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FUNCTIONAL DEPENDENCE BETWEEN EXPECTANCY OF PEOPLE'S LIFE AND MANKIND POPULATION DURING THE TIME OF DEMOGRAPHIC TRANSITION

FUNKCJONALNA ZALEŻNOŚĆ TRWAŁOŚCI ŻYCIA LUDZI OD LICZEBNOŚCI LUDZKOŚCI ŚWIATA W OKRESIE PRZEJŚCIA DEMOGRAFICZNEGO

Abstract: In this work is an attempt to mathematically prove the existence of the demographic transition taking into account one of its features, such as extension of human life dependent on the growth of the human population. Determined the functional form of this dependence, and the relationship between the probability of death, life expectancy, and social involving in the states of T (the influence of "traditional" values of concepts) and R (in the range of rules and possibilities of modern civilization).

Keywords: demographic transition, average life expectancy, world human population

One of the consequences of change in live style of human population and social groups from the "traditional" to "rational" (modern) type of existence was called as a demographic transition [1]. Demographic transition is characterized by the almost simultaneous, but not a parallel decrease in mortality and fertility, and intensive population growth. In his monograph [1] prof. Okolski, demographic transition theory sees as the most progressive achievements and specific paradigm of modern demography. At the same time he points out that some demographers wouldn't agree with global (universal character) of demographic transition.

If the demographic transition has a global character than the mathematical way of describing it's processes (in the present work is considered one of them - the life expectancy of people) should demonstrate a clear and explicit (functional) dependence on the size of the entire humanity.

Statistical data and the volume of their derivatives

Primary statistics used are given in Table 1. Baseline data were taken from [2].

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Table 1

Life expectancy of people (*W* [years]) and the human population (*N* [billion]) from 1960-2011

Year	**Age (*W*)**	***N***
1960	52.65	3.02
1965	55.79	3.33
1970	59.33	3.69
1975	61.36	4.07
1980	62.95	4.43
1981	63.24	
1982	63.53	
1983	63.75	
1984	64.00	
1985	64.24	4.83
1986	64.54	4.92
1987	64.79	5.00
1988	65.00	5.11
1989	65.22	5.20
1990	65.40	5.26
1991	65.58	5.39
1992	65.72	5.48
1993	65.82	5.57
1994	66.00	5.63
1995	66.18	5.67
1996	66.41	5.78
1997	66.65	5.86
1998	66.84	5.95
1999	67.01	6.03
2000	67.23	6.07
2001	67.46	
2002	67.66	
2003	67.86	
2004	68.13	
2005	68.34	6.45
2006	68.63	
2007	68.89	
2008	69.14	
2009	69.41	
2011		7.0

As can be seen from Table 1, the world population increases with time. At the same time the average length of human's life is increasing.

<u>The aim of present work was to</u>:

a) justify of fact that the main factor which mark the life expectancy of people is size of their population;

b) determine the functional form of the relationship between the size of the world's population and life expectancy of people.

<u>Justification of approach.</u> Actual population growth in different populations (societies) is shaped by three major factors: fertility, mortality and migration [1]. Real world population growth coincides with the birth rate and is a sum of the first two factors.

The rate of population change $\left(\frac{dN}{dt}\right)$ is the difference between birth $\left(\frac{dN_R}{dt}\right)$ and death $\left(\frac{dN_S}{dt}\right)$ rates of organisms from which it is composed of:

$$\frac{dN}{dt} = \frac{dN_R}{dt} - \frac{dN_S}{dt} = N \cdot f(N) \quad (1)$$

where: N - human population, $f(N)$ - functional rate of population change (*per capita*).

The Verhulst equation is a linear function, Gompertz's - logarithmic (for more details look at [3, 4]).

In modern ecology and biomatemathics [5, 6] an approach is proposed that a $f(N)$ is a binary function. First element of this function - which represents the fertility of the organisms is a positive and constant value (constancy may not be obligatory). Second element - which represents the mortality - is a negative and variable value (proportional to N (Verhulst's equation [3]) or ln$N$ (Gompertz's equation [4])). The result of this approach is an inaccurate describing of the population dynamics at low N - "the effect of Adam and Eve" - giving the possibility for the population to grow from initially small group of individuals; "Matuzaleus effect" - unreasonably inflated assessment of life expectancy and creating "population protoplast". Let us write equation (1) in more general form:

$$\frac{dN}{dt}\frac{1}{N} = \frac{dN_R}{dt}\frac{1}{N} - \frac{dN_S}{dt}\frac{1}{N} = f_N(N) = f_R(N) - f_S(N) \quad (2)$$

where: $f_R(N)$ and $f_S(N)$ are variable values of births and deaths rates (*per capita*) in population section of its changes from initial (N_0) to maximal number (N_m).

With: $N = N_0$, $f_n(N) > 0$ than: $f_R(N) > f_S(N) > 0$.

With: $N = N_m$, $f_n(N) = 0$ than: $f_R(N) = f_S(N)$.

For the human population is characteristic with increasing N (but nonparallel) simultaneous fall in values of the $f_R(N)$ and $f_S(N)$. Therefore we can assume that:

$$f_R(N_0) > f_R(N_m) \text{ and } f_S(N_0) > f_S(N_m)$$

In a population of size N, the average life expectancy W_N is equal to:

$$W_N = \left(\frac{dN_S}{dt}\frac{1}{N}\right)^{-1} = \left(f_N(N)\right)^{-1} \quad (3)$$

Therefore, further analysis of static data is reduced to determine the forms and parameters of the functional relation between the N and W_N, taking into account that changes in last values are limited by values: $f_S(N_0)^{-1} = W_0^{-1}$ (with $N \to 0$) and $f_S(N_\infty)^{-1} = W_\infty^{-1}$ (with $N \to \infty$).

Analysis of static data

Due to the fact that people life expectancy in the first decade of the twenty-first century approached W_∞, its value (the asymptote) can be calculated quite simply using common digital methods. Iteratively determined (from the data shown in Table 1) meaning of W_{∞} = 70.0 years.

Value $\left(W_N^{-1} - W_{\infty}^{-1}\right)^{-1}$ shows linear dependence (growing) relative to N^3 (see Fig. 1a). The regression line intersects the x-axis at the point: $\left(W_0^{-1} - W_{\infty}^{-1}\right)^{-1}$ = 29.1 years, value of it's angle slope tangent is $\left[\left(W_N^{-1} - W_{\infty}^{-1}\right)^{-1} \cdot N_{0.5}^{-3}\right] = 6.46$ billion^3 year^{-1}. Then: $W_0 = 20.5 – 19.5$ years, $N_{0.5}^3 = 4.5\left(N_{0.5} = 1.65 \text{ billion}\right)$. Then:

$$W_N^{-1} = W_{\infty}^{-1} + \frac{\left(W_0^{-1} - W_{\infty}^{-1}\right) N_{0.5}^3}{N_{0.5}^3 + N^3} \tag{4}$$

If our assumption is correct then the dependence on the coordinates $\left[\left(W_N^{-1} - W_0^{-1}\right)^{-1}, N^{-3}\right]$ shall also be linear. This can be seen in Figure 1b. The regression analysis of these data (Fig. 1b) shows that: $N_{0.5}$ = 1.55 billion.

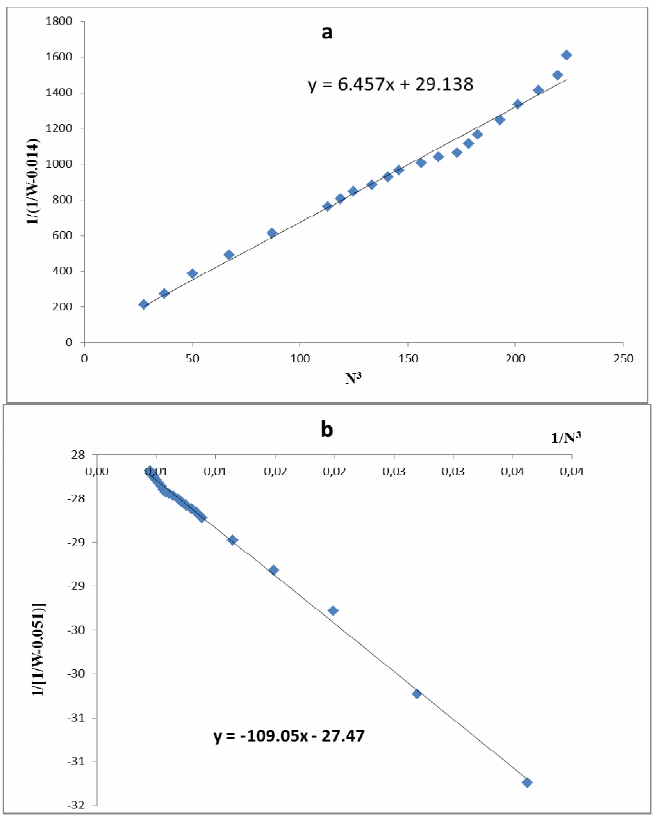


Fig. 1. Dependence of life expectancy (W) from the human population raised to the third power (N^3) (a) and its inverse volume $(N^3)^{-1}$ (b)

Analysis of statistical data presented in Figures 1a and 1b, leads to the possible dependency form between life expectancy W_N and human population size (N) which can be most simply written as an equation:

$$W_N^{-1} = W_0^{-1} \cdot \left(\frac{N_{0.5}^3}{N_{0.5}^3 + N^3} \right) + W_\infty^{-1} \left(\frac{N^3}{N_{0.5}^3 + N^3} \right) \tag{5}$$

The adequacy of the calculation results (in the correspondence to equation (4)) to the statistical data is shown on Figure 2.

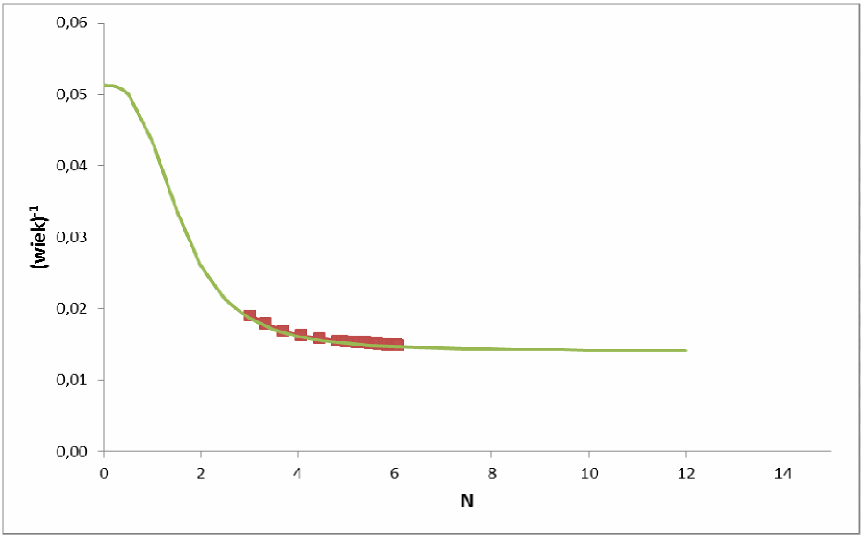
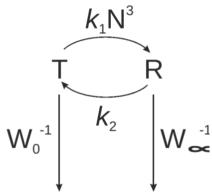


Fig. 2. The dependence of the inverse of life expectancy $(W_N)^{-1}$ of the human population size (N) (statistics - points, line corresponds to the dependence of life expectancy calculated according to equation (5))

Discussion and summary

Due to increased numbers of people and their material and spiritual creations, each of us is in an increasing influence of "rational" (modern) type of life. This type of life compared to "traditional" type indeed (in 3.6 times) increases the life expectancy of people. This effect is not rigid. Deliberately or accidentally, we can stay in the sphere of the influence of "traditional" values of terms (in the state T), or - operate in the rules and possibilities of modern civilization (in state R). The characteristic time for people in the state (T) is inversely proportional to world human population in the third power (see the diagram and equation (5) and (6)).

The characteristic residence time of people in the states of T and R is much lower than average living time in these states W_0 well $W_\infty = (k_1 N^3)^{-1}$ and $(k_2 N^n)^{-1}$.



Scheme (interpretation in text above)

Then, we assume:

$$
\begin{aligned}
&T + R = N \\
&Tk_1N^3 = Rk_2 \\
&\frac{k_2}{k_1} = N_{0.5}^3 \\
&R = \frac{N^4}{N_{0.5}^3 + N^3} \\
&R = \frac{N \cdot N_{0.5}^3}{N_{0.5}^3 + N^3} \\
&P_R = \frac{R}{N} = \frac{N^3}{N_{0.5}^3 + N^3} \\
&P_T = \frac{T}{N} = \frac{N_{0.5}^3}{N_{0.5}^3 + N^3} \\
&P_T + P_R = 1
\end{aligned}
\tag{6}
$$

where: P_T and P_R - the probability for individuals to stay in the states of *T* and *R*.

So the probability of death per unit time (in this text during the year) is:

$$W_N^{-1} = W_\infty^{-1} \cdot P_R + W_0^{-1} \cdot P_T \tag{7}$$

The functional relationship between the length of human life and the size of human population measured in the entire world's population support the hypothesis [1] of a global process called “demographic transition”.

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Abstrakt: Na podstawie zaproponowanych metod matematycznych została zanalizowana i opisana funkcjonalna zależność między wzrostem liczebności populacji ludzkiej a długością życia ludności świata. Analiza matematyczna pozwoliła na modelowanie dynamiki przejścia demograficznego populacji ludzkiej w zakresie jej liczebności od 3 do 7 mld i określenie wpływu racjonalnego (współczesnego) i tradycyjnego trybu życia na wzrostu długości życia ludzi.

Słowa kluczowe: przejście demograficzne, średnia długość życia, liczebność populacji ludzkiej