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A Mathematical Scheme for Calculating Flows and Pressure Drops in Lit and Unlit Cigarettes*

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SUMMARY

A computational methodology is presented for evaluating the flows and pressure drops in both lit and unlit cigarettes. The flows and pressure drops across rows of tipping-paper perforations are considered explicitly, as are the locations and relative sizes of the ventilation holes. The flows and pressure drops across air-permeable cigarette papers are included. The influence of plugwrap permeabilities on filter ventilation is developed. Lit cigarettes are mimicked by adding a "coal" pressure drop to the upstream end of the cigarette. The computational scheme is used to predict the effects of tobacco-rod length, puff volume, and vent blocking on cigarette ventilation and pressure drop. A derivation of the pressure-drop and flow equations for a cigarette with an upstream pressure drop is included in an appendix. [Beitr. Tabakforsch. Int. 19 (2000) 189–203]

ZUSAMMENFASSUNG

Ein mathematisches Modell zur Evaluierung der Luftströme und des Druckabfalls von angezündeten und nicht angezündeten Cigaretten wird vorgestellt. Die Luftströme und der Druckabfall durch die Ventilationslöcher des Mundstückbelags sowie die Position und relative Größe der Perforierungen werden besonders berücksichtigt, einschließlich der Ströme und des Druckabfalls durch luftdurchlässiges Cigarettenpapier. Der Einfluß der Permeabilität des Filterumhüllungspapiers auf die Filterventilation wurde untersucht. Eine brennende Cigarette wurde simuliert, indem an das Einströmende der Cigaretten ein „Kohle"-Druckabfall hinzugefügt wurde. Ein Rechenmodell wurde herangezogen, um die Auswirkungen der Länge des Tabakstranges, des Zugvolumens und des Abdeckens der Filterventilationslöcher auf Cigarettenventilation und den Druckabfall vorherzusagen. Eine Ableitung des Druckabfalls und der Flußgleichungen für eine Cigarette mit einem Druckabfall am Einströmende ist im Anhang enthalten. [Beitr. Tabakforsch. Int. 19 (2000) 189–203]

RESUME

Un modèle mathématique a été mis au point pour évaluer les débits d'air et les pertes de charge de cigarettes allumées et non allumées. Les débits d'air et les pertes de charge à travers les orifices du papier manchette sont pris en considération, de même que la localisation et la dimension relative des trous de ventilation. Les écoulements et les pertes de charge à travers le papier de cigarette perméable à l'air sont également étudiés. L'influence de la perméabilité du papier pour filtres sur la ventilation du filtre est évaluée. Les cigarettes en combustion ont été simulées par l'apport d'une perte de charge au niveau du dard en amont de la cigarette. Un modèle mathématique a servi à prédire les effets de la longueur du boudin de tabac, du volume de la bouffée et de l'obstruction des trous de ventilation sur la ventilation des cigarettes et la perte de charge. Une dérivation de la perte de charge et des équations des flux d'une cigarette ayant une perte de charge en amont est fournie en annexe. [Beitr. Tabakforsch. Int. 19 (2000) 189–203]

INTRODUCTION

The theoretical framework needed to describe the influence of component design on flows and pressure drops within cigarettes exists in the literature (2, 4, 6–9,

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11, 12). The objective of this work is to integrate those known relationships into numerical methods that allow relating cigarette design specifications and puffing conditions to cigarette performance. In developing the protocol described in this report, the equations have been generalized to account for new challenges such as unconventional smoking regimes and partial vent blocking. The reported methodology lends itself to computing puff-by-puff performance and accounting for the effects of component variabilities on performance variability. The performance measures addressed in this paper are limited to ventilation and pressure-drop considerations – cigarette-yield and puff-count predictions within this framework remain to be accomplished.

This report provides a scheme for calculating the flow and pressure drops in both lit and unlit cigarettes under a variety of conditions. For each cigarette segment, the flow in and out of that segment, and the pressure drop across it are determined. The pressure drop across filter vents (perforations) and the flow through the vents are calculated. The flow of air through permeable cigarette paper is also calculated. Example computations are included.

In measuring cigarette and cigarette-component pressure drops and filter ventilation levels, a constant flow of air is maintained at the outlet of the sample – in the case of a cigarette, a standard flow of 17.5 cm^3/s is maintained out of the filter. The same is true of filter ventilation measurements – a standard flow of air is maintained at the filter outlet, and the fraction of this flow that enters the filter through its perforations is monitored. Ventilated filters may include multiple rows of perforations in the tipping paper. The location of these rows of vents from the mouth-end of the filter establishes their position. In principle, the vent rows can be designed to have different hole sizes in each row. The *relative* open area per vent row can be measured and is included as a design specification.

In Part I of the calculation, the pressure drops and flow rates of each filter segment and the tobacco rod are calculated numerically for an *unlit* cigarette. Additionally, the pressure drops across each row of filter vents and the flow through them are determined. The unlit tobacco rod is partitioned into the overwrapped section and the exposed cigarette-paper section. The pressure drops and flow rates through these sections are calculated, as well as the flow of air through the cigarette paper (rod dilution). The results of Part I form the basis for computing the flow and pressure-drop characteristics of *lit* cigarettes in Part II. The coal is treated as an additional source of pressure drop. Part II also forms the computational scheme for determining the flows and pressure drops in lit cigarettes as functions of the tobacco rod length. Additionally, this computational scheme allows the determinations of the effect of partially blocking the filter vents, and the effect of puff volume on filter ventilation.

LIST OF INPUT DESIGN PARAMETERS FOR PART I

Symbol	Variable description
ΔP_{cig}	Pressure drop across the ventilated cigarette (cm water gauge [WG]) at standard flow, e.g., 11.0 cm WG
Q_o	Standard flow rate in cm^3/s, e.g., 17.5 cm^3/s
L_{cig}	Length of the cigarette in cm, e.g., 8.5 cm
L_{tp}	Length of the tipping paper in cm, e.g., 3.1 cm
L_F	Length of the filter in cm, e.g., 2.7 cm
ΔP_F^o	Pressure drop, at standard flow, across the unventilated filter in cm WG, e.g., 10.0 cm WG
nrow	Number of rows of filter perforations, e.g., 3
L_{vi}	Distance of the i^{th} vent row from the mouth-end of the cigarette in cm; e.g., 1.3, 1.5, 1.7 cm
w_i	Relative open area of the i^{th} vent row such that the sum over all values equals 1 (equation [IV.1.1]), e.g., 0.50, 0.25, 0.25
D_F^o	Total filter ventilation in fractional form, at the standard flow rate, e.g., 0.50 (50%)
α_{pw}	CORESTA permeability of the filter plug-wrap, e.g., 26,000 CORESTA
C_r	Tobacco column circumference in cm; e.g., 2.48 cm
α_{cp}	CORESTA permeability of the cigarette paper, e.g., 45 CORESTA
ΔP_{coal}^o	Pressure drop of the coal at standard flow in cm WG, e.g., 2.54 cm WG

CALCULATION PROTOCOL WITH EXAMPLES

PART I – UNLIT CIGARETTES

In this section, the flows and pressure drops throughout an unlit cigarette are calculated. Relationships are presented and solved between pressure drop across the filter and filter flow rates, and between pressure drops across filter vents and vent flow rates. Both the relative vent-row sizes and their positions are treated as variables. Additionally, the tobacco column is partitioned into a tipping-paper overwrapped section and an exposed cigarette-paper section. The flow and pressure drop of each section are calculated, as is the flow of air through the cigarette paper.

I.1 *Calculation of the impedance of the filter to air flow, k, from equation [I.1.1]* (2)

The pressure drop, ΔP_F, across the filter of length, L_F, at standard flow, Q_o, is:

$$\Delta P_F^o = kL_F Q_o$$

$$k = \frac{\Delta P_F^o}{L_F Q_o} = \frac{10.0 \text{ cm WG}}{2.7 \text{ cm} \cdot 17.5 \text{ cm}^3/\text{s}} = 0.2116 \,[\text{cm WG/cm}^4/\text{s}]. \qquad [I.1.1]$$

I.2 *Estimation of the flow Q_{vi} through each filter perforation band*

By definition, the total filter ventilation at standard flow is:

$$D_F^o = \frac{\sum_{i=1}^{nrow} Q_{vi}}{Q_o} \qquad [I.2.1]$$

where Q_{vi} represents the flow rate through the i^{th} row of filter vents. The row closest to the mouth end of the filter is considered the *first* row. To start the iterative calculation, set, $Q_{vi} = w_{vi} D_F^o Q_o$:

Q_{v1} = 0.50 × 0.50 × 17.5 cm³/s = 4.3750 cm³/s
Q_{v2} = 0.25 × 0.50 × 17.5 cm³/s = 2.1875 cm³/s
Q_{v3} = 0.25 × 0.50 × 17.5 cm³/s = 2.1875 cm³/s.

I.3 *Computation of the flow rate of each filter section Q_{Fi} using equation [I.3.1]*

$$Q_{Fi} = Q_o - \sum_{n=1}^{i-1} Q_{vn} \qquad [I.3.1]$$

with:

Q_{F1} = 17.5 cm³/s
Q_{F2} = 17.5 cm³/s – 4.375 cm³/s = 13.1250 cm³/s
Q_{F3} = 17.5 cm³/s – 4.375 cm³/s – 2.1875 cm³/s = 10.9375 cm³/s
Q_{F4} = 17.5 cm³/s – 4.375 cm³/s – 2.1875 cm³/s – 2.1875 cm³/s = 8.7500 cm³/s.

The flow rate out of the tobacco rod, Q_r, is equal to the flow rate out of the filter section abutting it:

$$Q_r = 8.7500 \text{ cm}^3/\text{s}.$$

I.4 *Estimation of the pressure drop of each cigarette section from equations [I.4.1] and [I.4.2]*

$$\Delta P_{Fi} = kL_{Fi} Q_{Fi} \qquad [I.4.1]$$

with:

ΔP_{F1} = 0.2116 [cm WG/cm⁴/s] × 1.3 cm × 17.5 cm³/s = 4.8148 cm WG
ΔP_{F2} = 0.2116 [cm WG/cm⁴/s] × (1.5 cm – 1.3 cm) × 13.1250 cm³/s = 0.5556 cm WG
ΔP_{F3} = 0.2116 [cm WG/cm⁴/s] × (1.7 cm – 1.5 cm) × 10.9375 cm³/s = 0.4630 cm WG
ΔP_{F4} = 0.2116 [cm WG/cm⁴/s] × (2.7 cm – 1.7 cm) × 8.7500 cm³/s = 1.8519 cm WG
ΔP_F = 4.8148 cm WG + 0.5556 cm WG + 0.4630 cm WG + 1.8519 cm WG = 7.6852 cm WG.

The pressure drop across the tobacco column is equal to the total cigarette pressure drop minus the pressure drop of the ventilated filter:

$$\Delta P_{rod} = \Delta P_{cig} - \Delta P_F \qquad [I.4.2]$$

with:

$P\Delta_{rod}$ = 11.0000 cm WG – 7.6852 cm WG = 3.3148 cm WG.

I.5 *Estimation of the pressure drop ΔP_{vi} across each band of perforations from equation [I.5.1]*

$$\Delta P_{vi} = \Delta P_{rod} + \sum_{n=i+1}^{nrow+1} \Delta P_{Fn} \qquad [I.5.1]$$

with:

$\Delta P_{v1} = \Delta P_{rod} + \Delta P_{F4} + \Delta P_{F3} + \Delta P_{F2}$ = 6.1852 cm WG
$\Delta P_{v2} = \Delta P_{rod} + \Delta P_{F4} + \Delta P_{F3}$ = 5.6296 cm WG
$\Delta P_{v3} = \Delta P_{rod} + \Delta P_{F4}$ = 5.1667 cm WG.

I.6 *Calculation of the initial value of j from equation [I.6.1] for each band of perforations*

$$\Delta P_{vi} = j \left(\frac{Q_{vi}}{w_i} \right)^{vexp} \qquad [I.6.1]$$

The vent flow exponent, vexp, is computed from the relationship:

$$vexp = \left[1 - 0.348 \exp\left(\frac{-7180}{\alpha_{pw}} \right) \right]^{-1}. \qquad [1.6.2]^a$$

For a 26,000 CORESTA plugwrap, equation [I.6.2] yields a value of 1.36 for the exponent.

Solving equation [I.6.1] for *j*:

j_1 = 6.1852 cm WG/(4.3750 [cm³/s]/0.50)$^{1.36}$ = 0.3238 [cm WG/[cm³/s]$^{1.36}$]

[a] This equation was developed in our laboratory and the work is unpublished.

Table 1.
Results of the calculations in Part I

Calculated variable	Iterations			
	1	2	3	4
Q_{v1} [cm^3/s]	4.375000	4.594121	4.590253	4.590321
Q_{v2} [cm^3/s]	2.187500	2.143477	2.144266	2.144252
Q_{v3} [cm^3/s]	2.187500	2.012403	2.015481	2.015427
Q_{v} [cm^3/s]	8.750000	8.750000	8.750000	8.750000
D_F	0.500000	0.500000	0.500000	0.500000
Q_{F1} [cm^3/s]	17.500000	17.500000	17.500000	17.500000
Q_{F2} [cm^3/s]	13.125000	12.905879	12.909747	12.909679
Q_{F3} [cm^3/s]	10.937500	10.762403	10.765481	10.765427
Q_{F4} [cm^3/s]	8.750000	8.750000	8.750000	8.750000
Q_{r} [cm^3/s]	8.750000	8.750000	8.750000	8.750000
ΔP_{F1} [cm WG]	4.814815	4.814815	4.814815	4.814815
ΔP_{F2} [cm WG]	0.555556	0.546281	0.546444	0.546441
ΔP_{F3} [cm WG]	0.462963	0.455551	0.455682	0.455679
ΔP_{F4} [cm WG]	1.851852	1.851852	1.851852	1.851852
ΔP_{F} [cm WG]	7.685185	7.668499	7.668793	7.668788
ΔP_{rod} [cm WG]	3.314815	3.331501	3.331207	3.331212
ΔP_{cig} [cm WG]	11.000000	11.000000	11.000000	11.000000
ΔP_{v1} [cm WG]	6.185185	6.185185	6.185185	6.185185
ΔP_{v2} [cm WG]	5.629630	5.638905	5.638741	5.638744
ΔP_{v3} [cm WG]	5.166667	5.183353	5.183059	5.183064
j_1 [cm WG/[cm^3/s]$^{1.36}$]	0.323759	0.302940	0.303287	0.303281
j_2 [cm WG/[cm^3/s]$^{1.36}$]	0.294679	0.303439	0.303278	0.303281
j_3 [cm WG/[cm^3/s]$^{1.36}$]	0.270445	0.303918	0.303270	0.303281
j_{av} [cm WG/[cm^3/s]$^{1.36}$]	0.296294	0.303433	0.303279	0.303281
j_{sd} [cm WG/[cm^3/s]$^{1.36}$]	0.026693	0.000489	0.000009	0.000000
Q'_{v1} [cm^3/s]	4.669665	4.588637	4.590349	4.590319
Q'_{v2} [cm^3/s]	2.178723	2.143511	2.144265	2.144252
Q'_{v3} [cm^3/s]	2.045494	2.014771	2.015439	2.015428
Q'_{v} [cm^3/s]	8.893882	8.746919	8.750054	8.749999

$$j_2 = 5.6296 \text{ cm WG}/(2.1875\ [\text{cm}^3/\text{s}]/0.25)^{1.36} = 0.2947\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}]$$
$$j_3 = 5.1667 \text{ cm WG}/(2.1875\ [\text{cm}^3/\text{s}]/0.25)^{1.36} = 0.2704\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}]$$
$$j_{av} = 0.2963\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}]$$
$$j_{sd} = 0.02669\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}$$

where j_{av} and j_{sd} are the average and standard deviation of the j_i.

I.7 *Use of j_{av} to calculate new Q_{vi} from equation [I.6.1]*

$$Q'_{v1} = (6.1852 \text{ cm WG}/0.2963\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}])^{1/1.36} \times 0.50 = 4.6697 \text{ cm}^3/\text{s}$$
$$Q'_{v2} = (5.6296 \text{ cm WG}/0.2963\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}])^{1/1.36} \times 0.25 = 2.1787 \text{ cm}^3/\text{s}$$
$$Q'_{v3} = (5.1667 \text{ cm WG}/0.2963\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}])^{1/1.36} \times 0.25 = 2.0455 \text{ cm}^3/\text{s}$$
$$Q'_{v} = 4.6697 \text{ cm}^3/\text{s} + 2.1787 \text{ cm}^3/\text{s} + 2.0455 \text{ cm}^3/\text{s} = 8.8939 \text{ cm}^3/\text{s}$$

I.8 *Renormalization of Q_{vi} to the original dilution*

The total filter-vent flow rate at the 50% dilution level is $0.50 \times Q_o = 8.75$ cm^3/s.

Therefore,

$$Q_{v1} = 4.6697 \text{ cm}^3/\text{s} \times (8.7500/8.8939) = 4.5941 \text{ cm}^3/\text{s}$$
$$Q_{v2} = 2.1787 \text{ cm}^3/\text{s} \times (8.7500/8.8939) = 2.1435 \text{ cm}^3/\text{s}$$
$$Q_{v3} = 2.0455 \text{ cm}^3/\text{s} \times (8.7500/8.8939) = 2.0124 \text{ cm}^3/\text{s}$$
$$Q_{v} = Q_{v1} + Q_{v2} + Q_{v3} = 8.7500 \text{ cm}^3/\text{s}.$$

I.9 *Going to step I.3 and repetition of the calculation until j_{sd} approaches zero*

We used a convergence criterion of less than 1×10^{-6}. Table 1 presents the results of this calculation through four iterations.

At this point, the flow out of the tobacco rod and the pressure drop across it have been calculated. However, the tobacco rod consists of two segments: the overwrapped segment covered by air-impermeable tipping paper, and the remainder of the tobacco column covered with air-permeable cigarette paper. Thus the tobacco rod is partitioned into two separate sections, each with a particular flow and pressure drop. Further, the amount of air entering the cigarette through the permeable wrapper is to be calculated. From the example solved above, the pressure drop across the tobacco column was calculated to be 3.3312 cm WG (Table 1) and the flow rate out of the column as 8.7500 cm^3/s. The lengths of the overwrapped portion of the rod, $L_{ow} = (L_{tp} - L_F)$ and the exposed portion of the tobacco rod $L_{er} = (L_{cig} - L_{tp})$ are known. The circumference of the tobacco rod and the permeability of the tobacco wrapper are known parameters.

I.10 *Estimation of initial values of the pressure drops across the exposed tobacco rod section ΔP_{er} and the overwrapped rod section ΔP_{ow}*

$$\Delta P_{er} = \left(\frac{L_{er}}{L_{rod}}\right) \Delta P_{rod} \qquad [I.10.1]$$

with:

ΔP_{er} = (5.4cm/5.8cm) 3.3312 cm WG = 3.1015 cm WG
$\Delta P_{ow} = \Delta P_{rod} - \Delta P_{er}$ = 3.3312 cm WG − 3.1015 cm WG = 0.2297 cm WG.

I.11 *Calculation of the encapsulated pressure drop of the exposed rod section ΔP^o_{re} and overwrapped rod section ΔP^o_{ow} at standard flow from equation [I.11.1]* (4)

$$\Delta P_{er} = \Delta P^o_{re}\left(0.04200\,Q_r + 0.0008653\,Q_r^2\right)\frac{\tanh(\Theta)}{\Theta} \qquad [I.11.1]$$

where:

$$\Theta = \left[\frac{\alpha_{cp} C_r L_{er} \Delta P^o_{re}}{600}\left(0.04200 + 0.001731\,Q_r\right)\right]^{\frac{1}{2}}. \qquad [I.11.2]$$

Note: In this report, as in Reference 4, the wrapper flow equations are based on a linear relationship between the pressure drop across the tobacco wrapper and the flow rate through it.
All the terms in equation [I.11.1] are known except ΔP^o_{re}. This was evaluated by rearranging the equation in the form:

$$\Delta P^o_{re} = \frac{\Delta P_{er}}{\left(0.04200\,Q_r + 0.0008653\,Q_r^2\right)\dfrac{\tanh(\Theta)}{\Theta}}.$$

The Θ terms include ΔP^o_{re}. A starting estimate for ΔP^o_{re} is $\Delta P_{er}/(1 - D_F)$ = 6.2030 cm WG. Substituting this value into the Θ terms on the right side of the equation yields a new estimate of ΔP^o_{re} = 7.9794 cm WG. This method of successive approximations converges after six iterations to a value of 8.2441 cm WG, as below.

Iteration	Input value (cm WG)	Output value (cm WG)
1	6.2030	7.9794
2	7.9794	8.2100
3	8.2100	8.2397
4	8.2397	8.2435
5	8.2435	8.2440
6	8.2440	8.2441
7	8.2441	8.2441

The pressure drop of the impermeable overwrapped section at standard flow is found from the equation:

$$\Delta P_{ow} = \Delta P^o_{ow}\left(0.04200\,Q_r + 0.0008653\,Q_r^2\right) \qquad [I.11.3]$$

$$\Delta P^o_{ow} = \frac{0.2297\ \text{cm WG}}{0.04200 \times 8.75\ \text{cm}^3/\text{s} + 0.0008653 \times (8.75\ \text{cm}^3/\text{s})^2}$$
$$= 0.5297\ \text{cm WG}.$$

I.12 *Calculation of the encapsulated pressure drop ΔP_{re} of the exposed-rod section at actual flow from equation [I.12.1]* (4)

$$\begin{aligned}\Delta P_{re} &= \Delta P^o_{re}\left(0.04200\,Q_r + 0.008653\,Q_r^2\right)\\ &= 8.2441\ \text{cm WG}\ (0.04200 \times 8.75\ \text{cm}^3/\text{s} + 0.0008653\ (8.75\ \text{cm}^3/\text{s})^2)\\ &= 3.5759\ \text{cm WG}\end{aligned} \qquad [I.12.1]$$

I.13 *Computation of the pressure drops across the rod segments at the actual flow rate out of the rod*

Since the overwrapped section of the tobacco rod is covered with air-impermeable tipping paper, its pressure drop is equal to its length ratio of the encapsulated-rod pressure drop:

$$\begin{aligned}\Delta P_{ow} &= \frac{L_{ow}}{L_{er}}\Delta P^o_{re}\\ &= (0.4\ \text{cm}/5.4\ \text{cm})\ 3.5759\ \text{cm WG}\\ &= 0.2649\ \text{cm WG}.\end{aligned} \qquad [I.13.1]$$

The pressure drop across the exposed rod section is equal to the total rod pressure drop determined in Part I minus the overwrap pressure drop:

$$\begin{aligned}\Delta P_{er} &= \Delta P_{rod} - \Delta P_{ow}\\ &= 3.3312\ \text{cm WG} - 0.2649\ \text{cm WG}\\ &= 3.0663\ \text{cm WG}.\end{aligned} \qquad [I.13.2]$$

Table 2.
Results of the calculations in Part I continued

Calculated variable	Iterations			
	1	2	6	7
ΔP_{ow} [cm WG]	0.229739	0.264880	0.261747	0.261747
ΔP_{er} [cm WG]	3.101474	3.066333	3.069465	3.069466
ΔP_{rod} [cm WG]	3.331212	3.331212	3.331212	3.331212
ΔP^{o}_{ow} [cm WG]	0.529658	0.610674	0.603452	0.603451
ΔP^{o}_{re} [cm WG]	8.244101	8.137066	8.146594	8.146595
ΔP_{re} [cm WG]	3.575875	3.529449	3.533581	3.533582
Q_{cp} [cm^3/s]				1.484105
Q_{coal} [cm^3/s]				7.265895
ΔP^{o}_{cig} [cm WG]				17.450500

I.14 *Going to step I.11 and iteration to self consistency*

We chose to continue the iterations until the change in ΔP_{re} between successive calculations was less than 1×10^{-6}. Table 2 presents the results of this calculation through seven iterations.

I.15 *Calculation of the air flow Q_{cp} through the cigarette wrapper using equation [I.15.1]* (4)

$$Q_{cp} = \frac{\Delta P_{re}\left[1 - \mathrm{sech}(\Theta)\right]}{\Delta P^{o}_{re}\left(0.04200 + 0.001731\ Q_r\right)} \qquad \text{[I.15.1]}$$

where Θ is given by equation [I.11.2]:

$$Q_{cp} = \frac{3.5336 \text{ cm WG}\left[1 - \mathrm{sech}(0.6838)\right]}{8.1466 \text{ cm WG}\left(0.04200 + 0.001731 \times 8.75 \text{ cm}^3/\text{s}\right)}$$

$$= 1.4841 \text{ cm}^3/\text{s}.$$

The flow rate into the upstream end of the tobacco rod is denoted Q_{coal}, and is equal to:

$$\begin{aligned} Q_{coal} &= Q_o - (Q_v + Q_{cp}) \\ &= 17.5000 \text{ cm}^3/\text{s} - 8.7500 \text{ cm}^3/\text{s} - 1.4841 \text{ cm}^3/\text{s} \\ &= 7.2659 \text{ cm}^3/\text{s}. \end{aligned}$$

I.16 *Calculation of the cigarette pressure drop P^{o}_{cig} for the case where the filter ventilation is zero*

The computation of the unventilated-cigarette pressure drop is a good check on the quality of the calculation. This experimental value is easily obtained, and its agreement with the predicted value tends to validate the computational approach. First, calculate the pressure drop of the exposed rod at the *standard* flow:

$$\Delta P^{o}_{er} = \Delta P^{o}_{re}\,\frac{\tanh(\Theta)}{\Theta} \qquad \text{[I.16.1]}$$

where:

$$\Theta = \left[\frac{\alpha_{cp} C_r L_{er} \Delta P^{o}_{re}}{600}\left(0.04200 + 0.001731\ Q_o\right)\right]^{\frac{1}{2}}. \qquad \text{[I.16.2]}$$

For this case, $\Theta = 0.7691$, $\Delta P^{o}_{re} = 8.1466$ cm WG, and $\Delta P^{o}_{er} = 6.8470$ cm WG. The pressure drop across the cigarette without filter ventilation is:

$$\begin{aligned} \Delta P^{o}_{cig} &= \Delta P^{o}_{F} + \Delta P^{o}_{ow} + \Delta P^{o}_{er} \\ &= 10.0000 \text{ cm WG} + 0.6035 \text{ cm WG} \\ &\quad + 6.8470 \text{ cm WG} \\ &= 17.4505 \text{ cm WG}. \end{aligned} \qquad \text{[I.16.3]}$$

PART II: LIT CIGARETTES

In this section, a lit cigarette is mimicked by adding a "coal" to the upstream end of the tobacco rod. This coal consists of an added source of pressure drop (2.54 cm WG in this example). This particular value is chosen simply for illustration and based on some unpublished work from the laboratories of Philip Morris. A lit cigarette behaves differently than an unlit one in part because the coal increases the total cigarette pressure drop, thereby increasing the amount of diluting air that enters through the wrapper and filter vents. Appendix I provides the derivation of the equations used in this section.

II.1 *Setting the initial values of the vent flow rates Q_{vi} equal to the values computed in Part I or using the method described in Part I.2*

$Q_{v1} = 4.5903$ cm^3/s
$Q_{v2} = 2.1443$ cm^3/s
$Q_{v3} = 2.0154$ cm^3/s

II.2 *Calculation of Q_{Fi} and ΔP_{Fi} as in steps I.2 and I.3*

Q_{F1} = 17.5000 cm³/s ΔP_{F1} = 4.8148 cm WG
Q_{F2} = 12.9097 cm³/s ΔP_{F2} = 0.5464 cm WG
Q_{F3} = 10.7654 cm³/s ΔP_{F3} = 0.4557 cm WG
Q_{F4} = 8.7500 cm³/s ΔP_{F4} = 1.8519 cm WG
Q_{ow} = 8.7500 cm³/s ΔP_{F} = 7.6688 cm WG
Q_{r} = 8.7500 cm³/s

II.3 *Calculation of the pressure drop of the overwrapped section of the tobacco column from equation [II.3.1]*

$$\Delta P_{ow} = \Delta P^{o}_{ow}\left(0.04200\,Q_r + 0.0008653\,Q_r^2\right) \quad \text{[II.3.1]}$$
$$= 0.6035 \text{ cm WG} \times 0.4337$$
$$= 0.2617 \text{ cm WG}$$

II.4 *Calculation of Q_{cp}, ΔP_{coal}, ΔP_{er}, and ΔP_{cig} for the lit cigarette*

The flow of air through the cigarette paper of a lit cigarette is derived in Appendix I. The equation is (equation [A5]):

$$Q_{cp} = \frac{\dfrac{\left(aQ_r + bQ_r^2\right)}{\left(a + 2bQ_r\right)}\{1 - \text{sech}(\Theta)\} + \dfrac{\sqrt{k\alpha_{cp}C_rL_{er}}\,\Delta P^{o}_{coal}Q_r}{Q_o\left[\Delta P^{o}_{re}\left(a + 2bQ_r\right)\right]^{1/2}}\tanh(\Theta)}{1 + \dfrac{\sqrt{k\alpha_{cp}C_rL_r}\,\Delta P^{o}_{coal}}{Q_o\left[\Delta P^{o}_{re}\left(a + 2bQ_r\right)\right]^{1/2}}\tanh(\Theta)} \quad \text{[II.4.1]}$$

where:

k = 1/600
a = 0.04200 s/cm³
b = 0.0008653 s²/cm⁶
$\Theta = \left[k\alpha_{cp}C_rL_{er}\Delta P^{o}_{re}\left(a + 2bQ_r\right)\right]^{\frac{1}{2}}$.

Substituting all the parameters into equation [II.4.1] yields Θ = 0.6838 and Q_p = 2.3008 cm³/s.
Assuming that the pressure drop of the coal is known under standard conditions (3) (ΔP^{o}_{coal} = 2.54 cm WG for this example), its pressure drop contribution to a ventilated cigarette is assumed to be linear with flow rate (equation [A1]):

$$\Delta P_{coal} = \Delta P^{o}_{coal}\frac{\left(Q_r - Q_{cp}\right)}{Q_o} \quad \text{[II.4.2]}$$
$$= 2.54 \text{ cm WG } \frac{8.7500 \text{ cm}^3/\text{s} - 2.3008 \text{ cm}^3/\text{s}}{17.5 \text{ cm}^3/\text{s}}$$
$$= 0.9361 \text{ cm WG}.$$

The pressure drop across the tobacco rod in a lit cigarette (equation [A8]) is given by:

$$\Delta P_{er} = \Delta P_{coal}\{\text{sech}(\Theta) - 1\} + \Delta P_{re}\frac{\tanh(\Theta)}{\Theta}$$
$$= 0.9361 \text{ cm WG } (-0.2000) \quad \text{[II.4.3]}$$
$$+\ 3.5336 \text{ cm WG} \tanh(0.6838)/0.6838$$
$$= 2.8864 \text{ cm WG}.$$

The total cigarette pressure drop is equal to the sum of the component pressure drops:

$$\Delta P_{cig} = \Delta P_F + \Delta P_{ow} + \Delta P_{er} + \Delta P_{coal}$$
$$= 7.6688 \text{ cm WG} + 0.2617 \text{ cm WG}$$
$$+\ 2.8864 \text{ cm WG} + 0.9361 \text{ cm WG}$$
$$= 11.7530 \text{ cm WG}.$$

II.5 *Calculation of the filter vent pressure drops ΔP_{vi} from those of the upstream components*

$$\Delta P_{vi} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \sum_{n=i+1}^{nrow+1}\Delta P_{Fn} \quad \text{[II.5.1]}$$

with:

$\Delta P_{v1} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \Delta P_{F4} + \Delta P_{F3} + \Delta P_{F2}$
= 6.9382 cm WG
$\Delta P_{v2} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \Delta P_{F4} + \Delta P_{F3}$
= 6.3918 cm WG
$\Delta P_{v3} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \Delta P_{F4}$
= 5.9361 cm WG.

II.6 *Determination of the vent flow rates Q_{vi} from the vent pressure drops*

$$\Delta P_{vi} = j\left(\frac{Q_{vi}}{w_i}\right)^{vexp}$$

Therefore,

Q_{v1} = (6.9382 cm WG/0.3033 [cm WG/[cm³/s]$^{1.36}$])$^{1/1.36}$ × 0.50 = 4.8601 cm³/s
Q_{v2} = (6.3918 cm WG/0.3033 [cm WG/[cm³/s]$^{1.36}$])$^{1/1.36}$ × 0.25 = 2.2823 cm³/s
Q_{v3} = (5.9361 cm WG/0.3033 [cm WG/[cm³/s]$^{1.36}$])$^{1/1.36}$ × 0.25 = 2.1564 cm³/s
Q_v = 4.8601 cm³/s + 2.2823 cm³/s + 2.1564 cm³/s
= 9.2987 cm³/s
$D_F = Q_v/Q_o$ = (9.3691 cm³/s)/(17.5 cm³/s) = 0.5314.

II.7 *Returning to step II.2 and iteration to self consistency*

Table 3 presents the results of this calculation through eight iterations.

Table 3.
Results of the calculations in Part II

Calculated variable	Iteration			
	1	2	7	8
Q_{v1} [cm^3/s]	4.590320	4.860072	4.808976	4.809118
Q_{v2} [cm^3/s]	2.144252	2.282274	2.258743	2.258814
Q_{v3} [cm^3/s]	2.015428	2.156363	2.136360	2.136428
Q_v [cm^3/s]	8.750000	9.298708	9.204079	9.204359
D_F	0.500000	0.531355	0.525947	0.525963
Q_{F1} [cm^3/s]	17.500000	17.500000	17.500000	17.500000
Q_{F2} [cm^3/s]	12.909680	12.639928	12.691024	12.690882
Q_{F3} [cm^3/s]	10.765428	10.357655	10.432281	10.432069
Q_{F4} [cm^3/s]	8.750000	8.201292	8.295921	8.295641
Q_{ow} [cm^3/s]	8.750000	8.201292	8.295921	8.295641
Q_r [cm^3/s]	8.750000	8.201292	8.295921	8.295641
ΔP_{F1} [cm WG]	4.814815	4.814815	4.814815	4.814815
ΔP_{F2} [cm WG]	0.546441	0.535023	0.537186	0.537180
ΔP_{F3} [cm WG]	0.455679	0.438419	0.441578	0.441569
ΔP_{F4} cm WG]	1.851852	1.735723	1.755751	1.755691
ΔP_F [cm WG]	7.668788	7.523981	7.549330	7.549255
ΔP_{ow} [cm WG]	0.261747	0.242983	0.246196	0.246187
ΔP_{er} [cm WG]	2.886450	2.686130	2.720497	2.720395
ΔP_{coal} [cm WG]	0.936056	0.878377	0.888334	0.888304
ΔP_{cig} [cm WG]	11.753040	11.331470	11.404357	11.404142
ΔP_{v1} [cm WG]	6.938226	6.516656	6.589542	6.589327
ΔP_{v2} [cm WG]	6.391784	5.981632	6.052356	6.052147
ΔP_{v3} [cm WG]	5.936105	5.543213	5.610778	5.610578
Q_{cp} [cm^3/s]	2.300793	2.149480	2.175511	2.175434

PART III: ROD LENGTH EFFECTS

The first application involves calculating the flows and pressure drops throughout a cigarette when the length of the lit tobacco column diminishes, as it would during smoking. Many of the input design features of the cigarette will not change with length; however, those dependent on the tobacco column will. As the rod length diminishes, the pressure drop of the total cigarette, the ventilation through the cigarette wrapper, and the filter ventilation will change. But the vent-flow exponent, vexp, and the pressure-drop equation coefficients k and j will not change. These characteristics, coupled with the assumption that the encapsulated pressure drop of the tobacco rod varies linearly with rod length, form the basis of the length calculations. Appendix II provides a methodology for calculating puff positions based on this application.
The calculation is similar to that described in Part II for a lit sample. The same cigarette design parameters used in Part II will be used here, including:

ADDITIONAL INPUT PARAMETERS FOR THE ROD LENGTH CALCULATION

Symbol	Variable description
ΔP^{o}_{ow}	Pressure drop across the overwrapped section [cm WG] at standard flow, e.g., 0.6035 cm WG
ΔP^{o}_{re}	Pressure drop across the encapsulated tobacco rod [cm WG] at full length and standard flow, e.g., 8.1466 cm WG
ΔP^{o}_{coal}	Pressure drop across the coal [cm WG] at standard flow, e.g., 2.54 cm WG
L_{cig}	Shortened cigarette length in cm, e.g., 7.5 cm
j	Vent pressure-drop coefficient in cm WG/[cm^3/s]$^{1.36}$, e.g., 0.3033 cm WG/[cm^3/s]$^{1.36}$

The encapsulated pressure drop of the exposed portion of the rod is linear with rod length. The value at the new rod length, L_i (7.5 cm – 3.1 cm), is equal to the ratio of this length to the original one, L_o (8.5 cm – 3.1 cm), multiplied by the full length encapsulated pressure drop:

$$\Delta P^o_{re}(L_i) = \Delta P^o_{re}(L_o)\frac{L_i}{L_o} \quad [III.0]$$
$$= 8.1466 \text{ cm WG } (4.4 \text{ cm}/5.4 \text{ cm})$$
$$= 6.6380 \text{ cm WG}.$$

III.1 *Setting the initial values of the vent flow rates Q_{vi} equal to the values computed in Part I*

Q_{v1} = 4.5903 cm^3/s
Q_{v2} = 2.1443 cm^3/s
Q_{v3} = 2.0154 cm^3/s

III.2 *Calculating of the filter flow rates Q_{Fi} and pressure drops ΔP_{Fi} as in steps I.2 and I.3*

Q_{F1} = 17.5000 cm^3/s	ΔP_{F1} = 4.8148 cm WG
Q_{F2} = 12.9097 cm^3/s	ΔP_{F2} = 0.5464 cm WG
Q_{F3} = 10.7654 cm^3/s	ΔP_{F3} = 0.4557 cm WG
Q_{F4} = 8.7500 cm^3/s	ΔP_{F4} = 1.8519 cm WG
Q_{ow} = 8.7500 cm^3/s	ΔP_{F} = 7.6688 cm WG
Q_{r} = 8.7500 cm^3/s	

III.3 *Calculation of the pressure drop ΔP_{ow} of the overwrapped section of the tobacco column from equation [III.3.1]*

$$\Delta P_{ow} = \Delta P^o_{ow}\left(0.04200\, Q_r + 0.0008653\, Q_r^2\right)$$
$$= 0.6035 \text{ cm WG} \times 0.4337 \quad [III.3.1]$$
$$= 0.2617 \text{ cm WG}$$

III.4 *Calculation of* ΔP^o_{re}, ΔP_{er}, Q_{cp}, ΔP_{coal}, *and* ΔP_{cig} *for the new rod length*

The flow of air through the cigarette paper of the shortened lit cigarette is (equation [A3]):

$$Q_{cp} = \frac{\dfrac{(aQ_r + bQ_r^2)}{(a + 2bQ_r)}\{1 - \text{sech}(\Theta)\} + \dfrac{\sqrt{k\alpha_{cp} C_r L_r}\,\Delta P^o_{coal} Q_r}{Q_o\left[\Delta P^o_{re}(a + 2bQ_r)\right]^{1/2}}\tanh(\Theta)}{1 + \dfrac{\sqrt{k\alpha_{cp} C_r L_r}\,\Delta P^o_{coal}}{Q_o\left[\Delta P^o_{re}(a + 2bQ_r)\right]^{1/2}}\tanh(\Theta)} \quad [III.4.2]$$

where:

k = 1/600
a = 0.04200 s/cm^3
b = 0.0008653 s^2/cm^6
$\Theta = \left[k\alpha_{cp} C_r L_{er} \Delta P^o_{re}(a + 2bQ_r)\right]^{\frac{1}{2}}$.

Substituting all the parameters into equation [III.4.2] yields Θ = 0.5572 and Q_{cp} = 1.7932 cm^3/s. Assuming that the pressure drop of the coal is known under standard conditions (2.54 cm WG for this example), its pressure drop contribution to a ventilated cigarette is considered to be linear with the flow rate (equation [A1]):

$$\Delta P_{coal} = \Delta P^o_{coal}\frac{(Q_r - Q_{cp})}{Q_o}$$
$$= (2.54 \text{ cm WG})\,(8.7500 \text{ cm}^3/\text{s} - 1.7932 \text{ cm}^3/\text{s})/(17.5 \text{ cm}^3/\text{s}) \quad [III.4.3]$$
$$= 1.0097 \text{ cm WG}.$$

The pressure drop across the tobacco rod in a lit cigarette (equation [A6]) is given by:

$$\Delta P_{er} = \Delta P_{coal}\{\text{sech}(\Theta) - 1\} + \Delta P_{re}\frac{\tanh(\Theta)}{\Theta}$$
$$= 1.0097 \text{ cm WG } (-0.1374) + 2.8792 \text{ cm WG } \tanh(0.5572)/0.5572 \quad [III.4.4]$$
$$= 2.4754 \text{ cm WG}.$$

The total cigarette pressure drop is equal to the sum of the component pressure drops:

$$\Delta P_{cig} = \Delta P_F + \Delta P_{ow} + \Delta P_{er} + \Delta P_{coal}$$
$$= 7.6688 \text{ cm WG} + 0.2617 \text{ cm WG} + 2.4754 \text{ cm WG} + 1.0097 \text{ cm WG}$$
$$= 11.4157 \text{ cm WG}.$$

III.5 *Calculation of the filter vent pressure drops ΔP_{vi} from those of the components*

$$\Delta P_{vi} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \sum_{n=i+1}^{nrow+1} \Delta P_{Fn} \quad [III.5.1]$$

with:

$\Delta P_{v1} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \Delta P_{F4} + \Delta P_{F3} + \Delta P_{F2}$
= 6.6009 cm WG
$\Delta P_{v2} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \Delta P_{F4} + \Delta P_{F3}$
= 6.0544 cm WG
$\Delta P_{v3} = \Delta P_{coal} + \Delta P_{er} + \Delta P_{ow} + \Delta P_{F4}$
= 5.5988 cm WG.

III.6 *Determination of the vent flow rates Q_{vi} from the vent pressure drops*

$$\Delta P_{vi} = j\left(\frac{Q_{vi}}{w_i}\right)^{vexp} \quad [III.6.1]$$

Therefore,

$Q_{v1} = (6.6009 \text{ cm WG}/0.3033\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}])^{1/1.36} \times 0.50 = 4.8152 \text{ cm}^3/\text{s}$
$Q_{v2} = (6.0544 \text{ cm WG}/(0.3033\ [\text{cm WG}/[\text{cm}^3/\text{s}]^{1.36}])^{1/1.36} \times 0.25 = 2.2594 \text{ cm}^3/\text{s}$

Table 4.
Results of the calculations in Part III

Calculated variable	Iterations			
	1	2	7	8
Q_{v1} [cm^3/s]	4.590320	4.740239	4.714259	4.714261
Q_{v2} [cm^3/s]	2.144252	2.221007	2.209176	2.209180
Q_{v3} [cm^3/s]	2.015428	2.093850	2.084038	2.084046
Q_v [cm^3/s]	8.750000	9.055097	9.007473	9.007486
D_F	0.500000	0.517434	0.514713	0.514714
Q_{F1} [cm^3/s]	17.500000	17.500000	17.500000	17.500000
Q_{F2} [cm^3/s]	12.909680	12.759761	12.785741	12.785739
Q_{F3} [cm^3/s]	10.765428	10.538753	10.576565	10.576559
Q_{F4} [cm^3/s]	8.750000	8.444903	8.492527	8.492514
Q_{ow} [cm^3/s]	8.750000	8.444903	8.492527	8.492514
Q_r [cm^3/s]	8.750000	8.444903	8.492527	8.492514
ΔP_{F1} [cm WG]	4.814815	4.814815	4.814815	4.814815
ΔP_{F2} [cm WG]	0.546441	0.540096	0.541195	0.541195
ΔP_{F3} [cm WG]	0.455679	0.446085	0.447685	0.447685
ΔP_{F4} [cm WG]	1.851852	1.787281	1.797360	1.797357
ΔP_F [cm WG]	7.668788	7.588276	7.601056	7.601053
ΔP_{ow} [cm WG]	0.261747	0.251275	0.252903	0.252903
ΔP_{er} [cm WG]	2.475423	2.378797	2.393832	2.393828
ΔP_{coal} [cm WG]	1.009735	0.975011	0.980434	0.980432
ΔP_{cig} [cm WG]	11.415693	11.193360	11.228224	11.228215
ΔP_{v1} [cm WG]	6.600878	6.378545	6.413409	6.413400
ΔP_{v2} [cm WG]	6.054437	5.838449	5.872214	5.872205
ΔP_{v3} [cm WG]	5.598757	5.392364	5.424529	5.424520
Q_{cp} [cm^3/s]	1.793161	1.727307	1.737571	1.737568

$$Q_{v3} = (5.5988 \text{ cm WG}/0.3033 \text{ [cm WG/[cm}^3\text{/s]}^{1.36}\text{]})^{1/1.36} \times 0.25 = 2.1331 \text{ cm}^3\text{/s}$$
$$Q_v = 9.2076 \text{ cm}^3\text{/s}$$
$$D_F = Q_v/Q_o = 0.5262$$

III.7 *Going to step III.2 and iteration to self consistency*

We found these calculations to converge slowly, so a variety of convergence accelerators were tried. An increase in the rate of convergence was obtained by defining the new value of Q_{vi} as being equal to the old value plus twice the new value – that sum being divided by three. For example, considering the values in Section III.1 as the old values, and those in Section III.6 as the new ones, the Q_v's were recalculated as:

$$Q_{v1} = [1(4.5903 \text{ cm}^3\text{/s}) + 2(4.8152 \text{ cm}^3\text{/s})]/(1 + 2) = 4.7402 \text{ cm}^3\text{/s}.$$

This procedure reduced the number of iterations required to achieve convergence. Table 4 presents the results of this calculation through eight iterations.

Appendix II illustrates how to calculate puff positions based on the volume of air entering the cigarette through the coal.

PART IV: FILTER VENTILATION EFFECTS

In this section, the influence of the flow rate during puffing on cigarette ventilation is examined, as well as the effect of covering a portion of the filter vents. Conventional Federal Trade Commission (FTC) smoking requires a 17.5 cm^3/s flow rate of two-second duration, but other puffing conditions may be of interest. In this section, the input design parameters and results of Parts I and II are used to calculate how the flow rate out of a filter affects the pressure drops, flow rates, and ventilation levels within the cigarette. In this section, the calculation scheme employed in Part II is used; only the initial conditions are changed. The first change involves redefining the flow rate out of the cigarette. Up to this point, the value of Q_o has been set to 17.5 cm^3/s. By setting this parameter to a different value, the calculation can proceed exactly as described in Part II.

	Filter ventilation (%)	
Q_o (cm^3/s)	ΔP_{coal} = 0	ΔP_{coal} = 2.54 cm WG
30.0	47.0	48.4
22.5	48.5	50.5
17.5	50.0	52.6
15.0	51.0	54.0
12.5	52.1	55.7
8.75	54.6	59.4

The model predicts that the filter ventilation will diminish as the flow rate through the (5, 7), as illustrated above.

Mimicking filter-vent blockage is also straightforward. This is accomplished within the calculational protocol by adjusting the filter vent-row weights to reflect the extent of the blockage. Once the weights are define cigarette increases d properly, one follows the calculation regime described in Part II. This case then becomes one of specifying appropriate vent weights.

The purpose of the vent-weight parameter is to allow designers to examine the effects of differing tipping paper hole sizes on cigarette performance. If, for example, one wanted to assess the effect of having the first row of vents being twice as large as those of the second and third rows (or having twice as many at the same size), the vent row weights could be specified as:

$w_1^/ = 2,\ w_2^/ = 1,\ w_3^/ = 1.$

For the computation we would normalize these values to sum to 1.0:

$$w_i = \frac{w_i^/}{\sum_{i=1}^{nrow} w_i^/} \qquad [IV.1.1]$$

with:

$w_1 = 2/4,\ w_2 = 1/4,\ +\ w_3 = 1/4,$
$w_1 + w_2 + w_3 = 1.$

These normalized values can be used directly in Parts I and II to find the flows and pressure drops throughout the cigarette. For vent blocking however, the weights are normalized such that their values sum to the fraction of holes left unblocked. For the case of 50% vent blocking, the fraction of holes blocked (f_{block}) in each row is 1/2. This constraint is accommodated by redefining equation [IV.1.1] as:

$$w_{ib} = \frac{\left(1 - f_{block}\right) w_i^/}{\sum_{i=1}^{nrow} w_i^/} \qquad [IV.1.2]$$

with:

$w_{1b} = 1/4,\ w_{2b} = 1/8,\ w_{3b} = 1/8),$
$w_{1b} + w_{2b} + w_{3b} = 1/2.$

The method of solving for partially blocked vents is:

- solving Parts I and II with f_{block} equal to zero;
- re-calculating the vent weights based on f_{block};
- solving as in Part II.

Using the same input parameters as in previous sections, the following results were obtained:

f_{block}	D_F (%)	ΔP_{cig} (cm WG)
0.0	50.0	11.0
0.2	43.8	11.8
0.4	36.1	12.7
0.5	31.7	13.3
0.6	26.8	13.9
0.8	15.1	15.4
1.0	0	17.4

CONCLUSIONS

Detailed numerical recipes have been presented for calculating the flows and pressure drops throughout cigarettes. The equations implemented were for the most part taken from the literature. Where gaps that precluded completing the model were found, equations were developed as well as numerical methods for solving the them. The results are presented without comparison to actual data. Such comparisons have been made and a paper presenting them is in preparation.

Initially, the design parameters, total pressure drop, and filter ventilation level of an unlit cigarette are used to evaluate performance characteristics needed for the lit-cigarette calculation. These characteristics include:

- k coefficient of equation [I.1.1] relating filter flow rate to filter pressure drop,
- j coefficient of equation [I.6.1] relating vent flow rate to vent pressure drop,
- ΔP^o_{re} pressure drop across the encapsulated tobacco column at standard flow,
- ΔP^o_{ow} pressure drop across the overwrapped portion of the tobacco column at standard flow.

These computed variables, along with the coal pressure drop, are used to calculate the flows (including the filter ventilation level) and pressure drops throughout a lit cigarette as functions of tobacco-rod length, puff volume, and filter-vent blocking.

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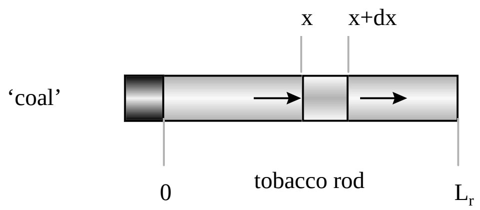
APPENDIX I
DERIVATION OF PRESSURE DROP AND FLOW EQUATIONS FOR A CIGARETTE WITH AN UP-STREAM PRESSURE DROP

Consider a volume element of a tobacco rod covered with an air-permeable paper, and having a coal at the upstream end. Mathematically, the effect of the coal is mimicked as a source of pressure drop upstream of the tobacco rod.



The derivation in this Appendix follows closely the derivation in Reference 4 for a cigarette without a coal. For brevity, we shall show only that which is different in this case and rely on the previous derivation for developing the equations. The equations numbered [A'] refer to the similarly numbered equations in the reference.

The change in the flow of air through the permeable paper in the element dx is:

$$dQ_p(x) = k\alpha C_r\left[\Delta P_r(x) + \Delta P_c\right]dx \qquad \text{[A'6]}$$

where ΔP_c is the pressure drop across the coal at ambient flow. ΔP_c is related to the coal pressure drop at standard flow, Q_o, by the relationship:

$$\Delta P_c = \Delta P_c^o \frac{\left[Q_r(L_r) - Q_p(L_r)\right]}{Q_o}. \qquad \text{[A1]}$$

It is further assumed that:

$$\frac{d\Delta P_c}{dx} = 0. \qquad \text{[A2]}$$

Equations [A'7] through [A'16] remain unchanged from those in the cited reference. However, the solution to the complete, second-order differential equation [A16] is now solved subject to the boundary conditions:

$$\bar{Q}_p(L_r) = 0 \qquad \text{[A'17]}$$

and:

$$\left.\frac{d\bar{Q}_p(x)}{dx}\right|_{x=0} = k\alpha C_r\frac{\Delta P_c^o}{Q_o}\left[Q_r(L_r) - Q_p(L_r)\right]. \qquad \text{[A'18]}$$

The solution, under these constraints, is:

$$Q_p = \frac{\dfrac{\left(aQ_r + bQ_r^2\right)}{\left(a + 2bQ_r\right)}\{1 - \mathrm{sech}(z)\} - \dfrac{\sqrt{k\alpha C_r L_r}\,\Delta P_c^o Q_r}{Q_o\left[\Delta P_{re}^o\left(a + 2bQ_r\right)\right]^{1/2}}\{1 - \exp(z)\,\mathrm{sech}(z)\}}{1 - \dfrac{\sqrt{k\alpha C_r L_r}\,\Delta P_c^o}{Q_o\left[\Delta P_{re}^o\left(a + 2bQ_r\right)\right]^{1/2}}\{1 - \exp(z)\,\mathrm{sech}(z)\}} \qquad \text{[A3]}$$

where:

$$z = \left[k\alpha C_r L_r \Delta P_{re}^o\left(a + 2bQ_r\right)\right]^{\frac{1}{2}}. \qquad \text{[A4]}$$

Equation [A3] can be written a little more compactly by substituting transcendental identities:

$$Q_p = \frac{\dfrac{\left(aQ_r + bQ_r^2\right)}{\left(a + 2bQ_r\right)}\{1 - \mathrm{sech}(z)\} + \dfrac{\sqrt{k\alpha C_r L_r}\,\Delta P_c^o Q_r}{Q_o\left[\Delta P_{re}^o\left(a + 2bQ_r\right)\right]^{1/2}}\tanh(z)}{1 - \dfrac{\sqrt{k\alpha C_r L_r}\,\Delta P_c^o}{Q_o\left[\Delta P_{re}^o\left(a + 2bQ_r\right)\right]^{1/2}}\tanh(z)} \qquad \text{[A5]}$$

which, in the limit of the coal pressure drop approaching zero, reduces to the solution derived in Reference 4 for a cigarette without a coal:

$$Q_p = \frac{\left(aQ_r + bQ_r^2\right)}{\left(a + 2bQ_r\right)}\{1 - \mathrm{sech}(z)\}. \qquad \text{[A'23]}$$

Solving for the pressure drop, equation [A'24] from the reference becomes:

$$\Delta P_r(x) + \Delta P_c = \frac{1}{k\alpha C_r}\frac{dQ_p(x)}{dx}. \qquad \text{[A'24]}$$

This equation leads to:

$$\Delta P_r + \Delta P_c = \frac{\left(Q_r - Q_p\right)\Delta P_c^o}{Q_o}\exp(z) - \left\{\frac{\left(Q_r - Q_p\right)\Delta P_c^o}{Q_o}\exp(z) - \frac{\Delta P_{re}}{z}\right\}\tanh(z). \qquad \text{[A6]}$$

Equation [A6] can be partitioned into the rod and coal (equation [A1]) pressure-drop contributions:

$$\Delta P_r = \frac{\left(Q_r - Q_p\right)\Delta P_c^o}{Q_o}\exp(z) - 1 - \left\{\frac{\left(Q_r - Q_p\right)\Delta P_c^o}{Q_o}\exp(z) - \frac{\Delta P_{re}}{z}\right\}\tanh(z). \qquad \text{[A7]}$$

This equation can be simplified using transcendental identities:

$$\Delta P_r = \Delta P_c\{\mathrm{sech}(z) - 1\} + \Delta P_{re}\frac{\tanh(z)}{z}. \qquad \text{[A8]}$$

In the limit of the coal pressure drop equal to zero, this equation reduces to the solution derived in Reference 4 for the case of a cigarette with no coal:

$$\Delta P_r = \Delta P_{re}\frac{\tanh(z)}{z}. \qquad \text{[A'27]}$$

Although today's computers obviate the need to simplify these equations, it is instructive to examine the functional form of their simplifications. Equations [A3] and [A4] were simplified by taking the first term of a series expansion of the transcendental functions, i.e.,:

$$1 - \operatorname{sech}(x) \cong \frac{x^2}{2}$$

$$1 - \exp(x)\operatorname{sech}(x) \cong -x$$

$$\frac{\tanh(x)}{x} \cong 1 - \frac{x^2}{3}.$$

Substituting these approximations into equation [A3] yields:

$$Q_p = \frac{\left(\dfrac{\Delta P_{re}}{2}\right) + \left(\dfrac{\Delta P_c^o Q_r}{Q_o}\right)}{\left(\dfrac{1}{k\alpha C_r L_r}\right) + \left(\dfrac{\Delta P_c^o}{Q_o}\right)} \qquad \text{[A9]}$$

where, in the limit that the coal pressure drop approaches zero:

$$Q_p = \frac{\Delta P_{re} k\alpha C_r L_r}{2}. \qquad \text{[A10]}$$

Expansion of the rod pressure-drop equation yields:

$$\Delta P_r = \Delta P_{re}\left(1 - \frac{z^2}{3}\right) - \Delta P_c \frac{z^2}{2}. \qquad \text{[A 11]}$$

APPENDIX II
METHODOLOGY FOR CALCULATING PUFF POSITIONS

The purpose of this Appendix is to describe a rationale and numerical scheme for determining the puff positions when performing puff-by-puff calculations. Initially, it is assumed that equal lengths of tobacco rod are consumed during each puff, the puff locations are estimated, and the coal volume at each puff position is calculated. Then the rod consumption during each puff is made proportional to that puff's coal volume, the puff positions are recomputed, as are the coal volume at the new locations. This procedure is repeated until the puff positions converge. An example calculation is included for clarity. The table below lists the initial cigarette-design variables used in the sample calculations.

INPUT DESIGN PARAMETERS

Symbol	Variable description
C_r	Tobacco column circumference in cm, e.g., 248 cm
L_{cig}	Length of the cigarette in cm, e.g., 8.5 cm
L_{tp}	Length of tipping paper in cm, e.g., 3.1 cm
L_F	Length of filter in cm, e.g., 2.7 cm
L_{butt}	Length to which the cigarette is smoked in cm, e.g., 3.4. cm
W_{tob}	Weight of tobacco in g, e.g., 0.725 g
N_{puf}	Number of puffs, e.g., 8.3
V_{puf}	Puff volume in cm^3, e.g., 35.0 cm^3
t_{puf}	Puff duration in s, e.g., 2.0 s
t_{int}	Interval between puffs in s, e.g., 58 s
MBR	Mass burn rate, g/s, e.g., 0.0010 g/s

1. Determination the weight of tobacco consumed during smoking

$$W_{con} = \frac{(L_{cig} - L_{butt})}{(L_{cig} - L_F)} W_{tob} = [(8.5\text{ cm} - 3.4\text{ cm})/(8.5\text{ cm} - 2.7\text{ cm})]\,0.725\text{ g} = 0.6375\text{ g} \qquad \text{[B1]}$$

2. Calculation of the number of intervals in the smoking process

$N_{int} = \downarrow N_{puf} = \downarrow(8.3) = 8$ — Non-integer puff counts (the *down arrow* represents the *floor* operator) [B2a]

$N_{int} = N_{puf} - 0.5$ — Integer puff counts [B2b]

3. Calculation of the weight of tobacco consumed during smolder

$$W_{smol} = N_{int}\,\text{MBR}\,t_{int} = (8)(0.001\text{ g/s})(58\text{ s}) = 0.464\text{ g} \qquad \text{[B3]}$$

4. Calculation of the weight of tobacco consumed during a puff

$$W_{puf} = (W_{con} - W_{smol})/N_{puf} = (0.6375\text{ g} - 0.464\text{ g})/8.3 = 0.0209\text{ g} \qquad \text{[B4]}$$

5. Calculation of the weight of tobacco consumed during the inter-puff interval

$$W_{int} = W_{smol}/N_{int} = 0.464\text{ g}/8 = 0.0580\text{ g} \qquad \text{[B5]}$$

6. Calculation of the linear density of the tobacco column

$$\rho_\ell = \frac{W_{tob}}{(L_{cig} - L_F)} = 0.725\text{ g}/(8.5\text{ cm} - 2.7\text{ cm}) = 0.125\text{ g/cm} \qquad \text{[B6]}$$

7. Calculation of the length of tobacco column consumed during a puff

$$L_{puf} = W_{puf}/\rho_\ell = 0.0209\text{ g}/0.125\text{ g/cm} = 0.167\text{ cm} \qquad \text{[B7]}$$

8. Calculation of the length of tobacco column consumed during the interpuff interval

$$L_{int} = W_{int}/\rho_\ell = 0.058\text{ g}/0.125\text{ g/cm} = 0.464\text{ cm} \qquad \text{[B8]}$$

9. Calculation of the puff positions

We consider the puff position as that at the *start* of each puff.

Puff no.	Butt length	Example	V_{coal}	L_{puf}
1	$L_1 = L_{cig}$	8.50 cm	12.2 cm^3	0.132 cm
2	$L_2 = L_1 - (L_{puf} + L_{int})$	7.87 cm	13.0 cm^3	0.141 cm
3	$L_3 = L_2 - (L_{puf} + L_{int})$	7.24 cm	13.9 cm^3	0.150 cm
4	$L_4 = L_3 - (L_{puf} + L_{int})$	6.61 cm	14.7 cm^3	0.159 cm
5	$L_5 = L_4 - (L_{puf} + L_{int})$	5.98 cm	15.6 cm^3	0.169 cm
6	$L_6 = L_5 - (L_{puf} + L_{int})$	5.34 cm	16.6 cm^3	0.180 cm
7	$L_7 = L_6 - (L_{puf} + L_{int})$	4.71 cm	17.6 cm^3	0.190 cm
8	$L_8 = L_7 - (L_{puf} + L_{int})$	4.08 cm	18.6 cm^3	0.201 cm
8.3	$L_{8.3} = L_8 - (L_{puf} + L_{int})$	3.45 cm	5.91 cm^3	0.064 cm
Total			128.11 cm^3	

10. Calculation of the volume of air that enters the cigarette through the coal, V_{coal}

This is accomplished using the mathematical scheme presented in Part III "Rod Length Effects", which shows how to calculate the flow rate through the coal of a lit cigarette as a function of the cigarette length. In this example, the same cigarette design parameters listed in the previous section are used, and the coal volume is calculated from the puff volume, V_{puf}, and total cigarette dilution, D_t, for each cigarette length given in the table above.

$$V_{coal} = V_{puf}\left(1 - D_t\right). \qquad \text{[B10]}$$

The fourth column presents those results.

11. Calculation of the length of tobacco column consumed per volume of air entering the coal

The last row of the table gives the total volume of air drawn through the cigarette during its 8.3 puffs, V_{Tcoal}. Our assumption is that the length of the tobacco column consumed during a puff is directly proportional to the amount of air entering the coal during that puff (1,10). The total length of tobacco column burned during puffing, $L_{PUFF,}$ is equal to:

$$\begin{aligned} L_{PUFF} &= N_{puf} L_{puf} \\ &= 8.3(0.167\ \text{cm}) \\ &= 1.39\ \text{cm}. \end{aligned} \qquad \text{[B11]}$$

The length of tobacco column consumed per volume of air entering the cigarette through the coal is:

$$\begin{aligned} \lambda &= \frac{L_{PUFF}}{V_{Tcoal}} \\ &= 1.39\ \text{cm}/128\ \text{cm}^3 \\ &= 0.0108\ \text{cm}^{-2}. \end{aligned} \qquad \text{[B12]}$$

Therefore, the length of tobacco column burned during any particular puff is equal to the coal volume for that puff × λ:

$$L_{puf} = \lambda V_{coal}. \qquad \text{[B13]}$$

The new values of L_{puf} are listed below under the heading Iteration 2. At this point, one goes back to step 9 and iterates to self consistency. As shown below, the puff positions converged after two iterations.

	Iteration 1			Iteration 2			Iteration 3	
Puff no.	L_{puf} (cm)	L_{cig} (cm)	V_{coal} (cm^3)	L_{puf} (cm)	L_{cig} (cm)	V_{coal} (cm^3)	L_{puf} (cm)	L_{cig} (cm)
1	0.167	8.50	12.20	0.132	8.50	12.20	0.133	8.50
2	0.167	7.87	13.00	0.141	7.90	13.00	0.142	7.90
3	0.167	7.24	13.90	0.151	7.30	13.80	0.150	7.30
4	0.167	6.61	14.70	0.159	6.68	14.60	0.159	6.68
5	0.167	5.98	15.60	0.169	6.06	15.50	0.169	6.06
6	0.167	5.34	16.60	0.180	5.43	16.50	0.180	5.43
7	0.167	4.71	17.60	0.191	4.78	17.50	0.190	4.78
8	0.167	4.08	18.60	0.202	4.13	18.50	0.201	4.13
8.3	0.167	3.45	5.91	0.064	3.46	5.91	0.064	3.46