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FRACTAL ANALYSIS OF LEAVES: ARE ALL LEAVES SELF-SIMILAR ALONG THE CANE?

ANNA GAZDA

Department of Botany and Nature Conservation, Forest Biodiversity Institute, University of Agriculture in Krakow, al. 29 Listopada 46, 31-425 Krakow, Poland; e-mail: rlgazda@cyf-kr.edu.pl

Abstract
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The fractal dimension can be used to quantify the shape of a natural curve. Curves with similar degrees of irregularity will tend to have the same fractal dimension. The fractal exponent describes the complexity of a shape and characterizes the scale-dependency of the pattern. This article presents an application of the fractal dimension in the analysis of leaves shape. In this paper I attempt to ask question if leaves of blackberry characterized by fractal dimension differ significantly in relation to the leaf's position along the cane. The fractal dimension of 49 leaves of blackberry from 8 primocanes, and 53 leaves from 19 lateral canes, from 9 individuals was estimated. The mean of D of a leaf is 1.12. There are no significant differences between D for leaves from two different cane types. Previous studies were focused on measurements of fractal dimension of leaves randomly chosen from one or a few individuals so there was necessity to measure fractal dimension all leaves growing along the same shoot, because usually leaf shape and size change more or less along a shoot. This research confirmed that fractal dimension is much more related to the shape complexity than to the size of leaves.

Key words: the fractal dimension, leaf morphology, blackberry.

Introduction

Shapes and patterns of natural objects hold the interest of morphologists, botanists and ecologists (Bradbury et al., 1984; Morse et al., 1985; Vlcek, Cheung, 1986; Palmer, 1988; Prusinkiewicz et al., 1988; Fitter, Stickland, 1992). Until recently the description of a shape have relied on applying qualitative "measures" or simplifying the natural objects using Euclidean geometry, which cannot be used to adequately represent complex, natural objects.

There is a growing interest in the numerical specification of shapes and patterns (Kincaid, Schneider, 1983; Borkowski, 1999). The shapes of many natural objects differ in detail but remain statistically self-similar. The fractal dimensions (Mandelbrot, 1983), can be used to quantify the shape of a natural curve. Curves with similar degrees of irregularity will tend to have the same fractal dimension. Mandelbrot defined a fractal as "a set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension" (Mandelbrot, 1983). In Euclidean geometry the dimension of any object is characterized by integer values. In the fractal geometry, the fractal dimension of an object is non-integer, and its value depends on object's degree of complexity. The fractal exponent describes the complexity of a shape and characterizes the scale-dependency of the pattern. Moreover this complexity of shape is reflected in the speed with which apparent dimension (the length, the perimeter, the area, the volume) changes as measurement scale changes. Very important concept in the fractal theory is the self-similarity and self-affinity. A lot of natural objects are statistically self-similar, it does mean that each portion can be considered as a reduced-scale image of the whole. Many attributes of natural objects could be very useful if they can be quantified using the fractal dimension, such as: (1) the complex shape of different objects (the numerical specification of form is important particularly in the study of morphogenesis and morphological patterns at all.; e. g. Vlcek, Cheung, 1986; Dicke, Burrough, 1988; Prusinkiewicz et al., 1988; (2) the branching structure of plants (as a tool to create architectural models); (3) or a quantification of the complexity of an object's structure (e. g. Fitter, Stickland, 1992; Sisó et al., 2001; Kalliokoski et al., 2010).

This article presents an application of the fractal dimension in the analysis of leaf shape. In this paper I attempt to ask question if leaves of blackberry (*Rubus hirtus* Waldst. & Kit. agg.) characterized by fractal dimension differ significantly. Are there any natural directions of changes of fractal dimension of leaves in relation to the leaf's position along the cane (from the oldest to the youngest leaves)?

Material and methods

Rubus hirtus Waldst. & Kitt. agg. belongs to the *Rosaceae* family, Genus *Rubus* L., Series *Glandulosi* (P. J. Mueller) (Tutin et al., 1968). *Rubus hirtus* is a clonal woody plant (Fig. 1). It propagates vegetatively mainly by tip

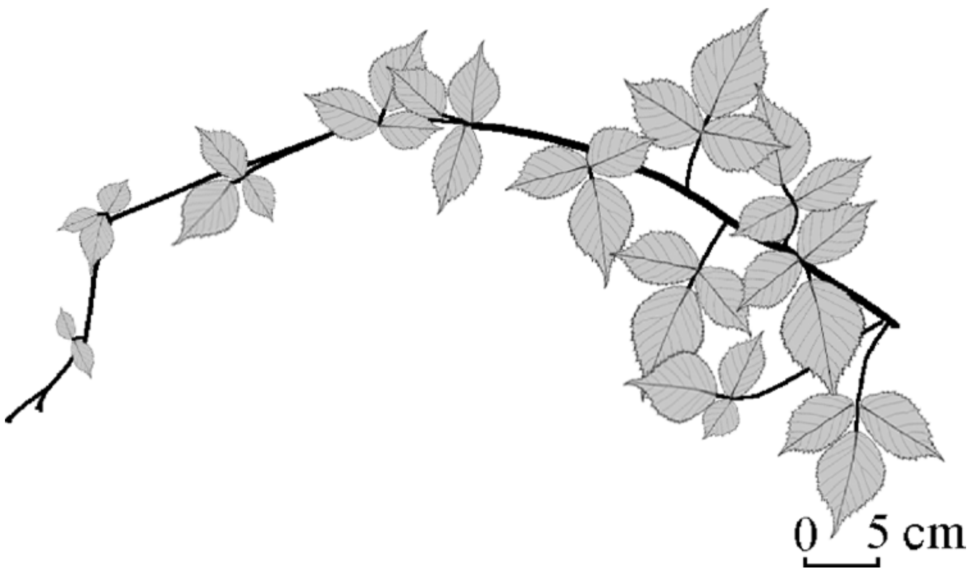

Fig. 1. The blackberry individual. This one created only one main primocane which had developed from crown as stoloniferous shoot.

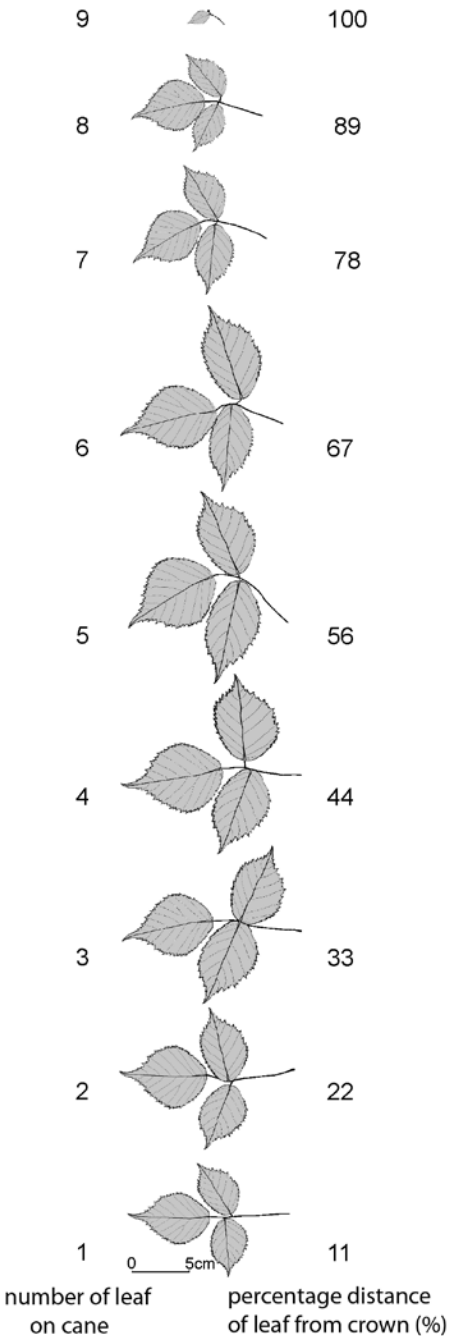

Fig. 2. Leaves from one of the blackberry main primocane.

rooting usually one of one or a few primocanes. It creates two main types of shoots which develop during one season: the primocanes and floricanes. Primocanes are one year old shoots and floricanes are two years old shoots with both lateral inflorescences and vegetative laterals. Main primocanes develop from crown at first as an erect shoot; next some of them change direction of growth from vertical to horizontal. Then few of stoloniferous primocanes are able to tip-rooting and forming new daughter plant. Blackberry is a very common species of the forest floor in southern Poland. Under a very dense forest canopy it is able to grow as a stunted plant (every year producing single, a very short cane which does not tip-root) for several years. However after cutting some trees it uses increase in abundance, by producing both more and longer canes, which are able to create new ramets, and next to dominate forest floor for a few years.

Investigation of blackberry was carried out in the Gorce National Park in the southern part of Poland. This sampling area was located in the natural forest within the area named the Łopuszna valley. At first during the autumn there were randomly chosen blackberry individuals, next the leaves from these individuals were picked up and labelled (Fig. 2). Then all leaves were scanned at 300 dpi resolution using an Epson table scanner and stored as PCX image file format.

The fractal dimension can be determined mathematically if the generating algorithm is known, or empirically if the object is measurable. There are many ways of estimating the fractal dimensions of fractal objects (for a summary of methods see: Peitgen, Soupe, 1988; Sugihara, May, 1990; Hastings, Sugihara, 1993; Gazda, 1996). One of them is the grid method since this method is highly sensitive to the variance, or dispersion, of any aggregation. Another method is the divider method which is more sensitive to the jaggedness of outline than the position of every the part of plant to each other. I decided to use the divider method to estimate the fractal dimension of leaves.

The fractal dimension of leaves was estimated using the program FAN (Tokarski, 1992) which was written for measuring the fractal dimension of outlines using, either the grid or the divider method.

T a b l e 1. The fractal dimension of blackberry leaves.

Number of individual	Number of leaf	Fractal dimension (D)	Cane type MC — main LC — lateral
1	1	1.14	MC
1	2	1.17	MC
1	3	1.17	MC
1	4	1.12	MC
1	5	1.14	MC
1	6	1.11	MC
1	7	1.13	MC
1	8	1.17	MC
1	9	1.1	MC
2	2	1.13	LC
2	3	1.1	LC
2	4	1.12	LC
2	5	1.08	LC
2	6	1.1	LC
3	2	1.13	LC
4	2	1.13	LC
4	3	1.08	LC
5	1	1.1	MC
5	2	1.13	MC
5	3	1.13	MC
5	4	1.11	MC
5	5	1.12	MC
5	6	1.1	MC
5	7	1.13	MC
5	8	1.13	MC
5	9	1.11	MC
5	10	1.11	MC
6	2	1.12	LC
6	3	1.09	LC
7	1	1.14	LC
7	2	1.12	LC
7	3	1.1	LC
7	4	1.07	LC
8	1	1.12	LC
8	2	1.13	LC
8	3	1.1	LC
8	4	1.05	LC
9	1	1.13	LC
10	1	1.18	MC
10	2	1.13	MC
10	3	1.14	MC
10	4	1.13	MC
10	5	1.12	MC
10	6	1.13	MC
10	7	1.08	MC
10	8	1.13	MC
10	9	1.14	MC
10	10	1.12	MC
11	2	1.13	LC
11	3	1.07	LC
11	4	1.12	LC

Number of individual	Number of leaf	Fractal di- mension (D)	Cane type MC — main LC — lateral
11	5	1.1	LC
12	2	1.12	LC
12	3	1.12	LC
14	1	1.14	LC
14	2	1.13	LC
14	3	1.07	LC
14	4	1.11	LC
14	5	1.04	LC
15	1	1.12	LC
15	2	1.16	LC
16	1	1.11	LC
17	1	1.06	LC
17	2	1.1	LC
17	3	1.05	LC
18	1	1.12	MC
18	2	1.12	MC
19	1	1.15	MC
19	2	1.14	MC
19	3	1.13	MC
19	4	1.13	MC
19	5	1.1	MC
19	6	1.11	MC
20	1	1.14	LC
20	2	1.13	LC
20	3	1.09	LC
21	1	1.12	MC
21	2	1.09	MC
21	3	1.11	MC
21	4	1.11	MC
21	5	1.13	MC
22	1	1.09	MC
22	2	1.15	MC
22	3	1.09	MC
22	4	1.15	MC
23	1	1.11	LC
23	2	1.07	LC
23	3	1.08	LC
24	1	1.15	LC
24	2	1.12	LC
24	3	1.09	LC
25	1	1.09	LC
25	2	1.12	LC
25	3	1.07	LC
26	1	1.12	LC
26	2	1.11	LC
26	3	1.11	MC
26	4	1.12	MC
25	5	1.12	MC
27	1	1.14	LC
27	2	1.13	LC
27	3	1.11	LC

Results and discussion

The fractal dimension of 49 leaves was estimated from 8 primocanes, and 53 leaves from 19 lateral primocanes, from 9 individuals (Table 1).

The mean of D is respectively 1.12 (range = 1.09—1.17) for the main primocane and 1.12 (range = 1.09—1.18) for lateral primocane (Fig. 3). There are no significant differences between D for leaves from different cane types (t-test: variance D_{MC} = 0.0003; variance D_{LC} = 0.0004; df = 52; P = 0.616; Wilcoxon test: z = 0.204, P = 0.8384). Figure 4 shows the changes of fractal dimension according to the sequence of leaf occurrence on its cane. There are no significant differences between the fitted curves to these data ($coefficient_{MP}$ = 0.09V0.02; $coefficient_{LC}$ = 0.10V0.03; F-statistic: 10.29 with 2 and 50 df; P = 0.0001802).

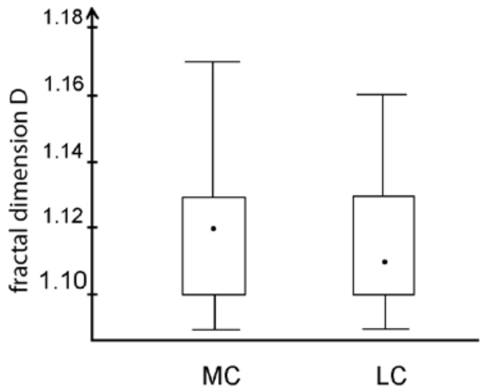

Fig. 3. The dispersion of the fractal dimension of blackberry leaves on two different types of cane: a main primocane and a lateral cane (MC — main primocane, LC — lateral cane).

The development of an organ or organism is characterized by a succession of stages each of which may involve distinctive morphological patterns. In blackberry, as in many others species, leaf shape and size change a little bit node-by-node along a shoot. The largest leaves (in the meaning either of the length, biomass and area of a leaf) are growing almost in the middle (¼–½) of blackberry cane. The fractal dimension of leaves changes according to the sequence of occurrence of leaves on the cane. The highest value of D is for leaves that are either the oldest or the youngest. To make the results easier to interpret, the number of leaves was expressed as a percentage leaf's distance from the crown. Next curves were fitted (Fig. 2). Coefficient "a" says us about the "rate" of change of values of the fractal dimension along the cane. An ANOVA shows that there are no significant differences between these coefficients for changes of D along the lateral canes or primocanes. Differences between the fractal dimension of leaves along a cane reflect their sensitivity to shape changes in the of leaf's outline. This process is caused by two phenomena: the different developmental stages of every leaf and the process of morphogenesis of the leaf (the shape of the leaf from every bud, from which it was developed, is coded by

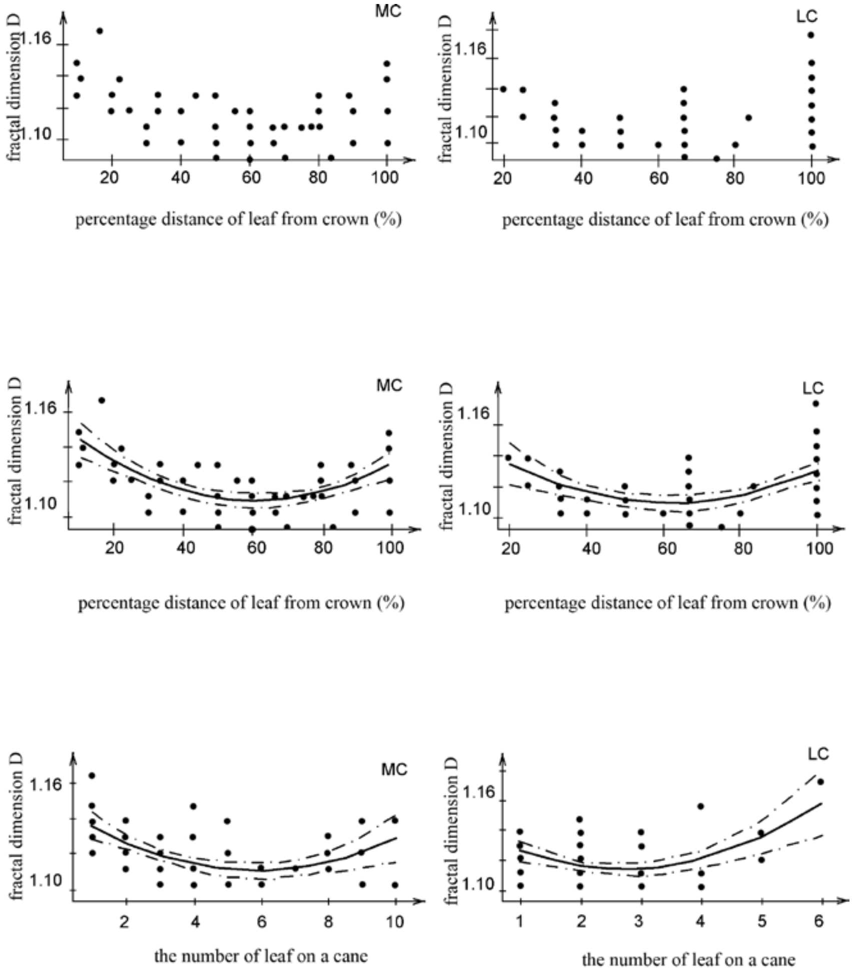

Fig. 4. The changes of the fractal dimension of blackberry leaves, along two types of cane: a main primocane (MC) and a lateral cane (LC): the x-axis shows distance of a leaf from a crown as percentage distance, and in sequence of leaves on a cane.

its genes and is usually a little different from each other). According to results of an analysis of variance and the mean value of D within these setsthere are no significant differences between the fractal dimension for leaves which occur on primocanes and lateral canes.

The results of this study are a very important because previous studies were focused mainly on measurements of fractal dimension of leaves randomly chosen from one or a few individuals of one species or of few species (e.g. Vlcek, Cheung, 1986; Borkowski, 1999). Now when quite a lot of researches want to apply fractal geometry both to generate shapes of different plants for modelling various both patterns and processes (e.g. Jonckheere et al., 2006; Gastner et al., 2009) and as a tool for morphometric studies and automated identification of plants (e. g. Borkowski, 1999; Bruno et al., 2008) there was necessity to measure fractal dimension not only of a few randomly chosen leaves, but all leaves growing along the same shoot, because usually leaf shape and size change more or less along a shoot. So the main aim of this study was to determine if it was any gradient or direction of changes of fractal dimension of consecutively leaves along any cane. This research confirmed that fractal dimension is much more related to the shape complexity than to the size of leaves.

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