DE GRUYTER OPEN

ECONOMIC THEMES (2018) 56(1): 17-34

DOI 10.2478/ethemes-2018-0002

THE APPLICATION OF THE MARKOWITZ'S MODEL IN EFFICIENT PORTFOLIO FORMING ON THE CAPITAL MARKET IN THE REPUBLIC OF SERBIA[1]

Milica Radović

Union University, Faculty of legal and business studies dr Lazar Vrkatić, Republic of Serbia

✉ *mizrad021@gmail.com*

Snežana Radukić

University of Niš, Faculty of Economics, Republic of Serbia

✉ *snezana.radukic@eknfak.ni.ac.rs*

Vladimir Njegomir

Union University, Faculty of legal and business studies dr Lazar Vrkatić, Republic of Serbia

✉ *vnjegomir@sbb.rs*

UDC 336.76 (497.11)

Original scientific paper

Abstract: The characteristics of the developing market to which the Serbian market belongs are: illiquidity, low turnover and non-transparency. The aim of this paper is to examine the possibility of application of the Markowitz's model for forming the set of efficient portfolios on the capital market in Serbia. The research is based on previous theoretical and empirical research in the world. Statistical methods of determining return and risk, matrix of variances and covariances for liquid shares have been applied. Portfolio optimization has been conducted by add-in programme Solver included in the Microsoft Excel package, respecting the limitations according to Markowitz's model. The results of the analysis have shown that the set of efficient portfolios, which meets the criterion that they give minimal risk for the given return, can be formed on the capital market in Serbia. The results show that Markowitz's model allows the investors to select the efficient portfolio, but only for liquid shares, depending on the risk they are ready to accept. The findings of this paper indicate that expressive illiquidity on the Serbian market and insufficient number of shares for diversification by sectors make the practical application of the research results difficult.

Received: 12.03.2018
Accepted: 02.04.2018

Keywords: Markowitz's model, portfolio, expected return, risk, efficient frontier, capital market, Republic of Serbia.

JEL classification: G01, G11, C22

[1] The paper presents a result of the project number III 44007 supported by The Ministry of Education, Science and Technological Development of the Republic of Serbia.

1. Introduction

Harry Markowitz (1952) in the article "Portfolio Selection" laid the foundations of the modern portfolio theory, presenting the model of portfolio optimisation. Presented model is called the Mean-Variance model (MV model), whose aim is not to find the portfolio by which the expected return is maximised, but to find the portfolio that establishes a balance between two basic parameters of the model, the return and the risk.

Markowitz's main idea is to form a mathematical model for portfolio selection, which provides the highest return at a certain level of risk. Such portfolio, which for given level of risk has the highest return, and/or the portfolio that for given return has the minimal risk, Markowitz has called the efficient portfolio. According to the postulate of Markowitz's theory, from the set of possible portfolios an investor decides for (chooses) the optimal or efficient portfolio that satisfies his/her risk preference, and/or the return. More conservative investor will choose the portfolio that has a lower risk, and at the same time lower expected return, while an investor inclined to risk will choose from the set of efficient portfolios, the portfolio with a higher risk that has higher expected return as well.

Markowitz has noted the significance of diversification and correlation among individual shares that are the part of portfolio's composition. This theory starts from the assumption that investors are not inclined to risk, but they want to increase their wealth at the end of the period. According to Markowitz, the methodology of portfolio selection takes place in three phases: forming of possible portfolios, determining the set of efficient portfolios and depending on investment preferences, the selection of optimal portfolio is made.

The Serbian capital market belongs to undeveloped, illiquid markets and therefore, the aim of this paper is the research of the possibility of applying Markowitz's model in portfolio selection, on the capital market in Serbia, and on the basis of the data on movement of share prices in the period from 2013 to 2017. The aim is to apply optimisation to form the portfolios that carry the lowest risk for given return by the application of optimisation. Two hypotheses have been set. The first hypothesis is that by the application of the Markowitz's portfolio theory, on the capital market in Serbia, the set of efficient portfolios can be formed. The second hypothesis is that for the set of efficient portfolios there is a linear correlation between portfolio return and the beta coefficient of portfolio.

The paper should contribute and estimate the possibilities of the application of modern the Markowitz's portfolio theory, but also it should identify the limiting factors for application on developing markets.

2. Literature review

The fundamentals of modern theory have been established by Markowitz's model of portfolio analysis, which starts from the assumption that investors make reasonable decisions based on relation between return and variance, and/or standard deviation, as

the risk measurement (Markowitz, 1952; Fabozzi, et al., 2007). Markowitz's model (Markowitz, 1991) quantified and described the risk. It has shown that unsystematic risk is reduced by diversification and portfolio forming. The basic assumptions of portfolio theory are: „investors maximize the return as they, on the past data, estimate future returns" (Radukić & Radović, 2014, p. 6) and risks and prefer to make decisions where for the same return they select the portfolio with minimal risk. This model has been mostly tested on developed, mature and liquid capital markets (Fama & French, 2004).

In accordance with Markowitz's methodology, portfolio diversification is the basis for forming the optimal portfolio (Black, 1993). The research papers have shown that diversification leads to an average result (Semmler, 2011). Diversification suits those investors on the financial market that cannot reliably estimate movements in the future, as well as those investors that have repugnance towards the risk (Rubinstein, 2002). The research papers have shown that weak correlation of securities reduces the risk without reducing the portfolio return (Jakšić, 2012). Markowitz's portfolio theory is the subject of research on different markets and in different periods of development, which shows that optimal portfolio forming is not the same in a different period (Agustini, 2016).

By the application of Markowitz's theory, Jakšić (2007) formed on the Croatian capital market the set of efficient portfolios in the period from 31 December 1999 to 1 July 2005. Formed efficient portfolios for given risk have the highest return. At the same time it is assessed that the market index CROBEX is not referential, unlike the index on developed markets such as S&P index on the Ameican capital capital.

Markowitz's theory is indispensable in theoretical considerations of validity and of assessment of its applicability on some capital markets. Numerous investors and portfolio managers rely on this model in making investment decisions.

3. Methodology

The objective of every investor is to find optimal combination of the securities from which the portfolio will be made, with the aim to realise the expected return with minimal risk (Sharpe, 1964; Lintner, 1965). Markowitz's theory in forming optimal portfolio starts from the calculation of expected return and risk for shares and portfolios.

3.1. The estimation of expected return of individual shares and portfolios

The return on some security investment represents all cash payments received by virtue of ownership and change in the market price, expressed as the percentage of the initial market price. Return investment is the amount for which invested capital is increased after the process of investing.

Expected return on the share *E(Ri)*, for the period of keeping in ownership, is the mean of average monthly returns in *T* period.

$$E(Ri) = \frac{1}{T}\sum_{t=1}^{T} R_{it} \tag{1}$$

T- number of months of holding security
Rit - average monthly return of *i* share in time *t*

Expected portfolio return *(Rp)*, which consists of *n* shares, is equal to the sum of expected returns of individual shares weighted with percentage participation of each share in portfolio (Alexander, 2008), which the expression (2) shows:

$$E(R_p) = \sum_{i=1}^{n} x_i E(R_i) \tag{2}$$

x_i - percentage participation of shares in portfolio
n - number of shares in portfolio

3.2. The estimation of expected risk of individual shares and portfolios

Risk presents the possibility when something that we plan will not be realised, and/or the state where there is the possibility of negative deviation from desired outcome that we expect or for which we hope. Therefore, it can be said that if the risk would exist, there must be: „possible, cause economic damage, financial determined, uncertain and accidental and there must be a large number of observations so as to determine the legality of occurrence“ (Njegomir, 2011, p. 79).

At the core of portfolio management is the idea that, by combining a certain number of investment instruments, the returns are diversified and the total risk of investors is reduced in regard to individual shares. The risk that can be reduced by diversification is called *specific or unsystematic* risk. There is the risk that cannot be avoided by diversification, it is called *systematic or market risk* (Radović, 2009; Radović & Vračarić, 2013).

Expected risk value (standard deviation of security (δ_i)) of individual share is calculated as the root from variance, where the variance (*Vi*) is square deviation of the actual from the expected return, and the standard deviation is square root from the variance (3)

$$V_i = \frac{1}{T}\sum_{t=1}^{T}(E(Rit) - E(Ri))^{2} \tag{3}$$

Expected porfolio risk (4) is determined by the risk of individual securities, which make the portfolio, expected covariance of return among securities and percentage participation of individual security in the portfolio,

$$V_{port} = \sum_{i=1}^{n} x_i^{2} V_i + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j Cov_{ij} \tag{4}$$

and/or standard deviation is:

$$\delta_{port} = \sqrt{V_{port}} \tag{5}$$

where:

V_{port} - portfolio variance

δ_{port} - standard deviation of portfolio

x_i - percentage participation of security in portfolio

V_i - security variance and,

Cov_{ij} - covariance of return rates of securities "i" and "j".

3.3. The calculation of covariance as the measurement of potential for portfolio diversification

Covariance is the measurement of correlation that presents the sum of weighted products of deviation from expected value of two values. Covariance shows how much two variables change together, whereas variance shows how much one variable changes. In other words, return covariance of i share and j share is equal to the product of their correlation and standard return deviations of the return of i and of j share. It is calculated on the basis of the expression:

$$Cov_{ij} = \sigma i \sigma j \rho ij \tag{6}$$

Cov_{ij} - return covariance of i and j share

ρij – correlation coefficient of shares i and j

$\sigma i \sigma j$ - standard deviation of the returns of i and j share

Return correlation is the measurement of influence of the change in return of one security on the change in return of the other security. The correlation coefficient is a standard statistical measurement of linear connection between two variables. It moves within the interval from -1 (perfectly negative correlation) over 0 (no correlation) to +1 (perfectly positive correlation, the returns of both shares increase).

3.4. Efficient portfolio

According to Markowitz's theory, efficient portfolio is the portfolio that for certain risk has the highest return or the portfolio that for given level of return has the lowest risk. Only one portfolio with highest return suits given risk and that is the efficient portfolio for that level of risk. There is the set of portfolios for different values of risk, which, presented in chart, makes the line of set of efficient portfolios, which we call the efficient frontier. For all portfolios on the efficient line, it is valid that from the set of possible portfolios there is no other portfolio that has the lowest risk for given return. Portfolios that are not on the efficient frontier are called inefficient portfolios. The

basic idea of Markowitz's theory is that the investor will, according to one's own inclination to risk, choose the balance between the return and the risk.

So as to form an efficient portfolio, one should determine percentage participation of each share in portfolio with the fulfillment of conditions that the expected return that carries a minimal risk has been given. By the application of the Markowitz's model of portfolio optimisation, the portfolios that have minimal variance for given return are singled out from the set of possible portfolios on the efficient frontier. Additional conditions are that the sum of participation of some shares in portfolio is equal to 1 and that the participation of individual share cannot be negative value and cannot be higher than 1 (Tomić-Plaziblat et al., 2006; Krneta, 2006).

The optimisation problem set out in this way is defined as follows:

$$\text{Min } V_{port} = \sum_{i=1}^{n} x_i^2 V_i + \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j Cov_{ij} \tag{7}$$

The solution to this problem should meet the following conditions:

$$E(R_p) = \sum_{i=1}^{n} x_i E(R_i) = \text{Const.} \tag{8}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{9}$$

$$x_i \geq 0 \text{ for each i = 1, 2,...,n} \tag{10}$$

$$x_i \leq 1 \text{ for each i = 1, 2,...,n} \tag{11}$$

Solutions of the optimisation problem are participations of individual shares in portfolio. This is the problem of square programming because the function of the aim which is minimised is square, with linear limitations of the type of equality and inequality.

3.5.The beta coefficient

The beta coefficient represents the factor that quantifies the relation between the return rate of security, and/or portfolio and systematic (market) risk. The beta coefficient expresses riskiness of the security in regard to the market risk, and/or it points to expected change of the return of the share in regard to the change of the market portfolio, but it does not point to the level of expected return of that share. The return of the share or the portfolio consists of the component that is a consequence of the change in the market return and the return that is independent of market movements. Linear equation of the return of the security has been given by the expression:

$$R_{jt} = \alpha_j + \beta_j R_{mt} + \varepsilon_{jt} \tag{12}$$

where

R_{jt} – return of the share j in the period t

α_j – constant member of linear regression, the so-called alpha-coefficient that represents the return specific for every individual share that is not of systemic character

β_j – beta coefficient for the share j
R_{mt} – return of the market, and\or of the market index in the period t
ε_{jt} – statistical residual whose $E(\varepsilon_j) = 0$ and represents inexplicable variation whose standard deviation is specific for every share with the assumption of normal distribution

The beta coefficient can be calculated by regression analysis of the return of the share and the return of the market in the specified period. In linear equation the return of the share is dependent variable, the return of the market index is independent variable, so that linear coefficient of the line equation represents the beta coefficient.

The beta coefficient of the portfolio is calculated on the basis of the values of the beta coefficient of the shares that make the portfolio and their participation in the portfolio:

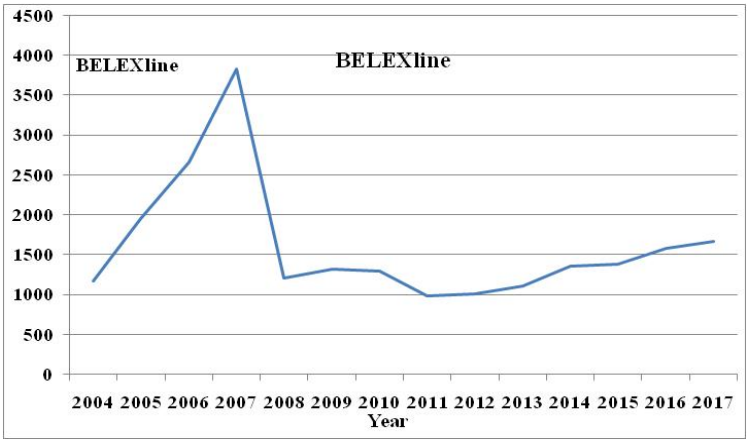
$$\beta_p = \sum_{j=1}^{n} x_j \beta_j \tag{13}$$

where
β_p - beta coefficient of the portfolio
x_j – percentage participation of the security in the portfolio
β_j – beta coefficient of the individual security
n- number of securities in the portfolio.

4. The empirical research

Capital market trend in Serbia had the period of growth from 2004 to 2007, and then of a sudden fall until 2009 (Radović, 2010) after which the period of consolidation was established until 2013. The period from 2013 to 2017 is characteristic by moderate but stable growth. The movement of index Belex-line in the period from 2004 to 2017 has been shown in Figure 1.

Figure 1. The movement of index Belex-line in the period from 2004 to 2017



Source: The Belgrade Stock Exchange, own representation

The research in this paper has been conducted on the shares that were traded on the Belgrade stock exchange in the period from 2013 to 2017.

4.1. Share selection on the capital market in the Republic of Serbia

The criteria for selection of the shares that are included in the portfolio composition are: liquidity, market capitalisation, level of turnover and sector diversification. Accordingly, the first selection of the shares of 21 companies has been done and those are: Aerodrom Nikola Tesla a.d. Beograd (AERO), NIS a.d. Novi Sad (NIIS), Komercijalna banka a.d. Beograd (KMBN), Galenika Fitoframacija a.d. Zemun (FITO), Metalac a.d. Gornji Milanovac (MTLC), Messer Tehnogas a.d. Beograd (TGAS), Energoprojekt holding a.d. Beograd (ENHL), Alfa-Plam a.d. Vranje (ALFA), Jedinstvo a.d. Sevojno (JESV), Sojaprotejin a.d. Bečej (SJPT) Impol Seval a.d. Sevojno (IMPL), Halkbanka a.d. Beograd (CCNB), Jumbes banka a.d. Beograd (JMBN), Energoprojekt Entel a.d. Beograd (EPEN), Kopaonik a.d. Beograd (KOPB), Veterinarski zavod Subotica a.d. Subotica (VTZS) Goša Montaža a.d. Beograd (GMON), Planinka a.d. Kuršumlija (PLNN), Imlek a.d. Beograd (IMLK), AIK banka a.d. Beograd (AIKB), Bambi a.d. Požarevac (BMBI).

Of 21 selected shares, the shares of IMLK in December 2015, BMBI in February 2017, PLNN in June 2017 and AIKB in December 2017 have been excluded from the market and that is why they have not participated in portfolio forming. These shares have been very attractive for the investors and liquid (only PLNN has had somewhat lower liquidity, but, as a rule, it has regularly paid high dividend). This indicates the problem of reduction in number of attractive shares on the Belgrade Stock Exchange.

Table 1. Liquidity of shares on the Belgrade Stock Exchange

	2013		2014		2015		2016		2017		average
	No. trade	%	No. trade	%	No. trade	%	No. trade	%	No. trade	%	%
AERO	252	100%	252	100%	252	100%	253	100%	252	100%	100%
NIIS	252	100%	252	100%	252	100%	253	100%	252	100%	100%
KMBN	138	55%	141	56%	170	67%	175	69%	193	77%	65%
FITO	130	52%	155	62%	151	60%	167	66%	143	57%	59%
MTLC	144	57%	130	52%	139	55%	159	63%	165	65%	58%
TGAS	121	48%	132	52%	150	60%	179	71%	146	58%	58%
ENHL	245	97%	245	97%	247	98%	240	95%	215	85%	95%
ALFA	153	61%	99	39%	137	54%	107	42%	101	40%	47%
JESV	124	49%	121	48%	130	52%	111	44%	123	49%	48%
SJPT	208	83%	221	88%	179	71%	180	71%	169	67%	76%
IMPL	78	31%	95	38%	78	31%	178	70%	184	73%	49%
CCNB	71	28%	36	14%	34	13%	38	15%	61	24%	19%
JMBN	80	32%	63	25%	49	19%	39	15%	73	29%	24%
EPEN	9	4%	24	10%	44	17%	90	36%	75	30%	19%
KOPB	28	11%	22	9%	19	8%	51	20%	104	41%	18%
VZAS	116	46%	122	48%	76	30%	77	30%	90	36%	38%
GMON	162	64%	123	49%	117	46%	88	35%	42	17%	42%

Source: Author's calculation

The selection of liquid shares has been done based on the data on trading on the Belgrade Stock Exchange for the period from 2013 to 2017. The liquidity has been considered for each year separately, mainly through turnover, number of transactions and number of trading days. The most significant indicator, which has been the basis for thr selection of shares, is the liquidity expressed through percentage of number of days in which trading has taken place in regard to total number of trading days in considered period for each share separately. Results of such determined liquidity of trading by years have been presented in Table 1.

It follws from Table 1 that in the five-year period only three shares are extremely liquid and that: AERO, NIIS and ENHL. The shares whose percentage of number of trading days is 60% are just KMBN and SJPT and close to that value, the shares FITO, MTLC and TGAS can be joined. Next group of shares to which ALFA, JESV and IMPL belong, have liquidity slightly below 50%. It is noticed that during five years IMPOL has had stable trend of liquidity growth (from 31% to 73%), while ALFA has had trend of liquidity reduction (from 61% to 40%). All other shares CCNB, JMBN, EPEN, KOPB, VZAS and GMON are illiquid, under 40%, and for that reason, they have been excluded from further consideration. Due to the lack of large number of liquid shares, the authors have decided to analyse 9 shares so that the portfolios would have a larger number of shares, which is the basic meaning of diversification and its forming.

Criterion of sectoral diversification could not be applied due to insufficient number of liquid shares. Review of selected shares that participate in portfolio and affiliation to sectors and market capitalization on 5 January 2018 is presented in Table 2.

Table 2. Percentage participation of selected shares by sectors and market capitalisation on the Serbian capital market

Share	**Name of the sector**	**Market capital. 10^6 din**	participation
AERO	H - Transportation and storage	50,954	20.8%
NIS	B - Mining	115,936	47.4%
KMBN	K - Financial and insurance activities	31,954	13.1%
FITO	C - Manufacturing industry	7,392	3.0%
MTLC	K - Financial and insurance activities	4,229	1.7%
TGAS	C - Manufacturing industry	12,886	5.3%
ENHL	K - Financial and insurance activities	10,931	4.5%
SJPT	C - Manufacturing industry	6,703	2.7%
IMPL	C - Manufacturing industry	3,581	1.5%
total		244,566	100.0%

Source: The Belgrade Stock Exchange.

The sector structure of the selected shares is as follows: four shares belong to the sector of the manufacturing industry, three to the sector of financial activity and only

one to mining and one to transportation. It is noticeable that high participation of mining sector of 47%, with only one company, is unfavourable.

4.2. The calculation of return and risk of individual shares

The empirical research has been carried out based on the mean of monthly returns of the index Belex-line and individual shares for the period from 2013 to 2017. The index Belex-line has been selected for the market portfolio in the analysis. Monthly returns have been calculated on the basis of market prices realised on the last trading day in the month. The values of share prices of MTLC and FITO have been corrected, because in May 2016 and in June 2015 retrospectively, the split of shares was carried out in a ratio 1:2, and/or each shareholder was assigned by the same number of shares he/she owned in the name of profit allocation and growth of initial capital.

The mean of return, variance, standard deviation (square root of variance) and Belex-line for each share have been determined by the application of statistical functions in *Excel*. The beta coefficient for each share in regard to the market index Belex-line has been determined by linear regression analysis of return of individual share with the return Belex-line (Ivković,1980). Results are presented in Table 3.

Table 3. Expected monthly return in %, standard deviation in %, variance, the beta coefficient and constant term of alpha linear regression

	BLX LINE	AERO	NIIS	KMBN	FITO	MTLC	TGAS	ENHL	SJPT	IMPL
average	0,88	2,25	0,11	0,76	1,5	1,58	2,58	1,13	-0,33	3,64
st dev	2,94	6,71	5,16	7,19	5,05	4,38	9,38	7,58	9,97	13,46
var	8,62	45,08	26,59	51,64	25,46	19,18	87,94	57,51	99,41	181,19
beta	1,00	1,53	0,83	1,24	0,88	0,38	1,31	1,32	1,5	1,89
intercept	0	0,9	-0,62	-0,34	0,73	1,24	1,42	-0,04	-1,65	1,98

Source: Authors's calculation. Data on trading have been downloaded from the website of the Belgrade Stock Exchange: www.belex.rs.

It follows from Table 3:

- Belex-line has expected monthly return of 0.88% with standard deviation of 2.94%,
- Six shares have the beta coefficient higher than 1, which means that they are more risky than the whole market, and that three shares are with the beta coefficient lower than 1.
- For three most liquid shares, AERO, NIS and ENHL, the higher beta coefficients correspond to higher monthly returns as well,
- TGAS and ENHL have approximate values of the beta coefficient, but their values of expected returns 2.58% and 1.13% are significantly different. It means that, in addition to systematic risk, other non-systemic factors influence on movement of share price as well.

- IMPL has the highest value of the beta coefficient and, at the same time, the highest expected monthly return,
- SJPT, despite high value of the beta coefficient, has a negative value of expected monthly return.

As the beta coefficient is the measurement of systematic risk, and variance is the measure of complete risk, it follows that three shares of the selected shares are less risky than the market, whereas six shares have the beta coefficient higher than one, which means that they are more risky than the market itself.

4.3. Matrix of variances and covariances of shares

Matrix of variances and covariances of shares is symmetrical square matrix with dimensions of 9 x 9, which is the number of shares that form portfolios. The calculation of matrix of variances and covariances of shares has been done by matrix calculation in Excel, and checks have been carried out on the basis of the expression (5). Its diagonal members are equal to variances of individual shares, and other elements are covariances of couples of shares (see Table 4).

Table 4. Matrix of variances and covariances of shares for portfolio forming

	AERO	NIIS	KMBN	FITO	MTLC	TGAS	ENHL	SJPT	IMPL
AERO	0.0045	0.0012	0.0008	0.0013	0.00004	0.0017	0.0016	0.0013	0.0022
NIS	0.0012	0.0027	0.0011	0.0005	0.0006	0.0007	0.0009	0.0011	0.0024
KMBN	0.0008	0.0011	0.0052	0.0009	0.0007	0.0006	0.0008	0.0018	-0.0002
FITO	0.0013	0.0005	0.0009	0.0025	0.0002	0.0014	0.0010	0.0017	0.0015
MTLC	0.00004	0.0006	0.0007	0.0002	0.0019	0.0007	0.0006	0.0006	0.0012
TGAS	0.0017	0.0007	0.0006	0.0014	0.0007	0.0088	0.0008	0.0006	0.0010
ENHL	0.0016	0.0009	0.0008	0.0010	0.0006	0.0008	0.0058	-0.0003	0.0036
SJPT	0.0013	0.0011	0.0018	0.0017	0.0006	0.0006	-0.0003	0.0099	0.0029
IMPL	0.0022	0.0024	-0.0002	0.0015	0.0012	0.0010	0.0036	0.0029	0.0181

Source: Author's calculation.

Positive values of covariance dominate in the matrix, which indicates that the majority of shares mutually positively covary. Covariances of return of the shares of IMPL i KMBN, as well as ENHL SJPT, have a negative sign. Covariances of return of MTLC and AERO have values close to 0. Simultaneously, covariances of the most liquid shares of AERO, NIIS and ENHL have positive values among themselves.

Matrix of variances and covariances, expected returns and standard deviation of shares present necessary parameters for forming the set of optimal portfolios.

4.4. Forming of the efficient set of portfolios

The participation of each share in the efficient portfolio has been determined on the basis of calculated parameters: vector of share returns, vector of variance and standard deviation of individual shares, matrix of variances and covariances and by the

optimisation process. Optimisation task of forming of the set of efficient portfolios has been defined by the expressions (6-10). Optimisation has been carried out by using Microsoft Excel Solver. The efficient portfolio has been formed by having determined the participation of each individual share in the portfolio for given return, as a parameter (see Table 5), then the risk, standard deviation and the beta coefficient of the portfolio have been calculated (see Table 6). The result of solving the optimization problem is presented as participations of individual shares in 15 portfolios.

It is noticed that portfolio No.5 has minimal variance and that its standard deviation is 3.17% with monthly return of 1.33% and the beta coefficient of portfolio of 0.740. The portfolio with minimal variance has been marked as MPV (Minimum Variance Portfolio). The portfolios from number 5 to 15 belong to the set of efficient portfolios and they lie on the efficient frontier.

Table 5. Percentage participation of share in porfolio

Porfiolio	1	2	3	4	MVP	6	7	8	9	10	11	12	13	14	15
AERO	3,7%	4,9%	6,7%	7,2%	8,1%	12,4%	18,2%	23,9%	29,5%	30,9%	31,9%	18,1%	0,0%	0,0%	0,0%
NIIS	23,8%	21,9%	19,0%	18,2%	16,7%	8,9%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
KMBN	3,9%	3,7%	3,6%	3,5%	3,4%	2,6%	1,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
FITO	24,8%	25,3%	25,8%	25,9%	25,8%	25,7%	24,6%	13,0%	0,7%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
MTLC	37,7%	38,9%	40,7%	41,2%	42,0%	46,0%	50,3%	44,4%	37,8%	22,0%	5,6%	0,0%	0,0%	0,0%	0,0%
TGAS	0,0%	0,1%	0,5%	0,7%	0,9%	2,1%	4,3%	10,8%	17,5%	24,4%	31,3%	36,9%	37,1%	13,6%	4,2%
ENHL	4,4%	4,1%	3,5%	3,3%	3,1%	2,2%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
SJPT	1,7%	1,1%	0,2%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
IMPL	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,7%	7,9%	14,5%	22,8%	31,1%	45,0%	62,9%	86,4%	95,8%
suma	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%

Source: Author's calculation.

Table 6. Parameters of optimal portfolios

Porfiolio	1	2	3	4	MVP	6	7	8	9	10	11	12	13	14	15
Var %	10,27	10,17	10,08	10,07	10,06	10,25	11,25	14,11	19,73	28,58	41,25	59,34	88,38	139,21	167,26
Stdev %	3,20	3,19	3,17	3,17	3,17	3,20	3,35	3,76	4,44	5,35	6,42	7,70	9,40	11,80	12,93
E %	1,15	1,20	1,28	1,30	1,33	1,50	1,75	2,00	2,25	2,50	2,75	3,00	3,25	3,50	3,60
alfa	0,49	0,54	0,63	0,65	0,68	0,84	1,06	1,17	1,28	1,35	1,42	1,58	1,77	1,90	1,95
beta	0,743	0,741	0,738	0,738	0,740	0,750	0,782	0,936	1,101	1,303	1,507	1,609	1,672	1,807	1,861

*Source:*Author's calculation.

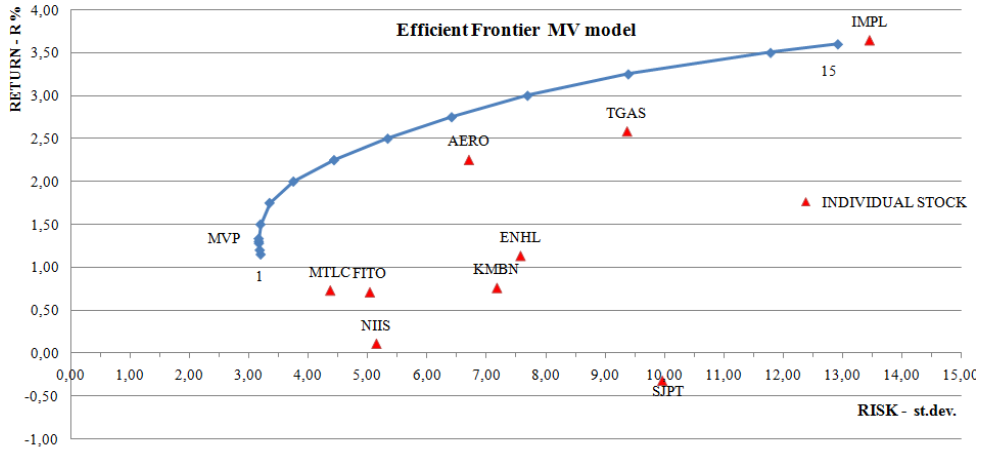
Based on Table 6 it follows that the portfolios from 1 to 4 have standard deviation the same as portfolio 5 and 6, which have higher return. The portfolios from 1 to 4 are below the efficient frontier and they do not belong to the set of efficient portfolios. The portfolio 1 is not efficient, though it has the return of 1.15%, and the standard deviation of 3.20%, because there is the efficient portfolio number 6 which has the same standard deviation, but higher return, and/or 1.50%. According to Markowitz's theory, a reasonable investor will select the portfolio No. 6, but not the portfolio No. 1.

Beta presents the measurement of systematic risk, and/or it expresses to what extent the portfolio is risky in regard to Belex-line. For portfolios on the efficient

frontier, from number 5 to 8, the beta coefficient is lower than 1, and the return is higher than of the market, which means that they are less risky than the index of Stock Exchange. For the portfolios from number 9 to 15, the beta coefficient is higher than 1, which means that they are more risky than the index of Stock Exchange. In other words, for less risky portfolios than the market, the rule is that if riskiness of the market increases for 1%, riskiness of that security will increase for less than 1%. On the capital market in Serbia, by the application of the Markowitz's model, the portfolio with higher returns can be formed, and with lower systematic risk than the market. However, it is noticed that the number of shares in the portfolio reduces with the increase of the beta coefficient, which indicates the insufficient number of liquid shares for portfolio forming.

Figure 2 that expresses dependence of portfolio return on the risk of that portfolio for certain set of shares has presented the line of set of efficient portfolios, which is called the efficient frontier.

Figure 2. The efficient frontier



Source: Author's calculation.

The efficient frontier is presented by the line from point MVP to portfolio 15. Each portfolio on that line presents the optimal portfolio for given level of risk. Process of determining those portfolios has been implemented by optimization, so that there is no portfolio that has higher return for corresponding risk or lower risk for given return.

Except for the efficient frontier, Figure 2 also presents the values of return and risk of the individual shares that participate in portfolio forming. It is noticed that couples of return and risk of individual shares lie below the efficient frontier. It indicates that an investor can select the portfolio that has the same risk, but higher return than of the individual shares. In other words, by portfolio forming, diversification, the return increases for the same risk or the risk reduces for the same return. Diversification reduces only unsystematic risk, whereas systematic risk cannot be reduced by divesifiaction.

It is noticed in Figure 2 on the example of shares that TGAS has the return of 2.58% with standard deviation of 9.38%, whereas, at the same time, portfolio number 11 with close return of 2.50%, as TGAS, has significantly lower risk expressed by standard deviation of 6.42%. In portfolio number 11, AERO participates with 31.9%, MTLC with 5.6%, TGAS with 31,3% and IMPL with 31.1%.

By the analysis of participation of some shares in optimal portfolios on the efficient frontier, the following is noticed:

- In the efficient portfolios, the shares of SJPT are not represented due to negative return and high risk.
- Shares of NIIS, ENHL and KBN are represented only in the portfolios with low returns, which is due to low return of NIIS, then of relatively high standard deviations of ENHL and KMBN, as well as of high value of covariance of the return of NIIS and ENHL.
- In portfolios on the efficient frontier with high returns, the participation of shares increases, primarily of TGAS and of IMPOL, which have high returns. Due to the lack of couples of shares with negative covariances, the efficient frontier for high risks converges towards the portfolio where IMPL is with dominant participation, due to its high value of monthly return.

In accordance with Markowitz's theory, a reasonable investor will select one of the portfolios from No.5 to No.15, depending on the risk that he/she is ready to take. In obtained results, the shares of SJPT are not represented in any efficient set of portfolios due to negative return, and significant risk. It confirms that the model gives logically correct results.

It has also proved that in efficient portfolios with lower return, the shares NIIS, MTLS and FITO are represented. In more risky portfolios with higher returns, the shares with higher return IMPL, TGAS, MTLC have been represented. It is noticed that the portfolio with higher risk in its structure, due to the lack of basic set of shares, converges towards higher participation of IMPL. This indicates logical and correct functioning of the model, but also the problem of insufficient number of shares with different individual characteristics of return and risk.

4.5. The analysis of correlation of return and the beta coefficient of efficient portfolios

For efficient portfolios, which lie on the efficient frontier, the value of expected return and the beta coefficient have been determined. On the sample of 11 efficient portfolios, the hypothesis will be tested that there is a linear dependence of expected return of portfolio on the beta coefficient as the measurement of systematic risk. It means that it will be statistically tested weather there is a linear correlation of expected return as dependent variable and the beta coefficient as independent variable on the sample of 11 efficient portfolios (see Table 7).

Table 7. Expected return and the beta coefficients for efficient portfolios

port	5	6	7	8	9	10	11	12	13	14	15	aver %	st dev %
E port %	1,33	1,50	1,75	2,00	2,25	2,50	2,75	3,00	3,25	3,50	3,60	2,38	0,71
beta port	0,74	0,75	0,78	0,94	1,10	1,30	1,51	1,61	1,67	1,81	1,86	1,22	0,39

Source: Author's calculation.

Zero hypothesis has been set up as *H0***:** Linear coefficient of linear regression of return and the beta coefficient is not statistically important. Alternative hypothesis is *Ha:* Linear coefficient of regression of return and the beta coefficient is statistically important.

Testing of the hypothesis on the sample of n =11 portfolios has been done with threshold of significance alpha =0.05 Testing has been conducted by the linear regression analysis, by the application of T-test that is usually applied on smaller sample and using statistical module in Excel (see Table 8).

Table 8. Results of regression analysis with threshold of significance of 0.05

coef. of regression	value	t-stat	p-value	R^2	cor. R^2	significance
intercept	0.184	1.617	0.14	0.981	0.978	NO
slope	1.806	21.3	5.208E-09			YES

Source: Author's calculation.

Statistical value p-value = 5.21 E-09 has been determined for the linear coefficient (slope). As obtained value p-value is significantly lower than adopted threshold of significance alpha=0.05, zero hypothesis *H0* is dismissed. It implies that alternative hypothesis *Ha* is accepted, and/or it has been statistically presented that there is statistically significant linear connection between return and the beta coefficient of portfolio. Obtained value p-value shows that the alternative hypothesis has been fulfilled and with threshold of significance of 0.001. The coefficient of determination (square of the correlation coefficient) for determined linear regression is even 98.1%. Corrected value of the coefficient is 97.8%. It means that the share of variations explained by the model in total variations of portfolio return is even 98%. This indicates that there is a statistically strong linear correlation. Extremely high value of coefficient of determination is explained by the fact that in formed portfolios, there are the shares that have significant participation in market index.

Conducted statistical T-test has shown that for 11 efficient portfolios there is statistically significant linear connection between portfolio return and the beta coefficient of portfolio.

5. Final considerations and limitations in the research

During the research, it has been demonstrated that illiquidity of the capital market in Serbia is the most dominant problem, which is the consequence of ownership concentration in the companies during privatisation. This problem is manifested in the shape of: lack of trading, low turnovers, great changes of price with low turnovers or high daily oscillations of prices. The lack of liquid shares in some sectors has proven to be a limiting factor for diversification and in the formation of the set of portfolios for analysis. The selected shares, which have been analysed, have insufficient sectoral diversification, because four shares belong to the sector of the manufacturing industry, three to the sector of financial activity, and only one to mining and one to transportation. High participation of mining sector of 47%, with only one company, is unfavourable as well. It makes impossible the analyses that would be based on various and much larger sample of shares.

In addition to specified limiting factors, this paper has demonstrated that, on the Serbian capital market, the Markowitz's model of portfolio theory can be applied for liquid shares and the set of efficient portfolios can be formed. The research has shown that unsystematic risk of individual shares is reduced by diversification and that portfolios have higher return than individual shares. By portfolio optimisation we have confirmed the first hypothesis that by investing in the Serbian securities we can, by diversification, compose the portfolio which carries minimal risk for given return. At the same time, the paper has confirmed that it is valid for efficient portfolios that the higher beta coefficients correspond to higher returns. By the statistical analysis, on the basis of the sample of 11 efficient portfolios, by the application of T-test, the second hypothesis has been confirmed that there is statistically positive linear correlation between return and the beta coefficient of the portfolio.

Numerous empirical research studies, including this research as well, do not offer unambiguous and reliable conclusion about (in)applicability of the Markowitz's portfolio theory. The findings of this paper create assumptions and incentive for additional empirical research and expert discussions on applicability of modern portfolio theory for the most liquid shares on developing markets.

References

Agustini, H. (2016). Analysis of dynamic portfolio allocation of Indonesian LQ45 during 2005-2011 following the Markowitz theory. *Journal The WINNERS*, 17(2), 91-101.

Alexander, C. (2008). *Quantitative Methods in Finance*. London, UK: John Wiley & Sons Ltd.

Black, F. (1993). Beta and Return. *The Journal of Portfolio Management*, 20(1), 8-18.

Fabozzi, F. J., Focardi, S. & Jonas, C. (2007). Trends in quantitative equity management: Survey results. *Quantitative Finance*, 7(2), 115-122. doi: 10.1080/14697680701195941

Fama, E. F., & French, K. R. (2004). The Capital Asset Pricing Model: theory and evidence. *Journal of Economic Perspectives*, 18(3), 25-46. doi:org/10.1257/0895330042162430

Ivković, Z. A. (1980). *Matematička statsistika*. Beograd: Naučna knjiga.
Jakšić, M. (2012). Upravljanje rizicima portfolija hartija od vrednosti. *Ekonomski horizonti*, 2012(3), 151-164. doi:10.5937/ekonhor1203151J
Jakšić S. (2007). Primjena Markowitzeve teorije na tržište dionica Zagrebačke burze. *Zbornik Ekonomskog fakulteta u Zagrebu*, 5, 332-343.
Krneta, S. (2006). *Portfolio hartija od vrednosti i strategije upravljanja porfoliom*. Beograd: Beogradska berza.
Lintner, J. (1965). Security prices, risk and maximal gains from diversification. *Journal of Finance*, 20, 587-616.
Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance*, doi: 10.1111/j.1540-6261.1952.tb01525.x
Markowitz, H. M. (1991). Fundations of Portfolio Theory. The *Journal of Finance*, 46(2), 279-287.
National Bank of Serbia. www.nbs.rs (Accessed Decembar 30, 2017).
Njegomir, V. (2011). *Osiguranje i reosiguranje - tradicionalni i alternativni pristupi*. Zagreb:Tectus.
Radović, M. (2009). *Finansijska tržišta institucije i instrumenti*. Novi Sad: Fakultet za pravne i poslovne studije.
Radović, M., (2010). Serbian stock market trend analysis from 1st March, 2006 TO 31st March, 2009. *Ekonomske teme*, Broj 1, str. 117-130.
Radović, M., Vračarić, V. (2013). Primena beta koeficijenta na tržištu kapitala u Srbiji. *CIVITAS,* 1(1), 127-143.
Radukić, S., Radović, M. (2014). Long Term Trend Analysis in the Capital Market – The Case of Serbia. *Journal of Central Banking Theory and Practice*, doi:10.2478/jcbtp-2014-0013
Reilly, F. K. & Brown, K. C. (2012). *Analysis of investments & management of portfolios* (10th ed). Australia: South-Western Cengage Learning.
Rubinstein, M. (2002). Markowitz's Portfolio Selections: a fifty – year retrospective. *Journal of Finance*, 57(3), 1041-1045.
Semmler, W. (2011). *Asset Prices, Booms and Recessions* (3rd ed). Berlin Heidelberg, Germany: Springer.
Sharpe, W. F. (1964). Capital asset price: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425-442.
Tomić-Plazibat, N., Aljinović, Z., Marosović, B. (2006). *Matematički modeli u financijskom upravljanju.* Split: Ekonomski fakultet.

PRIMENA MARKOWITZEVOG MODELA U FORMIRANJU EFIKASNIH PORTFOLIJA NA TRŽIŠTU KAPITALA U REPUBLICI SRBIJI

Apstrakt: Karakteristike tržišta kapitala u razvoju, kome pripada srpsko tržište, su nelikvidnost, mali prometi i netransparentnost. Cilj rada je da se ispita mogućnost primene Markowitzevog modela za formiranje skupa efikasnih portfolija na tržištu kapitala u Srbiji. Istraživanja su bazirana na prethodnim teorijskim i empirijskim istraživanjma u svetu. Primenjene su statističke metode utvrđivanja prinosa i rizika, matrice varijansi i kovarijansi za likvidne akcije.

Optimizacija portfolija sprovedena je potrogramom Solver ugrađenom u paket Microsoft Excel, uvažavajući ograničenja po Markowitzevom modelu. Rezultati analize su pokazali da se na tržištu kapitala u Srbiji može formirati skup efikasnih portfolija, koji ispunjavaju kriterijum da za zadati prinos daju minimalan rizik. Rezultati pokazuju da Markowitzev model omogućuje investitorima da izaberu efikasan portfolijo, ali samo za likvidne akcije, u zavisnosti od rizika koji su spremni da prihvate. Zaključci ovog rada ukazuju da praktičnu primenu rezultata istraživanja otežava izražena nelikvidnost na srpskom tržištu i nedovoljan broj akcija za diverzifikaciju po sektorima, te je neophodno dodatno ispitivanje.

Ključne reči: Markowitzev model, portfolijo, očekivani prinos, rizik, granica efikasnosti, tržište kapitala, Republika Srbija.

Authors' Biographies

Milica Radović is an associate professor at Union University, Faculty of legal and business studies dr Lazar Vrkatić in Novi Sad. She teaches the subjects Basics of economics and Financial markets on the study group: Business economics. She received her PhD at Faculty of Technical Sciences of the University in Novi Sad in the area of investment management. The subject of interest is: economy and financial markets. She has published a two books, one monograph, and numerous papers in scientific journals and proceedings of national and international conferences.

Snežana Radukić is an associate professor at Faculty of Economics, University of Niš, where she teaches Microeconomics and Theory and Policy of Pricing. She has obtained her PhD at University of Niš, Faculty of Economics. Her contemporary research interests encompass: microeconomics and markets of production factors. She has published a book, three monographs, and numerous papers in scientific journals and proceedings of national and international conferences. She is a researcher at the project No. 44007 supported by Ministry Education, Science and Technological Development of the Republic of Serbia.

Vladimir Njegomir is an associate professor at Union University, Faculty of legal and business studies dr Lazar Vrkatić in Novi Sad. He teaches the subjects Insurance, Risk management in insurance and Entrepreneurship on the study group: Business economics. He has two doctorates. One doctorate is in the area of structured finance, **namely** insurance risk management on the capital market, at Faculty of Technical Sciences of the University in Novi Sad. The second doctorate is in the area of entrepreneurship and insurance, at the University for Peace of the United Nations. The subject of interest is: risk management, insurance, reinsurance and entrepreneurship.