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ELASTICITY OF CONSUMER DEMAND: ESTIMATION USING A QUADRATIC ALMOST IDEAL DEMAND SYSTEM

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Abstract: This paper analyzes consumer demand using a Quadratic Almost Ideal Demand System (QUAIDS), which is an extension of an Almost Ideal Demand System (a complete demand system). Complete demand system models can be used to find the relationships that are impossible to discover using single-equation models. Additionally, the QUAIDS model can be used to model a nonlinear Engel curve. The research was based on household microeconomic data collected by GUS (the Polish Central Statistical Office) in the period 1999-2015. The complete demand model has been extended by demographic variable. The results show that the QUAIDS model does not reduce to the AIDS model and is an adequate tool to analyze consumer demand.

Keywords: complete demand system, Quadratic Almost Ideal Demand System, QUAIDS, elasticity of demand.

1. Introduction

Analysis of consumer demand is one of the most popular research areas in microeconomics. The first empirical research was conducted by Charles Davenant [1699], who presented a table of demand for wheat. In the next century, several empirical papers were published [Lloyd 1771; Verri 1771; Smith 1776], which stated the influence of demand and supply on price. This process is constantly being developed.

The Polish situation from 2000 to 2015 has changed significantly due to Poland joining the European Union. This has been proven by the Polish Central Statistical Office. The Gross Domestic Product in constant prices from 2000 to 2015 has increased by nearly 77%. The Polish literature concerning the analysis of demand is dominated by classical research involving mainly quantitative changes in demand or using single-equation models [Dudek 2011; Gulbicka, Kwasek 2006; Kwasek 2008; Stanisławska, Wysocki 2011; Stanisławska 2012]. However, single-equation models do not allow for complex analysis that describes and predicts the whole structure of demand. This is caused by the omission of a priori conditions, which results from economic theory and a lack of a demand function that analyzes additivity separately.

Complete demand systems are adequate tools for describing the whole structure of consumption. Construction of the demand function system can be used to find the relationships that are impossible to discover using single-equation models. In Polish literature, some studies use the complete demand system but only for a selected group of products, such as meat [Wolak 2008], alcohol [Gurgul, Wolak 2008] and basic food products [Dudek 2008]. There is a lack of research that accounts for the structure of all expenditure, including expenditure on clothing, recreation and transport. The latest research involving complete demand systems for the whole structure of consumption was conducted by Suchecki [2006] and Dudek [2011]. The main aim of this work is to determine the elasticity of demand for consumer goods using complete demand systems. The complete demand system was extended by incorporating demographic variable for Poland from 1999 to 2015.

2. Data

The analysis is based on a sample of household data derived from the Polish Household Budget Survey conducted by the Polish Central Statistical Office. Household Budget Surveys (HBS) are considered a basic information source according to income and expenditure. Additionally, the HBS includes demographic characteristics that allow for the modeling of the heterogeneity and better understanding of consumer demand.

Household expenditures were divided into groups according to the classification of expenditure based on COICOP/HBS [GUS 2011]. There were 12 groups of expenditure: food and non-alcoholic beverages (FOOD); alcoholic beverages, tobacco and narcotics (ALCO); clothing and footwear (CLOTH); housing, water, electricity, gas and other fuels (HOME); furnishings, household equipment, and routine maintenance of the house (EQUIP); health (HEALTH); transport (TRAN); communication (COMMU); culture and recreation (CULT); education (EDUC); restaurants and hotels (HOTE); and miscellaneous goods and services (OTHER).

The estimation of the complete demand system with the proposed expenditure groups requires the incorporation of the price indexes. The price indexes for these expenditure groups are available from 1999. The data from 2015 were the latest data available at the moment of writing this paper.

3. Quadratic Almost Ideal Demand System

Deaton and Muellbauer [1980] proposed the Almost Ideal Demand System (AIDS), which is one of the most popular versions of the complete demand system. They adopted a flexible functional form for the indirect utility function developed from a general class of Price-Independent Generalized Logarithmic (PIGLOG) models. Deaton and Muellbauer restricted preferences to be linear with respect to the logarithm of income.

But Banks, Blundell and Lewbel [1997] proved that the linear relation is too strong and investigated a generalized AIDS model that also includes a quadratic logarithmic income term. Their model is known as the Quadratic Almost Ideal Demand System (QUAIDS). By imposing particular restrictions on the parameters of this model, this can be reduced to the Almost Ideal Demand Model.

The QUAIDS model is based on the following indirect utility function:

$$\log\psi = \left\{\left(\frac{\log x - \log a(\mathbf{p})}{b(\mathbf{p})}\right)^{-1} + \lambda(\mathbf{p})\right\}^{-1}, \qquad (1)$$

where x is the total expenditure, and **p** is a vector of prices.

$$\log a(\mathbf{p}) = a_0 + \sum_{k=1}^{n} a_k \log p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n}\gamma_k^* \log p_k \log p_l,$$

$$b(\mathbf{p}) = \prod_{k=1}^{n} p_k^{\beta_k} = \exp\left(\sum_{k=1}^{n}\beta_k \log p_k\right),$$

$$\lambda(\mathbf{p}) = \sum_{k=1}^{n}\lambda_k \log p_k.$$

Substituting these price indexes into the QUAIDS indirect utility function ψ gives:

$$\log\psi = \left\{\left(\frac{\log x - (a_0 + \sum_{k=1}^{n} a_k \log p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n}\gamma_k^* \log p_k \log p_l)}{\prod_{k=1}^{n} p_k^{\beta_k}}\right)^{-1} + \sum_{k=1}^{n}\lambda_k \log p_k\right\}^{-1}, \qquad (2)$$

which corresponds to the following cost function:

$$\log c = a_0 + \sum_{k=1}^{n} a_k \log p_k + \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n}\gamma_k^* \log p_k \log p_l + \frac{u\prod_{k=1}^{n} p_k^{\beta_k}}{1 - u\sum_{k=1}^{n}\lambda_k \log p_k}$$

$$\log c = \log a(\mathbf{p}) + \frac{ub(\mathbf{p})}{1 - u\lambda(\mathbf{p})}. \qquad (3)$$

In the case of all $\lambda_i = 0$, this equation is reduced to the AIDS cost function [Moro, Sckokai 2000].

By applying Roy's identity to the indirect utility function, one can derive the expenditure share equation:

$$w_i = \alpha_i + \sum_{j=1}^{n} \gamma_j \log p_j + \beta_i \left(\log \frac{x}{a(\mathbf{p})} \right) + \frac{\lambda_i}{b(p)} \left(\log \frac{x}{a(\mathbf{p})} \right)^2, \tag{4}$$

where w_i is the share of expenditure.

The regularity condition resulting from economic theory implies the fulfillment of the following constraints on the parameters [De Agostini 2014]:

- Adding up:

$$\sum_{i=1}^{n} \alpha_i = 1, , \sum_{i=1}^{n} \beta_i = 0, \ \sum_{i=1}^{n} \lambda_i = 0, \ \sum_{i=1}^{n} \gamma_{ij} = 0, \quad j = 1, 2, \ldots, n,$$

- Homogeneity: $\sum_{j=1}^{n} \gamma_{ij} = 0$, $i = 1, 2, \ldots n$,
- Symmetry, by: $\gamma_{ij} = \gamma_{ji}$ $i, j = 1, 2, \ldots n$.

The elasticity coefficient can be calculated according to the following formula [Banks, Blundell, Lewbel 1997; De Agostini 2014; Castañon-Herrera, Urzúa 2011]:

- Uncompensated own and cross price elasticity coefficient:

$$e_{ij} = \frac{p_j}{q_i} \cdot \frac{\partial g_i(x, \mathbf{p})}{\partial p_j} = -\delta_{ij} + \frac{\mu_{ij}}{w_i}, \tag{5}$$

- Compensated own and cross price elasticity coefficient:

$$\tilde{e}_{ij} = \frac{p_j}{q_i} \cdot \frac{\partial h_i(u, \mathbf{p})}{\partial p_j} = -\delta_{ij} + \frac{\mu_{ij}}{w_i} + w_j \mu_i, \tag{6}$$

- Expenditure elasticity coefficient:

$$e_i = \frac{x}{q_i} \cdot \frac{\partial g_i(x, \mathbf{p})}{\partial x} = 1 + \frac{\mu_i}{w_i}, \tag{7}$$

where $\mu_i = \beta_i + \frac{2\lambda_i}{b(\mathbf{p})} \cdot \log\left(\frac{I}{a(\mathbf{p})} \right)$,

$$\mu_{ij} = \gamma_j - \mu_i \left(\alpha_j + \sum_{k=1}^{n} \gamma_k \log p_k \right) - \frac{\lambda_i \beta_j}{b(\mathbf{p})} \left(\log\left(\frac{I}{a(\mathbf{p})} \right) \right)^2,$$

δ_{ij} – Kronecker delta.

The Quadratic Almost Ideal Demand Model is based on the Seemingly Unrelated Regression (SUR). The estimation process was conducted in SAS 9.4 using the Proc Model procedure. To obtain the coefficients, the Iterative Feasible Generalized Least Squares was used. All the shares of expenditure sum up to one (adding up restriction), which causes the variance-covariance matrix to be singular [Gurgul, Wolak 2008]. To avoid this problem, one equation was deleted and the remaining coefficients were determined indirectly.

4. Results

Estimation of the complete demand system with the proposed expenditure groups requires the incorporation of the price indexes. This could be problematic because these price indexes are very often strongly correlated, which could cause a collinearity problem in the modeling process. Gostkowski [2015] described this problem extensively. Another problem when modeling the complete demand system, is households without expenditure for selected groups.

According to these facts, the number of estimated equations was limited to the six groups proposed by Gostkowski [2015]:

- W1: Home and equipment including expenditure for housing, water, electricity, gas and furnishings, household equipment.
- W2: Communication and education.
- W3: Miscellaneous goods and services including expenditure for alcoholic beverages, tobacco, expenditure for clothing and footwear and miscellaneous goods and services.
- W4: Transport and recreation
- W5: Food and health.
- W6: Restaurants and hotels.

Demographic variables play an important role in the analysis of consumer demand. They enable the understanding of consumer behavior in a much better way. Therefore QUAIDS was extended according to the number of people in a household. There are general procedures for taking demographic variables into account [Dudek 2011]. In the presented study, the demographic interpretation proposed by Pollak and Wales [1979] was applied. Statistical tests confirmed the importance of the incorporated variables (Table 1). The following modified QUAIDS model was used to calculate the demand elasticity:

$$\hat{w}_i = \alpha_i(z) + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i(z)\left(\ln x - \alpha_0(z) - \sum_{k=1}^{n} \alpha_k(z) \ln p_k - \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n} \gamma_{kl} \ln p_k \ln p_l \right) + \frac{\lambda_i(z)}{\prod_{k=1}^{n} p_k^{\beta_k(\mathbf{z})}} \left(\ln x - \alpha_0(z) - \sum_{k=1}^{n} \alpha_k(z) \ln p_k - \frac{1}{2}\sum_{k=1}^{n}\sum_{l=1}^{n} \gamma_{kl} \ln p_k \ln p_l \right)^2, \quad (8)$$

where z is the number of people in a household, and

$$\alpha_0(z) = \alpha_0 + \alpha_z z,$$
$$\alpha_i(z) = \alpha_i^0 + \alpha_i z,$$

$$\beta_i(z) = \beta_i^0 + \beta_i z,$$
$$\lambda_i(z) = \lambda_i^0 + \lambda_i z.$$

Table 1. Results of testing the hypothesis about the importance of demographic variable in the QUAIDS model

$\alpha_i z = 0,\ \beta_i z = 0,\ \lambda_i z = 0,$ $i = 1,2,3,4,5.$	Test statistic	*p-value*
Wald Test	W = 57284.1	<0.0001
Likelihood ratio test	LR = 35332.2	<0.0001

Source: own study.

The QUAIDS model is a generalized version of the AIDS model, so a question arises about which form is correct. It was mentioned that in the case that all λ_i the QUAIDS model reduces to the AIDS model [Moro, Sckokai 2000]. The results showed that all $\lambda_i = 0$ were statistically significant (Table 2). The Wald test and likelihood ratio test confirmed that the QUAIDS model does not get reduced to the AIDS model, and the QUAIDS model should be used to determine the elasticity coefficient (Table 3).

Table 2. Coefficients of selected parameters for QUAIDS model estimated for Poland. The remaining coefficients are in the attachment

Parameter	Estimate	Approx Std Err	*p-value*
λ_1^0	–0.0046	0.000234	<.0001
λ_2^0	–0.0007	0.000046	<.0001
λ_3^0	–0.0004	0.000116	0.0020
λ_4^0	0.0036	0.000170	<.0001
λ_5^0	0.0002	0.000148	0.1985

Source: own study.

Table 3. Results of testing the hypothesis about the reduction of the QUAIDS model to the AIDS model

$\lambda_i^0 = 0$ $i = 1,2,3,4,5.$	Test statistic	*p-value*
Wald Test	W=1011.6	<0.0001
Likelihood ratio test	LR=3294.8	<0.0001

Source: own study.

5. Elasticity of demand

One of the most popular measures in consumer demand is the income elasticity coefficient. It informs us about the percentage change in demand in response to a one percent change in consumer income. The QUAIDS model enables us to determine the expenditure elasticity coefficient, which shows the relative change of demand for a good or service in response to a one percent increase in all expenditure. In this paper, the demand means the total expenditure for specific goods or services.

Table 4. Expenditure elasticity coefficient estimated for a typical household in Poland based on the QUAIDS results. Standard error values were placed in parenthesis

	Home and equipment	Communication and education	Miscellaneous goods and services	Transport and recreation	Food and health	Restaurants and hotels
Elasticity coefficient	0.878 (0.0070)	0.669 (0.0080)	1.292 (0.0085)	1.697 (0.0085)	0.717 (0.004)	1.517 (0.0106)

Source: own study.

The most important group is clearly the food and health group, for which the expenditure elasticity coefficient is 0.717. This indicates that a one percent increase in all expenditure leads to a rise in demand for food and health by about 0.717 percent on average (*ceteris paribus*). Additionally for two groups (home and equipment, communication and education), the expenditure elasticity coefficients were smaller than one. Therefore, these groups can be interpreted as necessity goods. The highest expenditure elasticity coefficients were calculated for the transport and recreation group. This group can be interpreted as luxury goods together with the restaurants and hotels group as well as miscellaneous goods and services. Janský [2014], Dybczak et al. [Dybczak, Tóth, Voňka 2010], and Blacklow et al. [Blacklow, Nicholas, Ray 2010] presented similar results. However, it should be noted that these studies built models for other households, and any comparisons warrant caution.

The next important measure in demand analysis is the compensated price elasticity coefficient. It informs us about the percentage change in demand in response to a one percent change in price (*ceteris paribus*). It is worth noting that this coefficient measures only the price effect. The first parameter analyzed is the compensated own-price elasticity coefficient, which shows the percentage change in demand in response to its own price (*ceteris paribus*). The value of this parameter for the first group (e.g. home and equipment) yields −0.824. This means that a one percent increase in the price index of this group causes a drop in demand by 0.824% on average (*ceteris paribus*). According to the theory of economy, the price effect is usually negative, so the compensated own-price elasticity should be non-positive. This was proven for all expenditure groups. For three groups (communication and education, transport and recreation, restaurants and hotels), the elasticity of coefficients were greater than one, which means that the demand was elastic for these groups. The weakest dependence was observed for the food and health group, which is a normal situation.

Table 5. Compensated own and cross-price elasticity coefficient estimated for a typical household in Poland, based on the QUAIDS results

		W1	W2	W3	W4	W5	W6
		Price change					
W1	demand change	**–0.824*** (0,0139)	0.104* (0,0064)	0.091* (0,0098)	0.106* (0,0099)	–0.635* (0,0179)	0.246* (0,0108)
W2		0.554* (0,0281)	**–1.266*** (0,0321)	0.447* (0,0253)	1.008* (0,0272)	–1.496* (0,0415)	0.064 (0,0329)
W3		0.028 (0,0182)	0.143* (0,0107)	**–0.755*** (0,0165)	–0.274* (0,0161)	–0.291* (0,0243)	–0.100* (0,0114)
W4		–0.062* (0,0195)	0.392* (0,0122)	–0.390* (0,0171)	**–1.315*** (0,0281)	–0.010 (0,0301)	–0.216* (0,0127)
W5		–0.372* (0,0125)	–0.235* (0,0066)	–0.049* (0,0091)	0.056* (0,0107)	**–0.359*** (0,0220)	0.131* (0,0091)
W6		1.219* (0,0618)	0.052 (0,0427)	–0.423* (0,0351)	–0.712* (0,0367)	1.023* (0,0743)	**–2.650*** (0,1249)

* Means that the coefficient was statistically significant ($\alpha = 0.05$). In parenthesis there are standard error values.

Source: own study.

The second parameter analyzed is the compensated cross-price elasticity coefficient. It is measured as the percentage change in demand for the first good that occurs in response to a percentage change in the price of a second good. This measure enables us to determine whether two groups are complementary or substitute goods. Most of the compensated cross-price elasticity coefficients analyzed were small, which means that there is no strong dependence between the analyzed groups. Janský [2014] observed a weak dependence between the analyzed groups. For example, complementary dependence was observed between group W1 (home and equipment) and group W5 (food and health). In contrast, a substitute relation was observed between group W1 (home and equipment) and group W6 (restaurants and hotels).

6. Conclusion

This research showed that the complete demand system models are adequate tools for analyzing consumer demand. Using complete demand system models enables to find relations that cannot be revealed using single-equation models. Additionally, the results showed that the QUAIDS model does not get reduced to the AIDS model and the QUAIDS model should be used to analyze consumer demand.

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ELASTYCZNOŚĆ POPYTU KONSUMPCYJNEGO: ESTYMACJA NA PODSTAWIE QUADRATIC ALMOST IDEAL DEMAND SYSTEM

Streszczenie: W pracy podjęto próbę analizy popytu konsumpcyjnego z wykorzystaniem modelu Quadratic Almost Ideal Demand System (QUAIDS), który stanowi rozszerzenie modelu Almost Ideal Demand System (AIDS). Wykorzystanie kompletnych modeli popytu pozwala na odkrycie relacji, których modelowanie jest niemożliwe w przypadku modeli jednorównaniowych. Dodatkowo estymowany model umożliwia modelowanie nieliniowych krzywych Engla. Badania przeprowadzono na podstawie danych dotyczących polskich gospodarstw domowych zebranych przez Główny Urząd Statystyczny w okresie 1999-2015. Przeprowadzone postępowanie badawcze dowiodło, że model QUAIDS nie redukuje się do modelu AIDS, a model QUAIDS jest odpowiednim narzędziem do analizy popytu konsumpcyjnego.

Słowa kluczowe: kompletny model popytu, Quadratic Almost Ideal Demand System, QUAIDS, elastyczność popytu.

Attachment

Table 1. Parameter coefficients for the QUAIDS model estimated for Poland

Parameter	Estimate	Approx Std Err	Parameter	Estimate	Approx Std Err
α_1^0	0.3492	0.0013	α_1	–0.0262	0.0012
α_2^0	0.0607	0.0004	α_2	–0.0200	0.0009
α_3^0	0.0773	0.0009	α_3	0.0896	0.0024
α_4^0	0.0287	0.0010	α_4	0.1317	0.0039
α_5^0	0.4761	0.0016	α_5	–0.1927	0.0064
β_1^0	0.0237	0.0009	β_1	–0.0130	0.0004
β_2^0	–0.0132	0.0002	β_2	–0.0013	0.0001
β_3^0	0.0556	0.0006	β_3	–0.0036	0.0002
β_4^0	0.0775	0.0005	β_4	0.0022	0.0002
β_5^0	–0.1654	0.0010	β_5	0.0164	0.0004
λ_1^0	–0.0047	0.0002	λ_1	–0.0001	0.0000
λ_2^0	–0.0007	0.0000	λ_2	0.0000	0.0000
λ_3^0	–0.0004	0.0001	λ_3	–0.0002	0.0000
λ_4^0	0.0037	0.0002	λ_4	–0.0003	0.0000
λ_5^0	0.0002	0.0001	λ_5	0.0008	0.0000
γ_{11}	0.0405	0.0037	γ_{25}	–0.0879	0.0025
γ_{12}	0.0284	0.0017	γ_{33}	0.0480	0.0024
γ_{13}	0.0141	0.0026	γ_{34}	–0.0226	0.0023
γ_{14}	0.0144	0.0026	γ_{35}	–0.0482	0.0036
γ_{15}	–0.1608	0.0047	γ_{44}	–0.0037	0.0039
γ_{22}	–0.0162	0.0020	γ_{45}	–0.0178	0.0043
γ_{23}	0.0205	0.0015	γ_{55}	0.2704	0.0088
γ_{24}	0.0527	0.0017	α_z	1.8729	0.0504

Source: own study.