulation of the response to 8 management competences (C1 to C8) and the corporate evaluation (A), for each of the 10 "executive directors" (Randbetween function of Excel):

| Executive | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | *A* |
|---|---|---|---|---|---|---|---|---|---|
| 001 | 4 | 4 | 2 | 2 | 3 | 2 | 4 | 2 | 1 |
| 002 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 2 | 1 |
| 003 | 1 | 3 | 2 | 1 | 3 | 2 | 1 | 2 | 2 |
| 004 | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 2 |
| 005 | 2 | 4 | 3 | 4 | 3 | 3 | 1 | 3 | 3 |
| 006 | 1 | 4 | 1 | 2 | 1 | 2 | 3 | 3 | 3 |
| 007 | 1 | 1 | 1 | 1 | 4 | 4 | 3 | 3 | 2 |
| 008 | 2 | 4 | 1 | 4 | 3 | 2 | 2 | 3 | 2 |
| 009 | 3 | 1 | 2 | 2 | 4 | 1 | 4 | 3 | 1 |
| 010 | 4 | 4 | 2 | 3 | 4 | 2 | 4 | 3 | 2 |

**Figure 1. Table-example for 10 executives**

In spite of there not being indiscernibility in the RST analysis of the competences for each executive – it is possible to differentiate the executives by their self-evaluations (C1 to C8) – the Dominance principle (DRSA) identifies that there is an inconsistency, for example, when comparing executive "001" (corporate evaluation "1") with executive "004" (corporate evaluation "2"). In all the competences, executive "001" has self-evaluations which are higher or the same as the self-evaluations of executive "004". In this way, executive "001" should belong to a corporate evaluation class no worse than the class of executive "004". The data in this table (Figure 1) was then submitted to analysis by the software developed in Visual Basic for Applications (VBA/Excel) and which implements the DOMLEM algorithm for rules type "1", with the objective of identifying the correct classification of these executives, based on their executive competences and corporate evaluation. The results obtained were as follows:

- "downward" decision classes" ( $Cl_1^{\le}$ and $Cl_2^{\le}$ ) and "upward" decision classes ( $Cl_2^{\ge}$ and $Cl_3^{\ge}$ ):

$Cl_1^{\le}$ = {001, 002, 009}

$Cl_2^{\le}$ = {001, 002, 003, 004, 007, 008, 009, 010}

$Cl_2^{\ge}$ = {003, 004, 005, 006, 007, 008, 010}

$Cl_3^{\ge}$ = {005, 006}

- pairs of executives which violate the Dominance principle:
{001, 003}, {001, 004}, {004, 009} e {006, 010}.
- and, the lower and upper approximations for each decision class:

$\underline{C}(Cl_1^{\le})$ = {002}

$\overline{C}\ (Cl_1^{\leq}) = \{001, 002, 003, 004, 009\}$

$\underline{C}\ (Cl_2^{\leq}) = \{001, 002, 003, 004, 007, 008, 009\}$

$\overline{C}\ (Cl_2^{\leq}) = \{001, 002, 003, 004, 006, 007, 008, 009, 010\}$

$\underline{C}\ (Cl_2^{\geq}) = \{005, 006, 007, 008, 010\}$

$\overline{C}\ (Cl_2^{\geq}) = \{001, 003, 004, 005, 006, 007, 008, 009, 010\}$

$\underline{C}\ (Cl_3^{\geq}) = \{005\}$

$\overline{C}\ (Cl_3^{\geq}) = \{005, 006, 010\}$

Next, the decision rules with the greatest evaluation measures ($[e_i] \cap G|/|[e_i]$) were inferred:

if ((x,c3) ≥3), then x $\in Cl_3^{\geq}$ {005}

if ((x,c4) ≥3), then x $\in Cl_2^{\geq}$ {005, 008, 010}

if ((x,c6) ≥4), then x $\in Cl_2^{\geq}$ {007}

The first rule corresponds to rule "3", with the greatest evaluation measure, among the rules inferred for the approximation $\underline{C}\ (Cl_3^{\geq})$:

rule 1 = if ((x,c1) ≥ 2), then x $\in Cl_3^{\geq}$ {001, 005, 008, 009, 010}, evaluation measure: 0.20

rule 2 = if ((x,c2) ≥4), then x $\in Cl_3^{\geq}$ {001, 005, 006, 008, 010}, evaluation measure: 0.20

rule 3 = if ((x,c3) ≥3), then x $\in Cl_3^{\geq}$ {005}, evaluation measure: 1.00

rule 4 = if ((x,c4) ≥ 4), then x $\in Cl_3^{\geq}$ {005, 008}, evaluation measure: 0.50

rule 5 = if ((x,c5) ≥ 3), then x $\in Cl_3^{\geq}$ {001, 003, 005, 007, 008, 009, 010}, evaluation measure: 0.14

rule 6 = if ((x,c6) ≥ 3), then x $\in Cl_3^{\geq}$ {005, 007}, evaluation measure: 0.50

rule 7 = if ((x,c7) ≥ 1), then x $\in Cl_3^{\geq}$ {001, 002, 003, 004, 005, 006, 007, 008, 009, 010}, evaluation measure: 0.10

rule 8 = if ((x,c8) ≥ 3), then x $\in Cl_3^{\geq}$ {005, 006, 007, 008, 009, 010}, evaluation measure: 0.17

A similar procedure was used for the choice of the other rules in relation to the approximation $\underline{C}\ (Cl_2^{\geq})$, based on the rules inferred by the specific programme (in VBA/Excel). In this way, initially by Figure 1, executives "005" and "006" belong to the decision class $Cl_3$. However, when applying the Dominance principle and evaluating which executives can serve as models for the preparation of a training programme, it is inferred by rule "3" that executive "005" is a strong candidate for the position of model executive ("executive director"). In this case, by this rule, the importance of competence C3 ("Knowledge Management") in front of the other competences is made clear.

Next, the simulation for 25 executives ("heads of department") was carried out, maintaining the same competences (Table 5):

**Table 5. Table-example for 25 executives**

| Executive | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | *A* |
|---|---|---|---|---|---|---|---|---|---|
| **001** | 4 | 4 | 2 | 2 | 3 | 2 | 4 | 2 | 1 |
| **002** | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 2 | 1 |
| **003** | 1 | 3 | 2 | 1 | 3 | 2 | 1 | 2 | 2 |
| **004** | 1 | 1 | 2 | 2 | 2 | 1 | 1 | 1 | 2 |
| **005** | 2 | 4 | 3 | 4 | 3 | 3 | 1 | 3 | 3 |
| **006** | 1 | 4 | 1 | 2 | 1 | 2 | 3 | 3 | 3 |
| **007** | 1 | 1 | 1 | 1 | 4 | 4 | 3 | 3 | 2 |
| **008** | 2 | 4 | 1 | 4 | 3 | 2 | 2 | 3 | 2 |
| **009** | 3 | 1 | 2 | 2 | 4 | 1 | 4 | 3 | 1 |
| **010** | 4 | 4 | 2 | 3 | 4 | 2 | 4 | 3 | 2 |
| **011** | 1 | 4 | 2 | 4 | 4 | 3 | 1 | 3 | 2 |
| **012** | 3 | 2 | 1 | 3 | 4 | 4 | 1 | 3 | 3 |
| **013** | 3 | 4 | 3 | 4 | 4 | 1 | 1 | 2 | 2 |
| **014** | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 3 | 1 |
| **015** | 2 | 3 | 4 | 2 | 3 | 2 | 4 | 4 | 3 |
| **016** | 3 | 3 | 3 | 2 | 3 | 3 | 4 | 3 | 3 |
| **017** | 3 | 3 | 1 | 1 | 2 | 2 | 4 | 3 | 3 |
| **018** | 1 | 2 | 1 | 1 | 4 | 4 | 1 | 1 | 2 |
| **019** | 3 | 4 | 4 | 1 | 2 | 2 | 3 | 1 | 2 |
| **020** | 4 | 3 | 2 | 2 | 2 | 4 | 2 | 4 | 2 |
| **021** | 4 | 2 | 4 | 3 | 1 | 4 | 4 | 3 | 1 |
| **022** | 1 | 4 | 3 | 2 | 1 | 3 | 1 | 3 | 2 |
| **023** | 4 | 4 | 2 | 4 | 2 | 3 | 1 | 4 | 1 |
| **024** | 2 | 4 | 4 | 4 | 1 | 2 | 1 | 1 | 2 |
| **025** | 1 | 1 | 1 | 2 | 4 | 3 | 1 | 1 | 3 |

- "downward" decision classes ($Cl_1^{\leq}$ and $Cl_2^{\leq}$) and "upward" decision classes ($Cl_2^{\geq}$ and $Cl_3^{\geq}$):

$Cl_1^{\leq}$ = {001, 002, 009, 014, 021, 023}

$Cl_2^{\leq}$ = {001, 002, 003, 004, 007, 008, 009, 010, 011, 013, 014, 018, 019, 020, 021, 022, 023, 024}

$Cl_2^{\geq}$ = {003, 004, 005, 006, 007, 008, 010, 011, 012, 013, 015, 016, 017, 018, 019, 020, 022, 024, 025}

$Cl_3^{\geq}$ = {005, 006, 012, 015, 016, 017, 025}

- pairs of executives which violate the Dominance principle: {001, 003}, {001, 004}, {004, 009}, {004, 014}, {004, 023}, {006, 010}, {010, 017}, {011, 025}.
- the lower and upper approximations for each decision class:

$\underline{C}(Cl_1^{\leq})$ = {002, 021}

$\overline{C}\,(Cl_1^{\leq}) = \{001, 002, 003, 004, 009, 014, 021, 023\}$

$\underline{C}\,(Cl_2^{\leq}) = \{001, 002, 003, 004, 007, 008, 009, 013, 014, 018, 019, 020, 021, 022, 023, 024\}$

$\overline{C}\,(Cl_2^{\leq}) = \{001, 002, 003, 004, 006, 007, 008, 009, 010, 011, 013, 014, 017, 018, 019, 020, 021, 022, 023, 024, 025\}$

$\underline{C}\,(Cl_2^{\geq}) = \{005, 006, 007, 008, 010, 011, 012, 013, 015, 016, 017, 018, 019, 020, 022, 024, 025\}$

$\overline{C}\,(Cl_2^{\geq}) = \{001, 003, 004, 005, 006, 007, 008, 009, 010, 011, 012, 013, 014, 015, 016, 017, 018, 019, 020, 022, 023, 024, 025\}$

$\underline{C}\,(Cl_3^{\geq}) = \{005, 012, 015, 016\}$

$\overline{C}\,(Cl_3^{\geq}) = \{005, 006, 010, 011, 012, 015, 016, 017, 025\}$

Initially, some rules have been inferred with respect to the lower approximation of the class $Cl_3$:

rule 1 = if ((x,c1) $\geq$ 2), then x$\in Cl_3^{\geq}$ {001, 005, 008, 009, 010, 012, 013, 014, 015, 016, 017, 019, 020, 021, 023, 024}, evaluation measure: 0.25

rule 2 = if ((x,c3) $\geq$3), then x$\in Cl_3^{\geq}$ {005, 013, 014, 015, 016, 019, 021, 022, 024}, evaluation measure: 0.33

rule 3 = if ((x,c3) $\geq$4), then x$\in Cl_3^{\geq}$ {015, 019, 021, 024}, evaluation measure: 0.25

rule 4 = if ((x,c8) $\geq$4), then x$\in Cl_3^{\geq}$ {015, 020, 023}, evaluation measure: 0.33

The inference of the decision rule for the lower approximation of the class $Cl_3$, considering the previous rules and the greatest evaluation measures (rules 2 and 4), is as follows:

rule 5 = if ((x,c3) $\geq$ 3) and ((x,c8) $\geq$ 4), then x$\in Cl_3^{\geq}$ {015}, evaluation measure: 1.00.

Here, considering the lower approximation of class $Cl_3^{\geq}$, which corresponds to the set of executives {005, 012, 015, 016}, it can be inferred that these executives would possibly form the set of model executives. If, for example, the adoption of executive "015" as a model for executive training becomes a concrete reality, the previous rule "5" could serve as a standard suggestion for the identification of future executives ("head of department"). For this rule, the competences C3 ("Knowledge Management") and C8 ("Strategic Alignment") are shown to be in front of the other competences.

These examples show how it is possible to infer rules about executives and their competences. In this case, upper and middle management levels - "executive directors" and "heads of department", respectively, from the same organization. The purpose is to identify those executives who could be considered models for executive training - "coaching" and "mentoring" of executives.

## 5. Conclusions and future studies

The simulation of the multicriteria decision problem - with the executive competences considered as condition criteria and corporate evaluation as decision criteria, for the sets of 10 and 25 executives, with the application of the concepts of RST and DRSA - was demonstrated to be feasible when applied to the identification of executives who could be considered models for the preparation of training programmes. This training could be implemented, for example, by means of the "socialisation" of knowledge – transmission in the "tacit to tacit" form, that is, when successful experiences of executives with high evaluations are transmitted to the other executives in training. The analysis of the set of criteria composed of various competences obtained by self-evaluations and by independent appraisal (by an evaluation committee, for example), can become complex when checking the possibilities of pairs of executives which violate the DRSA, even if there are not inconsistencies from the perspective of RST. In this way, by means of DRSA, the concepts of RSA are extended so as to make them more wide-ranging when applied to a system or set of information.

The evaluation of the objects – in this study, the executives – considering their classifications in the respective decision classes ($Cl_n$), and applying DRSA, permits reclassifying them in other decision classes, when this is the case, at the end of the multicriteria analysis. In addition to this, it allows for the inferring of decision rules which will give support to the multicriteria decision by means of a more accurate analysis of those executives who could serve as models for the preparation of training programmes. For the study in question, the algorithm DOMLEM, of polynomial complexity, was applied for rules type "1" so that it was initially possible to identify those objects belonging to the lower approximation of decision classes of better evaluation ($Cl_3$). At first, this permitted the identification of the executives who really belong to this class of better evaluation, in other words, it determined the model executives from among the other executives. In addition, the inferring of decision rules also permits the identification of the relevant executive competences and could serve as a standard for future evaluations. The decision rules inferred, even though they are not minimums and may not cover all the objects, from the perspective of the concepts of the Dominance principle, permit support for the identification of probable model executives in the set of executives. The creation of executive training programmes is of vital importance when aligned to organizational strategy. This identification of attributes or criteria which will serve as "reducts" (minimums) for information systems or decision tables can become an NP-complete problem, bearing in mind the possibility of the existence of numerous reducts. As an additional proposal for future studies, the possibility of creating a "ranking" of the executives in the evaluation process was identified, supporting the establishment of criteria for a succession process for executives. It is also possible to increase the functions of the software programme in VBA to permit the inference of rules types "2" to "5", with the objective of applying it to large corporations.

Although other methods of uncertainty management could be used, including Dempster-Shafer theory of evidence [15], Bayesian inference, fuzzy sets, fuzzy logic, possibility theory, time Petri nets and evidence theory [8], the choice of RST and DRSA was chosen because it does not require any preliminary or additional information about the data. Furthermore, RST has been developed on solid mathematical foundations and offers effective methods that are applicable in many branches of AI. One of the advantages of RST is that programs implementing its methods may easily run on parallel computers [15]. Possibly, another opportunity to study is the use of a new dominance relation, called "limited dominance-based rough set", a model applicable to incomplete decision system [9].

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# Annex

DOMLEM algorithm [6]

**Procedure** DOMLEM
(**input:** $L_{upp}$ – a family of lower approximations of upward unions of decision classes:
    { $\underline{P}(Cl_t^{\geq})$, $\underline{P}(Cl_{t-1}^{\geq})$, … $\underline{P}(Cl_2^{\geq})$ }; **output:** $R_{\geq}$ set of $D_{\geq}$-decision rules);
**begin**
  $R_{\geq}$:= ∅;
  **for each** $B \in L_{upp}$ **do**
  **begin**
    **E:=find_rules**($B$);
    **for** each rule $E \in$ **E do**
    **if** $E$ is a minimal rule **then** $R_{\geq} := R_{\geq} \cup E$;
  **end**
**end.**

**Function find_rules**
(**input:** a set $B$; **output:** a set of rules **E** covering set $B$);
**begin**
  $G := B$; {a set of objects from the given approximation}
  **E** := ∅;
  **while** $G \neq \emptyset$ **do**
  **begin**
    $E := \emptyset$; {starting complex}
    $S := G$; {set of objects currently covered by $E$}
    **while** ($E = \emptyset$) **or not** ($[E] \subseteq B$) **do**
    **begin**
      $best := \emptyset$; {best candidate for elementary condition}
      **for** each criterion $q_i \in P$ **do begin**
        $Cond := \{(f(x,q_i) \geq r_{qi}) : \exists x \in S \ (f(x,q_i)=r_{qi})\}$;
        {for each positive object from $S$ create an elementary condition}
        **for** each $elem \in Cond$ **do**
      **if** *evaluate({elem}∪E)* is_better_than *evaluate({best}∪E)* **then** $best := elem$;
      **end**;{for}
      $E := E \cup \{best\}$; {add the best condition to the complex}
      $S := S \cap [best]$;
    **end**; {while not ($[E] \subseteq$ B)}
    **for** each elementary condition $e \in E$ **do**
      **if** $[E - \{e\}] \subseteq B$ **then** $E := E - \{e\}$;
    create a rule on the basis of **E;**
    **E** := **E** $\cup \{E\}$; {add the induced rule}
    $G := B - \cup_{E \in \mathbf{E}}[E]$; {remove examples covered by the rule}
  **end**; {while G ≠ ∅}
**end** {function}