



Book Review  
*Saving Truth from Paradox*, by Hartry Field

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To conclude, in an accurate, concise and clear way F&F critically addresses the main realist views on fictional entities, thus covering a wide range of literature on fiction. Moreover, it puts forward an interesting and original approach (although in need of further elaboration) to deal with traditional puzzles posed by fictional discourse, without, however, entering into a genuine ontological commitment to the exotic entities that realist theories need to postulate, which might prove to be helpful to fictionalist projects.

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**Saving Truth from Paradox**, by Hartry Field. New York: Oxford University Press, 2008, 432 pp.

In one of its versions, the liar paradox presents us with a sentence  $Q$  that can be shown to be logically equivalent to a sentence that asserts  $Q$ 's untruth:  $\neg \text{True}(<Q>)$ . By appealing to  $Q$ 's instance of Tarski's schema (a quite naïve assumption about truth),  $\text{True}(<Q>) \leftrightarrow Q$ , we easily reach the biconditional:

$$(*) \text{True}(<Q>) \leftrightarrow \neg \text{True}(<Q>),$$

which, in classical logic, leads to the contradiction:

$$(+)\text{True}(<Q>) \wedge \neg \text{True}(<Q>).$$

Most attempts at solving this paradox restrict Tarski's Schema (TS, from now on). Field takes a different route in *Saving Truth from Paradox* (*STFP* henceforth). According to him, the view that it is always preferable to restrict semantic principles like TS before revising classical logic should be regarded as logical dogmatism, for TS and other semantic principles are more basic than some principles of classical logic (p. 15 & ff., all references to *STFP*). Indeed, Field holds that the truth predicate is merely 'a device of infinite conjunction or disjunction (or, more accurately, a device of quantification)' (p. 210) and this means that it serves mainly *logical* purposes. From this defla-

tionary point of view, restricting TS need not be less harmful than restricting other logical principles concerning terms such as ‘or’, ‘and’ or ‘not’. In particular, Field claims (pp. 14–17) that TS is more basic for explaining our ordinary understanding of meaning than the Law of Excluded Middle (LEM, henceforth). A further reason for keeping TS unrestricted is that, usually, theories of truth that choose to restrict it have problems with strengthened versions of the Liar Paradox, which means that paradoxes do not necessarily go away by following that path.

Field’s goal in *STFP* is to offer a theory of truth for a language  $\mathfrak{L}^+$  containing its own truth predicate that is free of semantic paradoxes and meets two conditions: (i) it avoids strengthened versions of the Liar paradox; and (ii) it preserves, for all sentences  $A$  of  $\mathfrak{L}^+$  (including liar sentences like  $Q$ ), both, *all* instances of TS and the intersubstitutivity of  $T(<A>)$  and  $A$  in all non-opaque contexts. Meeting the second requirement will demand the definition of a non-material conditional.

*STFP* is divided in five parts: the first one introduces the problems posed by semantic paradoxes to the concept of truth and to related semantic notions, like satisfaction or validity (this part contains an interesting discussion about the unprovability of our naive notion of soundness). Field also presents here Kripke’s theory of truth and underlies its main virtues — what we will call, following him, ‘the transparency of truth’: the intersubstitutivity of  $\text{True}(<A>)$  and  $A$  in all non-opaque contexts — and defects — the lack of a reasonable conditional and the failure of TS. The second part of *STFP* contains a survey of the most popular solutions to the Liar Paradox. The third and fourth parts develop Field’s own paracomplete theory and the last one contains a critical comparison of paracomplete and dialetheist theories.

Field describes (p. 117) four strategies (some of them combinable) to block the Liar paradox: (a) giving up the notion of truth; (b) restricting the language in order to avoid the construction of liar sentences; (c) restricting TS; and (d) restricting classical logic so that  $(*)$  does not imply  $(+)$  (or, at least, does not trivially imply everything). We saw that Field dismisses (a). He also dismisses (b) because he does not want to impoverish in a drastic way the expressive resources of our language. Banning self-reference, for instance, would require dramatic changes in our language, as contingent cases of self-reference — like Epimenides’s utterance of ‘All Cretans are liars’ — show. For these reasons, Field mainly discusses strategies (c) and (d).

The second part of *STFP* is a critical survey of theories of truth that follow strategy (c). These theories, which try to validate as many classical logical principles as possible, are described by Field as ‘broadly classical.’ A relevant aspect of Field’s book is the way he classifies them. The use of different technical resources (fixed point theorems, revision rules, supervaluations, etc.) in order to model the semantics of the truth predicate is less relevant for Field than the way in which the theories restrict TS. Field examines in the book theories that restrict TS in a way that violates as few incoherence principles (out of a list offered in pp. 119–20) as possible. In this context, he discusses gap and glut theories (chapters 7–8), supervaluationist and revision rule theories (chapters 10–12), and stratified and contextual truth theories (chapter 14). The general objection he addresses to all these theories is their failure to preserve the transparency of truth in non-opaque contexts and the damage this causes to the logical function of the truth predicate (pp. 209–10, 227). But Field also examines other aspects of these theories. In particular, he profusely discusses the limitations that these theories find in order to use the truth predicate as a device for expressing agreement or disagreement with some of the thesis the theories are bound to accept or reject.

In parts three, four and five of *STFP*, Field focuses on solutions to the Liar that follow strategy (d). He examines paraconsistent dialetheist theories and his own paracomplete theory of truth. (We will focus on Field’s theory and leave out of this brief review the comparisons he draws between his theory and different dialetheist theories.) Field’s theory seeks to preserve all instances of TS and the intersubstitutivity of  $T(<A>)$  and  $A$  in all non-opaque contexts. Given that we can easily infer (+) from (\*) in classical logic, some of the principles or rules of inference used in (R1) must be restricted.

- (R1) LEM establishes  $\text{True}(<Q>) \vee \neg \text{True}(<Q>)$ . Given (\*) and the classical definition of the biconditional in terms of the conditional and the conjunction, a proof by cases which only uses the rule of modus ponens establishes (+).

Field wants to preserve the rules of proof by cases and modus ponens (as well as the classical definition of ‘ $\leftrightarrow$ ’) so he rejects the unrestricted validity of LEM and offers a *paracomplete* theory of truth: one in which there is at least a sentence  $A$  such that neither  $A$  nor  $\neg A$  are true and in which there is *no* sentence  $A$  such that  $\neg \text{True}(<A>)$  and

$\neg\text{True}(\langle\neg A\rangle)$  are both true. (He only calls *gap* theories those that explicitly *declare* untrue a sentence and its negation.)

The main feature of Field's theory is its conditional, which — unlike Kripke's — validates all instances of TS without altering a crucial feature of Kripke's theory of truth: the conservativeness requirement, which makes possible the intersubstitutivity of  $A$  and  $\text{T}(\langle A \rangle)$  in all non-opaque contexts (see pp. 262–64 and chapter 18). Moreover, Field's conditional enables him to define a determinately operator,  $D$ , that can be used to describe defective sentences in a semantic framework with infinitely many truth-values which are partially ordered. The ordering has a greatest and a lowest element:  $\mathbf{1}$  and  $\mathbf{0}$  respectively ( $\mathbf{1}$  is the only designated value). Whenever  $A$  is a defective sentence — i.e., a sentence that fails to be determinately true due to problems concerning paradoxes or vagueness — the sentence  $D(\langle A \rangle)$ , which Field defines (p. 236) as:

$$A \wedge \neg(A \rightarrow \neg A),$$

is assigned a truth-value closer to  $\mathbf{0}$  (to 'false,' so to speak) than that of  $A$ .

Using the new operator we can form now 'liar sentences' like  $Q_1$ , which is logically equivalent to  $\neg D\text{True}(\langle Q_1 \rangle)$ . A hierarchy of liar sentences can actually be generated by instantiating the following schema (see pp. 237, 254):

$$Q_\alpha =_{\text{df}} \neg D^\alpha \text{True}(\langle Q_\alpha \rangle),$$

where  $\alpha$  is a finite ordinal and ' $D^\alpha$ ' stands for  $\alpha$  concatenations of the  $D$  operator ( $Q_0$  is  $\neg\text{True}(\langle Q_0 \rangle)$ ). Field's theory can handle these sentences without paradox: there is no finite ordinal  $\alpha$  such that  $Q_\alpha$  and  $\neg Q_\alpha$  are true in it. However, the hierarchy can be extended beyond finite ordinals. For any formula  $A$  and limit ordinal  $\lambda$ , it seems possible to define  $D^\lambda A$  (the  $\lambda^{\text{th}}$  iteration of  $D$ ) as: 'for all  $\alpha < \lambda$ ,  $\text{True}(\langle D^\alpha A \rangle)$ .' The corresponding liar sentence would then be  $Q_\lambda =_{\text{df}} \neg D^\lambda \text{True}(\langle Q_\lambda \rangle)$ . But Field shows that this tentative definition is defective:  $D^\alpha A$  is meant to be (for *any* ordinal  $\alpha$ ) a sentence in a *countable* language  $\mathfrak{L}^+$ . Given that there are *uncountable* ordinals, the iteration of the  $D$  operator through transfinite stages cannot go on forever: some ordinals cannot be defined in  $\mathfrak{L}^+$  and, if  $\lambda$  is one of them,  $D^\lambda A$  is not a formula of  $\mathfrak{L}^+$ . After providing a rigorous inductive definition of the

transfinite iterations of the D operator that can be represented in the language of the theory (pp. 326–31), Field shows that his theory can handle the transfinite hierarchy of liar sentences (once all its elements are rigorously defined). Furthermore, he argues — in one of the most difficult and technical parts of the book (chapters 22–23) — that the theory does not suffer from ‘revenge problems’: all attempts he envisages at restating strengthened versions of the paradox by defining predicates that quantify over all possible iterations of the D operator fail, for they rest on defective definitions.

Unfortunately, there are also some bad news: if we want to block the inference from (\*) to (+), LEM is not the only thing that must go.

- (R2) Suppose  $\text{True}(<Q>)$  is the case. Given (\*) and the definition of ‘ $\leftrightarrow$ ’, we infer by modus ponens  $\neg\text{True}(<Q>)$  and then (+). This *reductio* of our initial supposition shows that  $\neg\text{True}(<Q>)$  is the case. However, a new application of modus ponens establishes now  $\text{True}(<Q>)$  from (\*), so we have again (+).

LEM plays no role in (R2) — which could perfectly be accepted by intuitionist logicians (who reject LEM, but endorse the *reductio* rule used there) — and there are still other derivations of (+) from (\*) (see pp. 8–9, fn 8, Field talks there about a paradox for properties, but what he says applies also to the Liar). Field’s logic needs to introduce restrictions that go beyond LEM. The most significant ones concern two basic rules of inference *Reductio ad Absurdum* (RA, see pp 8–9; 312–13) — which we used in R2 — and *Conditional Proof* (CP, see p. 269). Field will only accept RA and CP when they apply to sentences respecting LEM. Together with these rules, Field’s logic rejects the unrestricted validity of several laws involving conditionals (importation, exportation or contraction, for instance). But these restrictions are not just motivated by the Liar paradox, they are also meant to block Curry’s paradox. Indeed, one could see *STFP* as trying to offer the best possible solution to a ‘technical’ problem: how to preserve TS and the transparency of truth in a theory without paradoxes that modifies as little as possible classical logic. We are asked to contrast Field’s theory with its rivals and compare their respective costs and benefits. (This point becomes prominent, for instance, in the exchange that Field keeps with McGee and Shapiro in the *Précis* and subsequent discussion of *STFP* published in *Philosophical Studies* 147, pp. 415–70, 2010.)

In reading *STFP*, one gets the impression nevertheless that we are not carefully exploring (in the book) the importance of what we miss if we restrict — following most of Field's rivals — TS or the transparency of truth, or if we restrict — following Field — the validity of LEM, RA and CP. Many of the papers collected in Field's *Truth and the Absence of Fact* (New York: Oxford UP, 2001) could fill the gap concerning LEM or TS, but some questions are still in place: what is wrong with RA and CP? What are we giving up in restricting them? How does that affect or distort our understanding of negations and conditionals? *STFP* discusses (chapter 21) some of these issues concerning negation, but one has the feeling that more could be said. Sure, RA and CP can safely be applied when we reason about sentences that respect LEM, the restrictions are limited. But the question still remains: *what is wrong with them?* In *In Contradiction* (Oxford: Oxford UP, 2007, 2<sup>nd</sup> edition), Priest also restricts in a limited way the validity of rules as basic as RA, modus ponens (for the material conditional) or disjunctive syllogism: those rules only apply safely to a sentence A when it is *not* a dialetheia — a sentence such that both A and  $\neg A$  are true. But Priest explains why these rules should be restricted. He argues at length against the Principles of Non-Contradiction (PNC) and Explosion (PE). The above rules must be restricted *because* they assume the validity of PNC, or *because* they can be used to prove PE. Of course, one could question Priest's reasons for rejecting PNC and PE, but *those* reasons provide a philosophical motivation for restricting the above rules of inference. Presumably, Field could also offer a philosophical motivation for restricting RA and CP, but this motivation cannot be found in *STFP*.

Another relevant philosophical 'gap' concerns our understanding of paradoxes. Since Russell's 1908's seminal paper on paradoxes of self-reference, it has been quite a common goal among philosophers working on this field to provide (not only a solution to some paradoxes but) an answer to what Chihara once called *the diagnostic problem of the paradox*: '[t]he problem of pinpointing that which is deceiving us and, if possible, to explain how and why the deception was produced' (p. 590 of 'The Semantic Paradoxes: A Diagnostic Investigation,' *Philosophical Review* 88: 590–618, 1979). Russell's principle of vicious circularity, Kripke's notion of groundedness, Belnap and Gupta's studies of circular concepts or Priest's more recent appeal to the Inclosure Scheme are but some examples — among many — of philosophers who pursued to some extent the 'diagnostic problem.'

Field's discussion of semantic paradoxes in *STFP* departs significantly from this tradition. Field's goal is to offer a defence of the transparency of truth and the validity of TS by advancing a logic which respects both constraints on truth and fares better than several rival logics in many respects. But, by the end of the book, one does not get a clear idea of what is wrong with semantic paradoxes according to Field (apart from the obvious fact that they seem to force the acceptance of a contradiction).

A final issue concerns the complexity of Field's semantics for conditionals (see chapters 16–17). Field first uses a kripkean fixed point construction based on Kleene's strong truth tables to interpret the sentences of  $\mathfrak{L}^+$ , the language resulting from adding a truth predicate to a first order language  $\mathfrak{L}$  containing the usual logical constants plus Field's conditional: ' $\rightarrow$ '. Conditionals are treated initially as atomic formulas by valuations, for that reason their truth-values at the minimal fixed point  $X$  are not determined compositionally. In order to solve this problem, Field advances rules for revising the semantics of conditionals through a process that, starting from  $X$ , eventually yields a compositional three-valued semantics where each sentence gets an 'ultimate value': 0,  $\frac{1}{2}$ , or 1. However, under this framework a sentence may end up having ultimate value  $\frac{1}{2}$  for very different reasons. In order to capture this fact — and to interpret the D operator in a way that enables him to meet the challenges posed by higher-order vagueness and the liar hierarchy — Field designs a new semantic framework setting out from the previous one: an algebraic semantics where  $\frac{1}{2}$  is replaced with an infinity of truth-values. The details are quite involved and the basic insights make room for different concretions, but (even from this poor and brief sketch of some of Field's ideas) it seems clear that there is no obvious way to characterize a natural deductive system that matches Field's semantics. This point is stressed by Shapiro in the discussion of *STFP* mentioned above. He wonders about the scope of Field's project. Regardless of whether Field pursues the descriptive project of exploring a new logic or the rather normative project of reforming our logic in order to avoid paradox (or, more plausibly, something in between), it seems that his task should be accompanied by the specification of an intuitive set of inference rules we can appeal to in order to learn and teach how to use Field's conditional. But the logic proposed in *STFP* is described at a semantic level, no deductive system is advanced and this should be a drawback for someone who encourages us to adopt



this logic. The scope of this criticism is perhaps limited. After all, we are told that we can always use modus ponens and that CP applies whenever a sentence A respects LEM. Of course, this means that we should find out whether A respects LEM before applying CP (and this need not be an easy task), but perhaps things are like that. The existence of a simple set of rules of inference for conditionals that can effectively be applied in any situation and under any circumstances might be nothing but a chimera. We apply certain rules of inference by default, and usually we get things right, perhaps that's all we need and can hope for.

Let me say, to finish this review, that the achievements of Field's book are undeniable. He offers a paracomplete logic that preserves the principles and features that enable us to use the truth predicate as a logical device of quantification over sentences. His logic, moreover, validates an impressively high number of classical principles and contains an operator that can be used to characterize paradoxical sentences without — at least apparently — falling prey of new paradoxes. That is, doubtless, a remarkable feat and the use Field makes of fixed point constructions and revision rules, together with his discussion of well-known solutions to the Liar Paradox constitute a major contribution to the literature on this topic. If my remarks here pursue often what the book leaves out (rather than what it contains), that is, partly, because I think that the real philosophical significance of Field's achievements will emerge in discussing some of the issues that could not be developed in *STFP*.

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