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1 Introduction

The most general notion of object has its modern home in the first-order classical logic with identity. In this paper, we argue that the apparatus for speaking of objects finds its proper place among a wider logical setting. The purpose of this paper is to explain and defend this thesis. One of the benefits of our analysis is that it makes possible to isolate the logical principles that are characteristic of the notion of object.

The wider apparatus of logic is brought about in virtue of two contentions. The first one is that hypothetical and general statements are the fundamental and primitive notions that make reasoning possible. In section 5, we briefly try to argue for this claim. Our argument relies on a fastened view between logic and inference. The second contention is that concept quantification is a coherent and admissible form of logical expression. Concept quantification has been famously attacked, either for being incoherent, or as a form of (extensional) second-order quantification lying outside the province of logic. After a brief discussion of conventionalism in logic in section 2, we discuss some theses of Willard Quine that are pertinent to our discussion. This is done in section 3 and, forthwith, a defense of a form of second-order logic is mounted. In the following section 4, we make a small digression on the principle of induction and on the benefits of its enunciation with concept quantification.

In sections 6 and 7, we finally describe the logical principles that articulate the notion of object. A small concluding section closes the paper.

2 Conventionalism in logic

There is an interesting, albeit failed, account of logic. It is the view that explicates logic as the adoption of certain linguistic conventions. For instance, adopting (or not) *tertium non datur* would be a matter of convention within a particular linguistic framework. The conventionalist strategy can be compared to the discovery of non-Euclidean geometries. Denying the parallel postulate is not a contradiction in terms or a failure to describe correctly some reality, but rather adopting another kind of geometry. If mathematicians came to accept different geometries side by side, why shouldn't philosophers accept also different logics side by side? Why can't we be free to adopt different logics, different ways of reasoning, in order to pursue more efficiently some inquiries?

A conventionalist view of logic would, in one single sweep, explain why the truths of logic are of a different kind from empirical truths, and why they are analytic and *a priori*. Of course, this would be an important step in the enterprise of logical positivism. A logic would merely be an adopted calculus where, by conventional stipulation, some inferences are permitted and some principles are asserted. The stipulations would be seen as implicit definitions of the terms of the calculus.

Conventionalism in logic was refuted by Quine more than seventy years ago. The difficulty lies in that logic itself is presupposed in establishing non-trivial conventions.[1] Suppose that we want to establish conventions for the *if*-idiom. We may start by laying down the following convention:

> *If x and z are true sentences, y is a sentence and z is the result of substituting the letter 'p' for x and the letter 'q' for y in 'If p then q', then y is true.*

Note, however, that the convention uses the *if*-idiom itself, as well as the *and*-idiom and, implicitly, the *all*-idiom. If we do not understand these idioms beforehand and know how to operate with them, the above convention does not get off the ground. I include this Quinean analysis here because, later in the paper, I will argue that both the *if*-

[1] See specially part III of Quine 76. Cf. also the short and witty article by Carroll 1895.

idiom (the idiom of hypothetical claims) and the *all*-idiom (the idiom of general claims) are the fundamental and primitive notions which make logic possible.

3 Concept quantification

The above argument of Quine was one of his first attacks on logical positivism, an attack which culminated in the famous *Two Dogmas of Empiricism* paper (see Quine 1953b) and its attendant rejection of the analytic-synthetic distinction. However, and this is important to notice, the paper is actually only an attack on a certain class of analytic statements, viz. the ones that can be turned into a logical truth by putting synonyms for synonyms. The class of logical truths is not itself subjected to attack in Quine's paper. Quine's real target is the reification of the notion of meaning, an attack mounted through the notion of synonymy. Presumably, if the notion of synonymy were acceptable, meanings could be reified: indeed, they could be defined as equivalent classes of terms under the relation of synonymy. The attack on synonymy had great success but, as we will see, combined with another thesis of Quine, had an unfortunate and important collateral damage.

There are many problems with the notion of reified proposition (as meanings of sentences) but Quine is famous for rejecting an ontology of propositions on the basis that it is hard to account for their identity conditions. It is part and parcel of the notion of object that the *same* object can be referred to by means of different descriptions or from different perspectives. Identity, as stressed by Gottlob Frege, is an important part of the apparatus of linguistic reference to objects. Quine also stresses quantification. At any rate, the most general notion of object finds its modern home in first-order classical calculus with identity. If one takes seriously the view that the notion of object is a quintessential metaphysical notion, then first-order classical logic with identity must be deeply ingrained with metaphysical presuppositions. I will recount a 'logico/metaphysical story' on how the notion of object is brought into logic. The aim is to make explicit, via inferential articulations, the metaphysical presuppositions of this notion.

Speaking of objects, as it is remarked in Parsons (2008: 10), just is using the linguistic devices of singular terms, predication, identity and quantification to make serious statements. Meanings and propositions do not qualify as objects. According to Quine, they do not so qualify because they miss at least one of the requirements, the one on identity. However, as Quine 1953a is careful to point out, he does not deny that words and statements are meaningful. He just denies that they *have* a meaning, if this 'having' is interpreted as more than a mere figure of speech. We, humans, *use* meaningful statements all the time. The point is that using them is not the same thing as naming them, nor does it presuppose that we can name them (i.e., that propositions can be treated as objects).

However, Quine goes a step too far when he defends that quantification is not only a *necessary* condition for speaking of objects but that it is also a *sufficient* condition. *To be is to be the value of a bound variabl*e, in Quine's famous, but ill-conceived, dictum. Quine argues for this thesis in *On what there is?* (Quine 1953a: 15) where he also lays down his doctrine of ontological commitment. The argument of Quine proceeds via an analysis of complex descriptive names which do not refer. He analyzes these names via Russell's theory of descriptions and points out that the ontological commitment is carried out by the bound variables. Notice, however, that Russell's theory relies on the full apparatus of first-order classical logic with identity: it needs predication, quantification and identity. Therefore, what is argued by Quine is that in the presence of this linguistic apparatus (the apparatus for speaking of objects), ontological commitment is carried out by the bound variables. So far, so good. However, to conclude from this argument that bound variables carry ontological commitments in the absence of the full apparatus is a *non sequitur*.

Quine's emphasis on quantification is right in one direction - speaking of objects presupposes the availability of the apparatus of quantification (with ontological commitment) - but wrong-headed in the other - that the mere use of the apparatus of quantification signals ontological commitments. The collateral damage is, of course, that quantifying over propositions (or, in general, over concepts) does not make sense. As I said above, I will present a 'logico-metaphysical' story on how the notion of object is brought into logic. In this story, the notion of generality is primitive and conceptually prior to the notion of object and, by itself, does not signal ontological commitments. It is merely a means of expression.

Before I tell my story, let me make a few remarks. I start with an example: '$\forall P(P \rightarrow P)$', where 'P' is a second-order propositional bound variable. This expression, I submit, makes perfectly good sense.[2] On our view of propositional generality, an instantiation of a universal (second-order) propositional quantification is *not* obtained by converting each occurrence of the quantified variable into an expression that *names*, but rather into an expression that *propounds* (i.e., a meaningful sentence). That which can be propounded is essentially open-ended and unfinishable, depending on linguistic and conceptual resources of whose limits we have no real conception. It is impossible to survey all propositions or concepts because 'the attempt to survey reason itself fails: reason can transcend whatever it can survey' (Putnam 1998: 119).

It is important to point out that the role given to meaningful sentences in the conclusion of the elimination rule for universal propositional quantification is one in which they appear in positions of *use*. Let me give an example: 'Bustopher is a fat cat $\rightarrow$ Bustopher is a fat cat' is an instantiation of '$\forall P(P \rightarrow P)$'. Do notice that the expression 'Bustopher is a fat cat' appears in positions of use in the sentence 'Bustopher is a fat cat $\rightarrow$ Bustopher is a fat cat'. Even though the notion of proposition does not have the right content to allow proposi-

[2] During a presentation of this work, I mentioned that it makes perfectly good sense to say that every proposition implies itself. With his customary politeness, Ricardo Santos pointed to me that I probably did not want to use the word 'implies' here. Implication is reserved for inference, as when one says that the sentence 'The cat is on the mat and the dog is in the garden' implies the sentence 'The dog is in the garden'. The word 'implies' is part of the metalanguage and cannot function as a sentential operator. For this function, one should use instead the conditional 'if … then …' as in 'If the cat is on the mat and the dog is in the garden then the dog is in the garden'. Of course, this gives rise to a *grammatical* problem: to say 'every so is such that if so then so' is not proper English. Locutions like 'every' or 'for all' demand to be followed by noun phrases. Grammar pushes us to say something like 'every proposition is such that if it holds, then it holds'. In a nutshell, in our example, grammar demands ordinary language to treat propositions as objects (and truth as a property thereof). This grammatical objection is taken very seriously by some authors. See, for instance, the critique of Burgess (2005: 211ff). Grammar and 'common usage' do not have, however, a good repute as a guide to philosophical or scientific inquiry (see the caustic Russell 1953).

tions to function *as objects* in the range of a quantification, I am arguing that it has a content which permits their use within the *all*-idiom in the manner which I have just described. In actual instantiations, the question of whether a sequence of symbols counts as expressing a proposition can only be answered by way of interpretation and public agreement. It is, irredeemably, a matter of interpretation. It is certainly right to say that, in meaningful exchange, interpretation always lurks.[3]

4 Digression on the natural numbers

Given the coherence of propositional and concept quantification, a refusal to accept this kind of quantification is tantamount to a prohibition to engage in concept building and expression. It is an unwillingness to go on in the direction of a greater linguistic expressiveness. Let me make a brief digression and discuss an example from mathematics: the principle of mathematical induction:

$$\forall H\,[H(0) \wedge \forall n(H(n) \rightarrow H(n+1)) \rightarrow \forall n H(n)]$$

The use of this principle is, directly or indirectly, behind almost all of the mathematics that is indispensable for science. In the light of the discussion of the last section, I purposely enunciate the principle with a (second-order) concept quantification because this is a way to express its universality, in the sense that the properties to which the principle of induction applies are not parochial to a fixed language. The principle rather applies to properties whose expression is part of

[3] The notion of propositional quantification has some superficial affinities with substitutional quantification. There are two crucial differences, however. The instantiations of a universal propositional quantification are not obtained from sentences of a given fixed formal language. As a consequence, propositional quantification is not amenable to semantic ascent and cannot be reformulated as quantification over a certain fixed domain of sentences together with the concurrence of a truth (or satisfaction) predicate. Moreover, in our notion of quantification, sentences formed using the very apparatus of propositional quantification can themselves be used as instantiations of universal quantifications. In other words, contrary to substitutional quantification, propositional quantification accepts impredicativity and, when generalized to concept quantification, it allows for the formation of impredicative concepts.

an open-ended process.

The expert reader knows that the mathematics indispensable for science only requires enunciations of the principle of induction within particular formal languages.[4] We can certainly opt for this kind of parochialism with respect to science as it is currently practiced. Nevertheless, the principle as enunciated above not only has a regulative force - in the sense that the evidence for the truth of its formal (schematic) enunciations are best seen as coming from the general principle[5] - but it also sheds light on the relationships between provability and truth in mathematics.

Arithmetic, when developed in a formal language, is subjected to Gödel's first incompleteness theorem. According to this theorem, under mild technical conditions, a consistent theory of arithmetic is necessarily incomplete, i.e., there are always sentences of the formal language which are neither provable nor refutable (their negations are not provable). If we further see the system of arithmetic as aiming at proving true arithmetical sentences, we can say that there are always true sentences of arithmetic which are not provable in the given formal system.

The proof of Gödel's theorem relies essentially on the complete formalization of the arithmetical theory. If the principle of induction is formulated as above, with concept quantification, Gödel's theorem simply does not apply. This state of affairs should not come as a surprise because in any formal deductive system the statement of the principle of induction is, by necessity, *restricted*.[6] Not only is this

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[4] For instance, in the language of set theory. Mathematical logicians have studied in detail what exactly is necessary - in terms of induction or set comprehension - for proving ordinary theorems of mathematics (for a reference, see Simpson 1999). For the connection of this kind of work with science and the indispensability arguments, I recommend Feferman 1998.

[5] This simple point was given its due weight by Kreisel (1967: 148).

[6] The observation about the open-endedness of concept formation is quite plain. Ditto for the observation that the principle of induction is open-ended. However, the following question and (especially) its answer are rather deep. Let be given a formal language of arithmetic. It has a delimited apparatus of concept formation and, therefore, a restricted amount of induction. Would it nevertheless be possible to set up a (consistent) theory in the formal language that would either prove or refute each sentence of that very same formal (and, hence, restricted)

observation in line with the open-endness of concept formation, but it also finds rigorous mathematical support in the theorem that the truth predicate for a formal language of arithmetic cannot be defined within the language itself.[7]

Wherein is exactly located the presumed gap between truth and provability intimated by Gödel's first incompleteness theorem? I suggest that the gap, if it is a gap, lies in the open-endness of concept formation and, in our case, in the open-endness of numerical concept formation.

5 The centrality of hypothetical and general statements

Inference is central to logic and reason. Without being able to move from premises to a conclusion, there is no reasoning. Logical truth is a particular case (or a degenerate case, as when we say that a point is a degenerate circle) of inference: one without premises. The degenerate case is no substitute for the general case since logical truth alone does not account for the *moves* from statements to statements which characterize logic. It is an obvious point that it is impossible to set up a logical system without at least one inference rule. Without inference, there are only isolated proclamations.[8]

Suppose that the primitive man concludes that he is in danger from the information that there is a lion in the vicinity. A reflection on what he *does*, i.e., the *move* of concluding that he is danger from the knowledge that there is a lion in the vicinity, and the will (and capability) to express this movement, takes him to say 'if there is a lion in the vicinity then I am in danger'. Hypothetical (or conditional) statements express, within language, the *sanction* of the linguistic act of drawing a conclusion under certain conditions. They are a fundamental form of logical expression and make possible a linguistic ascent, from linguistic act into linguistic expression, without which

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language? Gödel's first incompleteness theorem says that this is impossible.

[7] This is Tarski's undefinability of truth theorem.

[8] Such proclamatory discourse would marry well with the notion of language as consisting only of *descriptions of reality*. Interestingly, in Sellars 1997 it is argued that even perceptual statements can only count as assertions insofar as they are inferentially articulated.

there is no explicit reasoning.[9,10]

In the example above, a measure of *generality* is implicit. The conditional sentence 'if there is a lion in the vicinity then I am in danger' operates like an *inference ticket* which can be cashed in *all* the appropriate situations.[11] In the complete absence of generality, even though there is more in discourse than mere isolated proclamations, we are - in a sense - only a finite number of steps away from inferring all that can be concluded. Let us advance an argument for this thesis. Arguably, the main role of the elimination rules of a connective is to make possible the use of premises in which this connective occurs as the principal connective. It is plain that if the conditional is the *only* logical connective present in the premises of an argument, we would be only a finite number of steps away from inferring all that could be inferred via the elimination rule (i.e., via *modus ponens*).[12,13] On the

[9] The content of hypothetical statements results from some distinguished roles in inference. More precisely, these are the roles carried out by the introduction and elimination rules of the natural deduction calculus. This calculus was invented by Gerhard Gentzen in 1935, and subsequently studied and expanded in Prawitz 1965.

[10] The attentive reader can point that there are inferences with more than one premise, and that this fact calls for the notion of conjunction as well. This is correct, but not terribly interesting. There is a further factor that explains our simplified account: technically, as it will be pointed out, conjunctive claims can be expressed using hypothetical and general claims.

[11] This point was made a long time ago by Ryle 1950.

[12] The above argument is not as strong as the argument for the centrality of hypothetical statements. The caveat 'in a sense' is needed because the introdution rule for implication does allow the inference of infinitely many sentences (uninteresting as they may be). Nevertheless, we believe that it carries a certain weight. It would be nice to advance stronger and more perspicuous arguments for the centrality of general statements.

[13] Frege 1984 also assigns a central role in logic to generality. Frege speaks of the scientific need to express *laws* and says that 'in point of fact the distinction between law and particular fact cuts very deep'. Frege, on the other hand, does not accord a centrality to hypothetical statements. He sees them as truth functional and, therefore, replaceable by the combination of negation and conjunction (cf. Frege 1980). Of course, Frege is right: truth-functional conditionals (as any truth functional connective) can even be written with the Sheffer stroke only. However, our analysis of truth-functionality is not yet in place. Our rendition of

other hand, generality releases reasoning from a pre-fixed enclosure into a boundlessness of inferential moves.

The sentence '$\forall P(P \rightarrow P)$' is an example of a (true) general sentence. Notice that the number of immediate inferences which can be drawn from it (via the elimination rule) is unbounded. General claims, as hypothetical claims, are also a primitive and unexplained notion, fundamental to logical expression. These claims can be made not only with respect to predicate positions, but also with respect to name positions. However, it is methodologically interesting to start with propositional generalities - as we did - because doing so only requires the notion of proposition (i.e., of a meaningful sentence) and does not rely yet on any particular analysis of this notion.

At this juncture, it is worth remarking that the familiar propositional connectives can be introduced with the apparatus of hypothetical and general propositional claims:[14]

$$\neg A =_{df} A \rightarrow \forall P.P$$
$$A \wedge B =_{df} \forall P\,((A \rightarrow (B \rightarrow P)) \rightarrow P)$$
$$A \vee B =_{df} \forall P\,((A \rightarrow P) \rightarrow ((B \rightarrow P) \rightarrow P))$$

These are the *expressive* definitions of the above connectives.[15] It is known that the attendant notion of consequence gives rise to intuitionistic logic.[16]

logic is still on a prior and more general footing.

[14] The definitions appear informally in sections 18 and 19 of Russell 1996. The formal definitions are due to Prawitz 1965.

[15] For the reader unfamiliar with these definitions, let us discuss the definition of disjunction. In natural deduction, the elimination rule of disjunction is the following: If 'P' can be inferred from 'A' and if 'P' can be inferred from 'B', then 'P' is a consequence of '$A \vee B$' (this is also known as *discussion by cases*). Given that concept quantification is allowed in our language, the definition of disjunction above mirrors the elimination rule just described. *Mutatis mutandis* for the other connectives. The introduction rules of the natural deduction calculus follow from the definitions.

[16] The notion of consequence is the one originating from the rules of introduction and elimination (for the conditional and the universal quantifier) of the natural deduction calculus (cf. footnote 9). It is a mathematical observation that these rules give rise to intuitionistic logic.

6 The *alethic* ingredient

The previous section recounts the first part of our 'logico/metaphysical story'. The subsequent story narrates the appearance of the notion of object and, in general, of the descriptive component of logic. We have already introduced the expressive ingredient of logic.[17] Let us now introduce two other ingredients: firstly, the *alethic* ingredient (i.e., the logical apparatus for the notion of bivalent truth) and, in the next section, the *ontic* ingredient (i.e., the logical apparatus for the notion of object).

The *alethic* ingredient of logic only comes into play when we are dealing with a linguistic base whose semantics is taken to be unproblematically bivalent. We start with a initial bit of language whose propositions are true or false, but not both (the principle of bivalence). For convenience, let us call them atomic propositions.[18] We take that atomic propositions come in pairs, one called the *opposite* of the other (with switched truth values). The opposite of an opposite is the original atomic proposition. This is our rendition of bivalence. The *inferential explication* of bivalence can be done through the following laws:

$$(\mathrm{ALE})_{1\text{-}}\ \ \forall P((A \rightarrow P) \rightarrow ((\underline{A} \rightarrow P) \rightarrow P))$$
$$(\mathrm{ALE})_{2\text{-}}\ \ \forall P(A \rightarrow (\underline{A} \rightarrow P))$$

where the letter 'A' stands for a sentence that expresses an atomic proposition (an atomic sentence), and '$\underline{A}$' stands for its opposite. The first law is a form of *tertium non datur* and can be elucidated by saying that an atomic proposition and its opposite exhaust the space of logical possibilities in the following sense: Whenever we want to infer a given proposition, it is enough to infer it from an atomic proposition

[17] Generalizations with respect to name positions (which will be discussed below) are also part of the apparatus of the expressive component of logic, but they presuppose a previous analysis of the structure of predicative statements. For the record, the expressive definition of '$\exists x\, A(x)$' is the formula '$\forall P(\forall x(A(x) \rightarrow P) \rightarrow P)$'.

[18] This is just a convenient way of speaking. There are no metaphysical connotations in the use of this terminology (not, for instance, connotations to Tractarian *Elementarsätze* or semantical atomism) beyond what we have said: atomic propositions are either true or false, but not both.

and to infer it from its opposite. The second principle is a form of *ex falso quodlibet*.[19]

If ‘A’ stands for an atomic sentence, it is easy to prove both that $\neg A \leftrightarrow \underline{A}$ and that $\neg \underline{A} \leftrightarrow A$.[20] In other words, for atomic propositions, expressive negation is equivalent to (bivalent) opposition. Furthermore, if we consider the sentences of the language built up from atomic sentences by means of propositional connectives (the conditional, and negation, conjunction, disjunction, as defined above) - originating *propositional sentences* - it is easy to argue by induction on the build up of sentences, that the statements of the form

$$\forall P\ ((A \rightarrow P) \rightarrow ((\neg A \rightarrow P) \rightarrow P))$$

where ‘A’ now stands for a propositional sentence, are provable. Note that the above says that $A \vee \neg A$. It is now clear that the propositional fragment of the language obeys the laws of classical logic.

Our definitions of the propositional connectives are done in terms of propositional quantifications. It may cause some uneasiness the fact that the apparatus of propositional quantification is present in the propositional calculus (via the very definitions of the propositional connectives). The uneasiness is, however, uncalled for. In a sense, the apparatus of propositional quantification present in the midst of the propositional fragment is trumped by the alethic principles. The familiar (self-enclosed) set up of the propositional calculus via truth tables may, on this view, be considered an ‘in your face’ manner of displaying the *result* of this trumping.

In the same vein, it is worth remarking explicitly that the alethic trumping mentioned above explains the compositional nature of the classical propositional calculus. The like-minded reader must have noticed that the semantics of propositional quantification cannot be fully compositional. The reason for this lies in the impredicativity of concept quantification and the attendant consequence that one must

[19] With the definitions of the propositional connectives above, the sentence in $(ALE)_{1.}$ is equivalent to ‘$A \vee \underline{A}$’, and the sentence in $(ALE)_{2.}$ is equivalent to ‘$\neg(A \wedge \underline{A})$’.

[20] The claim that $A \leftrightarrow B$ should be understood as an abbreviation for the pair of claims that $A \rightarrow B$ and that $B \rightarrow A$.

deny that the sense of a second-order generalization is intelligible only if its instantiations are intelligible in advance of the generalization itself.[21] This observation effectively precludes a compositional semantics *in general*. However, in the presence of the alethic principles, the restricted area of the propositional calculus is fully rooted in its bivalent beginnings, disarming - as it were - the role of the concept quantifications in its midst and, therefore, making it possible to cohere with a (compositional) truth-functional semantics.

7 The *ontic* ingredient

In the previous section, we brought into discussion a linguistic base whose semantics was taken to be unproblematically bivalent. For convenience, we called the elements of this linguistic base 'atomic propositions'. The expressive devices of hypothetical and generality claims create their own cargo by permitting the expression of new propositions which, themselves, can be subjected to the expressive devices again. And so on. A quite *diverse* cargo is created. Even if the atomic base has an unproblematic bivalent semantics, the application of these devices need not maintain language within such bivalent confines. What we have argued in the previous section is that they do if we restrict ourselves to the propositional calculus.

In this section, we further suppose that these atomic propositions have a predicative structure. For instance, we may say that Bustopher is a fat cat or that Mungojerrie and Rumpelteazer are twin cats. Proper names and predicate symbols mark places in sentences which are suitable for generalizations. They indeed mark different sorts of places, since proper names do not fit into predicate places nor vice versa. A sentence of the form '$\forall H(H(\beta)\rightarrow H(\beta))$, is a generalization of the sentence 'Bustopher is a fat cat → Bustopher is a fat cat', where 'β' denotes Bustopher. It is a generalization with respect to the predicate place (a concept quantification). We could further generalize in

[21] A second-order generalization may have instantiations which are not simpler than the generalization itself since the bound second-order variable may be instantiated by formulas of arbitrary complexity. In this essay we are not discussing the semantics of concept quantification but it is clear that it must rely on a species of a rule-following semantics. The reader can find an entertaining discussion of impredicativity in Ferreira 2006.

the name place and write '$\forall x \forall H(H(x) \to H(x))$'. First-order quantification is also a device for the expression of generalizations, through which a new manner of formation of concepts is allowed.

In order to deal with the first-order classical predicate calculus, we must extend our analysis of the atomic case to atomic *formulas*. In analogy with the last section, if the letter 'A' stands for a predicative symbol (unary: to simplify), then '$\underline{A}$' stands for its opposite. The extended alethic principles take the form:

$$(\mathrm{ALE})_1 \quad \forall x \forall P((A(x) \to P) \to ((\underline{A}(x) \to P) \to P));$$
$$(\mathrm{ALE})_2 \quad \forall x \forall P(A(x) \to (\underline{A}(x) \to P)).$$

Disregarding the identity axioms, the *inferential explication* of the notion of object can completed through the following principle:

$$(\mathrm{ONT}) \quad \forall M \forall K\, [\forall x \forall P((M(x) \to P) \to ((K(x) \to P) \to P)) \to \forall P((\forall x M(x) \to P) \to (\forall x (K(x) \to P) \to P))].$$

Here is an elucidation of this principle: If for each object x, reasoning from the fact that x falls under M and from the fact that x falls under K exhausts the space of logical possibilities, then reasoning from $\forall x M(x)$ and from a generic example falling under K also exhausts the space of logical possibilities.

Arguably, the principle (ONT) is a bit opaque on a first reading,[22] but we can frame it in a more familiar terminology: With the definitions of the propositional connectives and of the existential first-order quantifier in footnote 17, (ONT) is equivalent to

$$\forall M \forall K\, [\forall x (M(x) \vee K(x)) \to \forall x M(x) \vee \exists x K(x)].$$

It is also worth remarking that if there are only finitely many objects in the range of x, (ONT) is provable. For instance, if there are only two objects α and β, the principle takes the form

$$\forall M \forall K\, [(M(\alpha) \vee K(\alpha)) \wedge (M(\beta) \vee K(\beta)) \to (M(\alpha) \wedge M(\beta)) \vee K(\alpha) \vee K(\beta)].$$

[22] There is, however, no reason why the inferential explication of the notion of object must be prosaic.

The rendition of the notion of object is given by the inferential rules which regulate the use of the 'linguistic devices of singular terms, predication, identity and quantification'. (ONT) is part of this rendition. If I were pressed to elucidate its particular role in the notion of object, I would say that (ONT) conveys the notion of quantification over a *closed totality.*[23]

It is a simple exercise (via an inductive argument on the complexity of formulas) to show that (ONT), together with the alethic principles, entails that first-order formulas satisfy *tertium non datur.*[24] The principles of classical logic for the first-order fragment follow suit.

8 Coda

Discourse about objects finds its modern home in first-order classical logic with identity. The main thesis of this paper is that logic is not a uniform terrain where all truths lie on a par. We have analyzed first-order classical logic and showed that it decomposes into two main ingredients: a deeper and wider expressive component and, on top of it, a narrower descriptive component. We argued that hypothetical and general claims are fundamental to logic and part of its expressive component. In a sense, our argument is transcendental: these types of claims are what make logic possible. Another face of logic, formed by the alethic and ontic ingredients, flattens - so to speak - the terrain on top of the expressive component giving us the descriptive language of first-order classical logic with identity. It is an important region in the landscape of language, lying among and on top of the larger and original expressive foundation.[25]

[23] As we have noticed, (ONT) is provable if there is only a fixed finite number of objects in the range of 'x'. In this case, the notion of quantification over a closed totality is given automatically by stating that the domain of quantification has the given finite number of elements. For instance, if there are only two objects, we can write '$\exists y \exists z (y \neq z \wedge \forall x (x = y \vee x = z))$'. It is only when the domain is limitless that (ONT) is needed.

[24] Note that a particular case of (ONT) is that $\forall M\, [\forall x(M(x) \vee \neg M(x)) \rightarrow \forall x M(x) \vee \exists x \neg M(x)]$.

[25] Versions of this paper were read at Universidade Nova de Lisboa, Universidade do Porto (on the occasion of the conference *The Logical Alien at 20*) and at

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