



Wittgenstein on Dedekind's Cut

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APONTAMENTO

WITTGENSTEIN ON DEDEKIND'S CUT

What is the value of Wittgenstein's discussion of the theory of Dedekind's cut in *Remarks on the Foundations of Mathematics*, IV, 29-40?

Although Kreisel and Dummett do not mention the topic in their essays on Wittgenstein's book, Bernays in his "Comments on Wittgenstein's *Remarks on the Foundations of Mathematics*" dismisses Wittgenstein's discussion as being unsatisfactory. He refutes what he sees as Wittgenstein's main critical argument against the theory of Dedekind's cut. He interprets Wittgenstein as saying that in the theory the extensional approach is mixed up with the intensional. Wittgenstein's criticism then has a simple rejoinder: one can use just the extensional approach, provided that one accepts as meaningful not only the concept of number, but also the concept of a set of natural numbers.

But the point Wittgenstein is making is not about a mixture of the two approaches, but also the use of the assumption of the set of real numbers as given, independently of our knowledge of it. This is the force of 37, made very obvious by the choice of words at its beginning: "The misleading thing about Dedekind's conception...". The passage in the discussion mentioned by Bernays is 34, where the blending of the two approaches is treated. But there is no implication in the text that this blending is a fault of Dedekind's theory. It is actually described as being a fault of Dedekind's theory. It is actually described as being a difficulty with the concept of cut, but Wittgenstein does not reject the concept because of this difficulty.

There is an overestimation of the distinction between the extensional and the intensional points of view in the whole discussion. This can be seen in the English translation of the very interesting passage of the same paragraph, where Wittgenstein says that a cut is an "extensive *Vorstellung*", that is, a conception of an extension (of a procedure, from a domain to another domain). Miss Anscombe's translation as "The cut is an extensional *image*" captures neither the literal nor the intended meaning of the passage.

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The extension that Wittgenstein is referring to is that of the conception of a rational cut (taken to be meaningful) to the conception of a real cut. (It is also uncertain that the notion of carrying out such an extension produces an image, or is at least accompanied by a mental image.) There is no textual evidence for a rejection by Wittgenstein of the theory of Dedekind's cut. The wording of 40 implies the opposite, since it says that Dedekind does provide a general expression-scheme, one that one could call the logical form of inference. Wittgenstein is aware that there is something profound about this theory (precisely that which a child could not understand) and the question he asks in 31, how does it get to have a profound content, he answers in 37 when he describes Dedekind's conception as assuming the real numbers to be spread out in the line, whether we know them or not.

What Wittgenstein does reject is the definition of irrational numbers in terms of solutions of geometrical problems, e.g., as the measure of the diagonal of the unit square. But this is the rejection already known from *Principles of Mathematics*, 267, and perfectly consistent with the aim of theory in providing an arithmetical proof of the existence of the irrational numbers. Therefore it seems that on the whole Wittgenstein analyses the theory of Dedekind's cut to prevent vividly a model of Platonism.

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