

# RATING OF DYNAMIC COEFFICIENT FOR SIMPLE BEAM BRIDGE DESIGN ON HIGH-SPEED RAILWAYS

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## Abstract

The aim of the work is to improve the methodology for the dynamic computation of simple beam spans during the impact of high-speed trains.

Mathematical simulation utilizing numerical and analytical methods of structural mechanics is used in the research.

The article analyses parameters of the effect of high-speed trains on simple beam spanning bridge structures and suggests a technique of determining of the dynamic index to the live load. Reliability of the proposed methodology is confirmed by results of numerical simulation of high-speed train passage over spans with different speeds. The proposed algorithm of dynamic computation is based on a connection between maximum acceleration of the span in the resonance mode of vibrations and the main factors of stress-strain state. The methodology allows determining maximum and also minimum values of the main efforts in the construction that makes possible to perform endurance tests. It is noted that dynamic additions for the components of the stress-strain state (bending moments, transverse force and vertical deflections) are different. This condition determines the necessity for differentiated approach to evaluation of dynamic coefficients performing design verification of I and II groups of limiting state. The practical importance: the methodology of determining the dynamic coefficients allows making dynamic calculation and determining the main efforts in split beam spans without numerical simulation and direct dynamic analysis that significantly reduces the labour costs for design.

## Keywords:

Bridge;  
Simple beam;  
HSR;  
High-speed;  
Dynamic interaction.

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## 1 Introduction

An approach to assignment of dynamic coefficients determining the totality of the phenomena of interaction of the "bridge-train" system, presented in most modern standards for design of bridge structures, is controversial from the point of view of the methodology and physical nature and does not have a due theoretical basis. The long-term practice of registration of dynamic effects in bridge design, built on an empirical approach, has a very limited field of application and cannot be used in the bridge structures design for train speeds more than 200 km/h [1]. Attempts to introduce new recommendations for determination of the dynamic coefficient while registering of high-speed load for the bridges calculation were made in [2].

Dynamic processes accompanying the movement of the train across a bridge materially determine the construction, building material, cross-sectional dimensions, rigidity of individual elements and a structure in total [3, 4, 5, 21]. Since absence of Russian experimental and experienced data, designing of bridge structures on high-speed railways is currently applied on the basis of calculations and numerical simulation [6, 7]. It should be noted that the analysis of dynamic problems in software systems implementing the finite element method (FEM), requires considerable time spending and depends on a choice of a solution method.

## 2 Problem statement

While train is moving across the bridge variable force effect is transmitted from vehicle through wheel pairs to deck. At high train speeds the force action frequency can coincide with the frequencies of natural oscillations, that can lead to resonance oscillations of structures [7, 8, 9]. The described dynamic effects lead not only to an efforts increase in the elements of bridge structures, but also have an adverse effect on the stability of the span and the comfort of passengers during the train movements across bridges on high-speed railways [10, 11]. The need for continuous monitoring of the dynamic response of span structures during the movement of high-speed trains with velocities up to 350 km / h was placed at the basis of the monitoring system for engineering structures at the Moscow-Kazan HSR in Russia [12].

Thus, the primary tasks in the design of bridge structures on high-speed railways are:

- pursuance of criteria of deck stability (limiting maximum vertical acceleration of the span structure),
- ensuring the comfort of passengers while driving a train across a bridge [13, 14],
- pursuance of criteria of an effort amount at the contact "wheel – rail" [10],
- determination of the dynamic coefficients to the temporary load while calculating the elements of the bridge structure [19, 20].

Since the basic type of span structures in the construction of HSR is simple beam spans, for its calculation a load model in a form of a group of constant forces appropriating to the scheme of a high-speed train (the "mobile forces on a structure" model) is used. The results quantitatively and qualitatively describe the dynamic work of the structural elements of the span and require minimal labour [7]. Verifications on the safety criterion (rationing of the minimum amount of force on the "wheel – rail" contact) and the comfort of passengers (rate setting of the maximum vertical accelerations in the train) in this case are justified by limiting the vertical rigidity of the span structures [15, 16].

## 3 Analysis of the dynamic effect of high-speed trains in calculation of bridge structures

High-speed trains with the length  $X_n$  (the distance between the first and the last axis) can be represented as a sequence of  $n$  loads  $P_i$ , whose abscissa is denoted by  $x_i$  (the distance between the load  $P_i$  and the first axis). For the research, the excitation function due to the moving axial loads can be expanded in a Fourier series like [18]:

$$P(t) = \frac{2}{X_n} \int S_0(\lambda) \left( \cos \frac{2\pi Vt}{\lambda} + \sin \frac{2\pi Vt}{\lambda} \right) d\lambda, \quad (1)$$

where  $\lambda$  is the excitation wavelength,

$$S_0(\lambda) = \sqrt{\left( \sum_{i=1}^n P_i \cos \frac{2\pi x_i}{\lambda} \right)^2 + \left( \sum_{i=1}^n P_i \sin \frac{2\pi x_i}{\lambda} \right)^2}, \quad (2)$$

where  $S_0(\lambda)$  is the excitation of a train and can be regarded as a dynamic characteristic of a train.

With this representation of the excitation function, it is possible to simplify the procedure of dynamic calculations for simple beams. Using the calculated parameters, it is possible to obtain the maximum vertical acceleration in the middle of the span when the train moves with velocity  $V$ . This technique can be used only for the resonant mode with respect to the first bending mode of oscillations.

The maximum value of the acceleration  $a_{\max}$  is determined by the product of the following terms [18]:

$$a_{\max} = C \cdot A(L/\lambda) \cdot G(\lambda), \quad (3)$$

where  $C$  is a parameter that depends on the mass of the span structure:

$$C = \frac{4}{m\pi}, \tag{4}$$

$A(L/\lambda)$  depends only on the length of the span  $L$  and it is a dynamic influence line defined like:

$$A(L/\lambda) = \left| \frac{\cos \frac{\pi L}{\lambda}}{\left(\frac{2L}{\lambda}\right)^2 - 1} \right|. \tag{5}$$

$G(\lambda)$  depends on the so-called dynamic characteristic of the train and the damping coefficient ( $\xi$ ) of the span structure. This parameter determines the range of the effect of the train and units are expressed in kN/m.

Knowing the spectral response, it is possible to determine the acceleration in the middle of the span for simple split beams for the critical speeds of the train.

$$G(\lambda) = \text{Max} \left[ \frac{1}{\xi X_i} S_{0,i}(\lambda) \left( 1 - \exp\left(-2\pi\xi \frac{X_i}{\lambda}\right) \right) \right], \tag{6}$$

where  $X_i$  is the length of the train part, including the  $i$ -th number of axes.

It should be noted that using models of high-speed trains A1 - A10 is justified in cases when at the time of designing of engineering structures the type of high-speed train is not defined. Thus, the envelope values constructed on the maximal values of the spectral characteristics of A1 - A10 trains for the whole range of excitation length ( $\lambda$ ) exceed the corresponding values for real domestic and foreign high-speed trains (Fig. 1). It means that in order to perform dynamic calculations it is sufficient to use only A1 - A10 trains, and not all 22 trains regulated by [17].

Spectrum investigation of dynamic effects of European and domestic trains allows concluding that the frequencies of the impact have a wide range of values. Thus, the design of engineering structures for all possible trains is irrationally and does not allow optimizing structures in order to minimize the dynamic impact and consequently the materials consumption. In this case the requirement to determine the list and characteristics of a high-speed train intended for operation in the design task is the main concern. The principle of universality of the high-speed line in this case goes against economic considerations, which certainly should be taken into account at the earliest stages of the project, in justifying the investment and determining the main goal of the HSR construction.

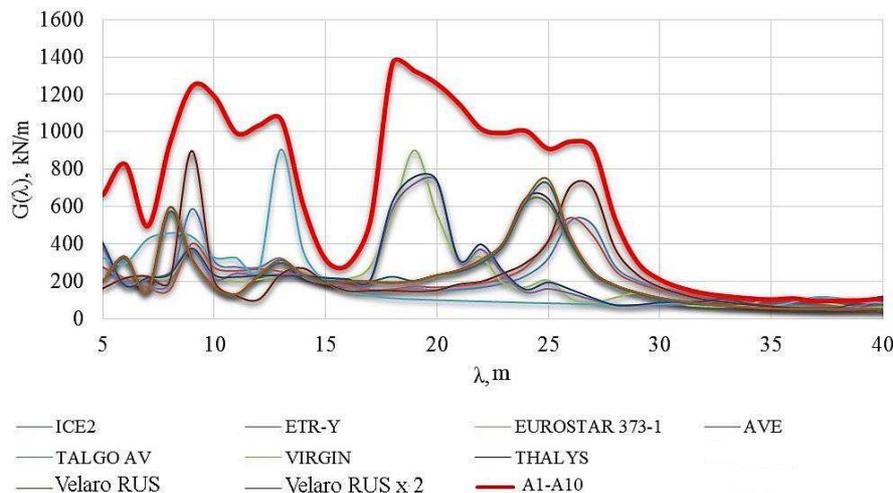


Fig. 1: Comparison of the spectral characteristics of real high-speed trains and the envelope for A1 – A10 trains.

#### 4 Determination of the main efforts and deformations and also the dynamic coefficients to the load under the influence of high-speed trains

As is known, the increase of the dynamic effect of the live load is due to the appearance of significant inertia forces, which actually determine the value of the dynamic coefficient, i.e. the increase of the component of the stress-strain state (bending moments, transverse force, and displacement). Thus, it is necessary to investigate the distribution of inertia forces along the span structure length, which are obtained by the product of the running mass by the acceleration value of the corresponding section of the beam.

Accelerations in the middle of the span at resonance oscillations reach their maximum value at the moment of the train's getting off the span. The diagram of accelerations can be approximated with sufficient accuracy as a half-wave of a sinusoid [19]:

$$a(x) = a_{\max} \sin\left(\frac{\pi x}{L}\right), \quad (7)$$

where  $a(x)$  are the vertical accelerations for resonance oscillations in the cross section of a beam with a coordinate  $x$ ;  $a_{\max}$  is the maximum vertical acceleration at resonant oscillations in the middle of the beam.

It is also worth noting that after the train departs from the span structure, the acceleration diagram is replaced by a reverse sign with almost equal values. This circumstance is important in determining the minimum effort in the design for endurance testing.

It follows from the above reasoning that in case of a single-valued line of influence of the stress-strain state factor of the structure (bending moment in the middle of the beam, transverse force in the reference section, etc.), the maximum dynamic additive will be obtained with the full load of the span structure by the linear force of inertia:

$$k(x) = m \cdot a_{\max} \sin\left(\frac{\pi x}{L}\right), \quad (8)$$

where  $k(x)$  is the inertia force for resonance oscillations in the section of the beam with the coordinate  $x$ ;  $m$  - linear mass of the span structure, taking into account the weight of the bridge,  $t/m$ .

Loading a triangular single-line influence by an inertial load (Fig. 2), we find a dynamic addition to the force:

$$\Delta N = m \cdot a_{\max} \left( \int_0^s \frac{c}{s} x_1 \sin \frac{\pi}{L} x_1 dx_1 + \int_0^{L-s} \frac{c}{L-s} x_2 \sin \frac{\pi}{L} x_2 dx_2 \right). \quad (9)$$

We apply the method of integration by parts and introduce a substitution:

$$\sin \frac{\pi}{l} (l-s) = \sin \frac{\pi}{l} s, \cos \frac{\pi}{l} (l-s) = -\cos \frac{\pi}{l} s.$$

Having completed the integration and trigonometric transformations, we finally obtain:

$$\Delta N = \frac{m \cdot a_{\max} \cdot c \cdot L^3}{s \cdot \pi^2 \cdot (L-s)} \sin\left(\frac{\pi}{L} s\right). \quad (10)$$

Thus, for the maximum bending moment in the middle, we will substitute the span into expression:

$$c = \frac{L}{4}, s = \frac{L}{2}, \quad (10.1)$$

and obtain:

$$\Delta M_{0.5} = \frac{m \cdot a_{\max} \cdot L^2}{\pi^2} \tag{11}$$

Similarly, by loading the triangular influence line, an expression for the transverse force in the reference section was obtained:

$$\Delta Q_0 = \frac{m \cdot a_{\max} \cdot L}{\pi} \tag{12}$$

To determine the dynamic deflection of the span structure, the curvilinear influence line should be loaded (Fig. 2).

By loading the curvilinear unambiguous influence line (Fig.2), by inertial loading, we find the dynamic addition to the deflection:

$$\Delta \delta = 2 m \cdot a_{\max} \int_0^{L/2} \left( \frac{x^3}{12EJ} - \frac{L^2 x}{16EJ} \right) \sin \frac{\pi}{L} x \, dx \tag{13}$$

Completed the integration and trigonometric transformations, we finally obtain:

$$\Delta \delta = \frac{m \cdot a_{\max} L^4}{\pi^4 EJ} \tag{14}$$

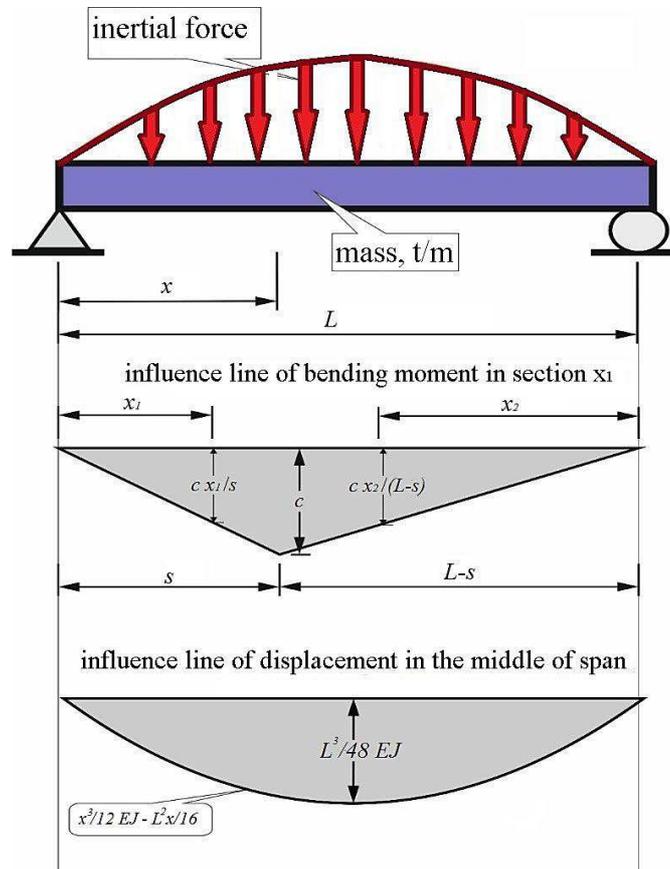


Fig. 2: The scheme for loading the span structure by inertial load. Influence lines of the main factors of the stress-strain state of a span structure.

Expressions are proposed for the dynamic coefficients ( $1 + \mu_1$ ) to the time load from high-speed trains for various factors of the stress-strain state [19].

When calculating deformations:

$$1 + \mu_1 = 1 + 0,788 \frac{m \cdot a_{\max}}{V_{0.5}} \quad (15)$$

When calculating the bending moment:

$$1 + \mu_1 = 1 + 0,811 \frac{m \cdot a_{\max}}{V_{0.5}} \quad (16)$$

When calculating the transverse force:

$$1 + \mu_1 = 1 + 0,637 \frac{m \cdot a_{\max}}{V_0} \quad (17)$$

where  $a_{\max}$  is the maximum vertical acceleration at the resonance oscillations of the span structure in the middle of the span;  $m$  - linear mass of the span structure taking into account the weight of the bridge,  $t/m$ ;  $v_{0.5}$  and  $v_0$  is the equivalent uniformly distributed load from high-speed trains (kN/m), with  $\alpha = 0.5$  and  $0.0$ , respectively (see Attachment I [17]).

## 5 Conclusion

The suggested provisions of the article establish a connection between the maximum acceleration for resonance oscillations of the span structure and dynamic additions to the components of the stress-strain state. It is established that the dynamic coefficients for the time load in determining the bending moments, transverse force and vertical deflections are different. This circumstance determines the necessity for graded approach to definition of dynamic coefficients while performing calculating verifications of I and II groups of limiting state. Moreover, the obtained expressions allow us to determine not only the maximum values of the unknown factors, but also their minimum values that make it possible to perform endurance tests. Taking into account the abovementioned provisions, it is possible to perform a dynamic calculation and determine the main efforts in beam span under the influence of high-speed trains without using of a numerical simulation apparatus and direct dynamic analysis.

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