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# A Generalized Erlang-C Model for the Enhanced Living Environment as a Service (ELEaaS)

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***Abstract:*** *In this article, a full-access waiting multi-server queue with a state-dependent arrival and departure processes is investigated and suggested for use as a generic traffic model of the novel concept of the Enhanced Living Environment as a Service (ELEaaS). The generalized arrival and service flows with nonlinear state dependence intensities are used. The idea is based on the analytical continuation of the Poisson arrival process and Bernoulli service process, and the classic M/M/n queuing system. Birth and death processes and state-dependent rates are applied. The suggested new queuing system is of a M(g)/M(g)/n/k type (in Kendal notation) with a generalized arrival and departure processes M(g). The input and output intensities depend nonlinearly on the system state with defined parameters – the so-called "peaked factors". The state probabilities of the system are obtained using the general solution of the birth and death processes. The influence of the peaked factors on the queuing behavior is evaluated showing that state-dependent arrival and service rates may change significantly the characteristics of the queuing system. The simplicity and uniformity in representing both peaked and smooth behavior make this queuing model also attractive for future networks' analysis and synthesis.*

***Keywords:*** *Enhanced Living Environment (ELE), Enhanced Living Environment as a Service (ELEaaS), multi-server queue, M(g)/M(g)/n/k queuing system, generalized state-dependent arrival and departure processes.*

## 1. Introduction

The Enhanced Living Environment as a Service (ELEaaS) is an emerging topic for Research & Development (R&D) due to the spreading use of the Enhanced Living Environments (ELE) supported by cloud technologies. The continuing development of the smart sensor technologies and the possibility to interconnect different kind of ad-hoc networking devices allow software and hardware developers to collect data from various traffic sources, perform data acquisition, analysis and mining, and present generic Ambient Assisted Living (AAL) services in the cloud that forms the ELEaaS [1, 2]. The ELEaaS concept covers many different aspects, starting from the general system architecture [3] and going to specific details like cooking assistance [4] and ontology that could describe all system dependences. Furthermore, data traffic models [5], obtained from measurements in experimental networks, demonstrate completely different characteristics and behavior in comparison to the classical traffic models used in Public Switched Telephone Networks (PSTNs). This phenomenon requires identification of new flexible traffic models that can take into account all features of new data flow and dynamically adapt themselves to changes in the flows in an almost ad-hoc manner. The creation of the ELEaaS will virtualize the access and edge parts of the network allowing technological interoperability and vast development of standardized interfaces and protocols.

Simple models like the classical full-access waiting systems can be used often to obtain comprehensive results, e.g., to predict the global traffic behavior. Packet arrival and departure processes are often assumed to have a Poisson distribution because of its attractive theoretical properties.

The basic characteristic of traffic flows in modern telecommunications networks is that they can be smooth, or regular, or peaked. In these three cases, the variance of the number of arriving packets is respectively smaller, equal, or bigger than the mean value. The typical transmission of packets in telecommunications networks is in a burst (a large amount of data sent in a short time). The burstiness is caused by the nature of data being exchanged and it is the main reason for the peaked traffic flows in these networks. That is why there are many studies that describe complicated queuing systems with specific behavior as long-range dependence, peakedness, self-similarity, and heavy-tail distributions. One way to describe this behavior is to generalize the queuing system by state-dependent arrival and service rates.

In the last few years, increasing interest in developing models and methods of classical queuing systems (especially the Erlang-C formula) for studying the Internet has led to many extensions of previously published results. In 2007, Vint Cerf listed seven research problems concerning the Internet [6]. The second problem relates to the so-called Internet Erlang Formula. In this regard in [7], a robust and efficient algorithm for evaluating multi-service, multi-rate queuing systems is presented, including a finite buffer system and a loss system, based on the Erlang formulae.

In [8], the Erlang delay formula is demonstrated to link the demand, capacity and performance of the Internet. It provides an upper bound of the probability that a given peak rate flow will suffer from degradation when a bandwidth sharing max-min fairness is applied. It is shown that the Erlang-C formula may have in fact a much more general application in the Internet.

In [9-11], the idea of utilization of the Erlang formulae in Asynchronous Transfer Mode (ATM) networks and IP networks is addressed. Through the Erlang-C formula, one can estimate the probability of delay, which usually occurs in IP networks. The use of Erlang models in IP networks gives opportunities to monitor the Quality of Service (QoS) parameters. The simplicity of the Erlang formulae is a strong advantage when compared to other methods for traffic description. The requirements for precise modeling motivate the future research in this field.

A possibility for the utilization of the Erlang-B and the Erlang-C formulae in Next Generation Networks (NGN) is presented in [12].

A model, based on the queuing theory for service performance estimation of cloud computing systems, is presented in [13]. The model is based on event-driven simulations and demonstrates the scheduling capabilities of heterogeneous and non-dedicated clouds. It is based on the classical *M/M/m* queuing system. The results demonstrate the usefulness of the presented simulation models for the design of cloud computing systems with QoS guarantees.

Queuing systems with arrival and/or service rates, which depend on the state of the system, are used in various application areas. For instance, state-dependent features are present in congestion control protocols used in packet-switched networks. In [14], a Transmission Control Protocol (TCP) – like linear-increase multiplicative-decrease congestion control mechanism is presented. The authors consider a Poisson process for batch arrivals and congestion signaling. The service times in the queuing model depend on the workload of the system and the transmission rate cannot exceed a certain maximal value.

In [15], the burstiness of the total arrival process is characterized in packet-switched network performance models by the dependences among successive inter-arrival times, successive service times, and between service and inter-arrival times. It is applicable to telecommunications protocols with a correlation between client states and server states. The dependence is demonstrated analytically by considering a multiclass single-server queue with batch-Poisson arrival processes.

Important issues of the network planning process for multi-service IP networks are discussed in [16]. The presented ideas and concepts are implemented in an overall planning framework. In order to do so, the IP QoS mechanisms are categorized and a systematic approach for classification and modeling of the Internet traffic is suggested.

The feedback information on the buffer state provides the basis for the TCP protocol to regulate carefully the transmission rate of Internet flows. To evaluate this behavior, a G/G/1-type queue with a workload-dependent **arrival** and service **rates** is considered in [17].

In [18], a generalized Poisson arrival process by state-dependent arrival rates is introduced and evaluated. The proposed multi-server delay system provides a unified framework to model peaked and smooth traffic, which makes it attractive for use in network analysis.

In [19], a queuing system, where feedback information on the level of congestion is given immediately after the arrival instants, is presented. If the amount of work to be done right after an arrival is smaller or larger than a given number, the server starts to work at lower or higher speed, respectively. In addition to this, the authors have considered the generalization to the *N*-step service speed function.

In [20], a state-dependent bulk service queue with departure of the impatient customers and server vacations is investigated.

There are many studies about queues with workload-dependent service speeds. In many papers, it is assumed that the server speed is adapted to the buffer occupation continuously. In many practical situations, however, service speed adaptations are made only at particular points in time, such as the arrival epochs, because feedback information about the buffer state may only be available at such epochs.

Simple models of user-provided networks had been studied in [21]. There are many detailed analyses on mobile networks of moving objects presented from different perspectives in [22]. Mobility prediction in Long Term Evolution (LTE) networks is shown in [23]. Medical information integration for ELE is studied in [24]. Attention to scheduling is paid in [25]. Scheduling in Software Defined Networks (SDN) is demonstrated in [26, 27]. Data fusion is presented in [28]. Specific ELE models are shown in [29, 30]. A Vehicle-to-Infrastructure (V2I) model in vehicular ad hoc networks, based on the complex network theory, is proposed and analyzed in [31].

In this article, queuing systems with adaptable arrival and service rates, based on the amount of work which needs to be done right after the arrival or departure of a customer request, are considered. Between these events, the arrival and service rates are assumed to be fixed, and may not be changed before the arrival or departure of the next customer request. The classical multi-server delay queuing systems, used for analyzing call centers [32], are generalized. Generalized arrival and departure flows with nonlinear state dependence intensity are used. The idea is based on the analytical continuation of the Poisson and Binomial distributions and the classic *M/M/n* queuing system. Birth and death processes and state-dependent service rates [33, 34] are applied. The presented generalized model can be used to analyze the stochastic behavior and performance of today's cloud-based systems.

The article is organized in eight sections. The next section presents the ELEaaS model and attempts to parameterize the traffic model for ELEaaS access. A fixed number of servers is considered. The service time depends on the type of the service, and could be adapted to the traffic load and congestion level. The third section is dealing with the Erlang's delay *M/M/n* queuing system as per [35]. The fourth section presents a generalized multi-server queue implementation for ELEaaS, whereas the fifth section demonstrates a generalized discrete first Erlang distribution. The performance metrics used are defined in section 6 as per [36]. The numerical results are presented in section 7. Finally, section 8 concludes the article with comments on the traffic model's applicability, adaptation capabilities, and correctness.

## 2. ELEaaS traffic model

The ELEaaS model is depicted on Fig. 1. Various sensor data is collected continuously from different types of networks – e.g., Body Area Networks (BANs), Car Area Networks (CANs), Home Area Networks (HANs), Internet Area Networks (IANs), environment networks, social networks, etc. – and after some preliminary processing is stored in the cloud. For simplicity, the servers that support the ELEaaS in the cloud are considered identical. They are distributed among different physical hosts and sites. Virtual machines' migration, and physical machines' and virtual machines' shutting down and staring up could change the model dynamically. This new paradigm is specific for all green technologies.

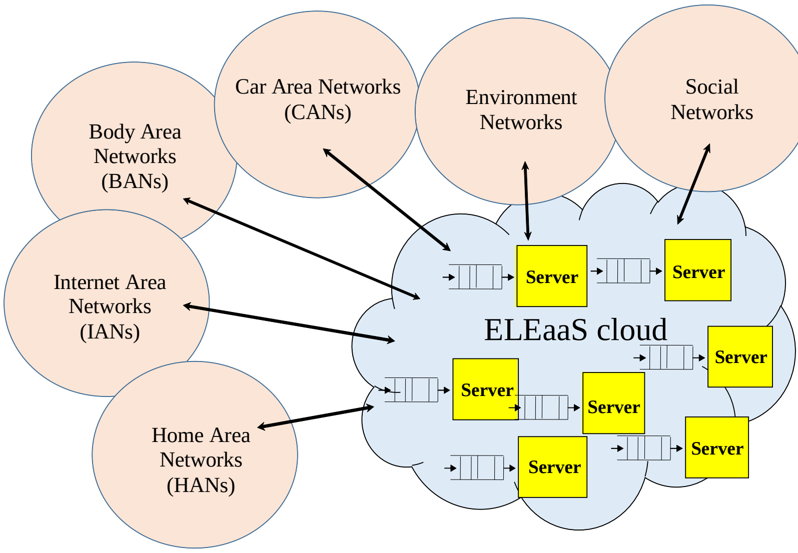


Fig. 1. The model of the enhanced living environment as a service

The corresponding traffic model is shown in Fig. 2. The multi-server queues in the ELEaaS cloud (on the right hand side of the figure) are considered identical. The presence of the common queue in front of all queues before the ELEaaS servers is optional. Access servers and application servers could start up and shut down dynamically depending on different factors like green energy availability, traffic load, and other QoS parameters.

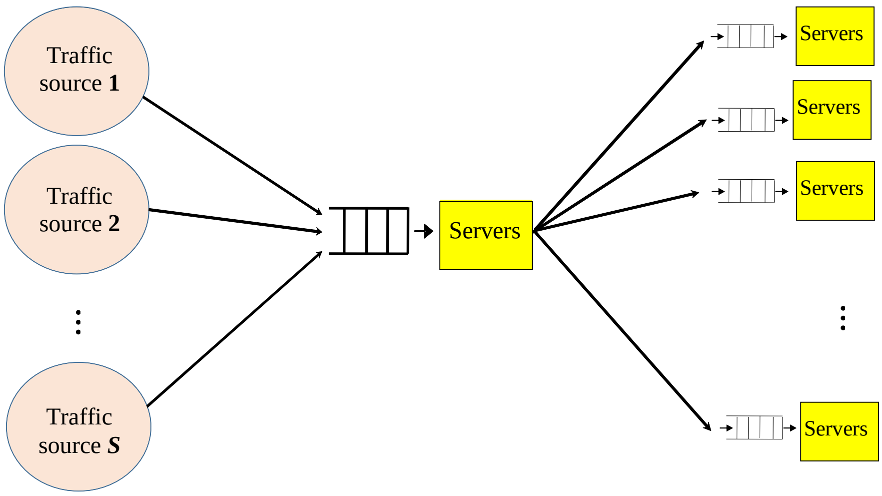


Fig. 2. The traffic model of the enhanced living environment as a service

The presented structure allows the application of a multi-server queuing analytical model for the calculation of performance parameters. A generalized multi-server queuing system $M(g)/M(g)/n/k$ is applied by any single block of a queue and multiple servers behind it (Fig. 3). The corresponding performance analysis is presented in Section 7.

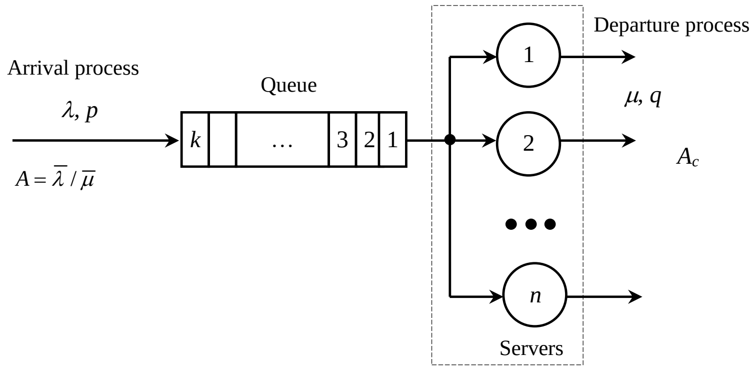


Fig. 3. A generalized multi-server queue – $M(g)/M(g)/n/k$

## 3. Erlang's delay system *M/M/n*

We consider a system with a Poisson arrival process ($M$), exponential service times ($M$), and $n$ identical servers (Fig. 3 when $p = q = 1$). The customer requests are served from the queue in the order of their arrival. When all $n$ servers are busy, a newly arriving customer request joins the queue and waits until a server becomes idle.

The probability that an arbitrary arriving customer request will wait in the queue is proportional to the time during which all servers are busy. As shown in [35], this probability could be calculated by using the following formula:

(1) $$P(t_w > 0) = E_2(n, A) = 1 - \sum_{j=0}^{n-1} P_j = \frac{\frac{A^n}{n!}\frac{n}{n-A}}{\sum_{j=0}^{n-1}\frac{A^j}{j!} + \frac{A^n}{n!}\frac{n}{n-A}}, \qquad A < n,$$

where $A$ is the offered traffic (Erl),

$n$ is the number of servers,

$t_w$ is the waiting time (a random variable),

$P(t_w > o)$ is the waiting probability.

This formula has several names: the Erlang-C formula, the Erlang's second formula, or the Erlang formula for waiting time systems. Initially, it was derived to dimension the number of operators managing a call center [8, 10]. In [32], the robustness and usefulness of a relatively simple theoretical model is demonstrated, namely the *M/M/N+M* (Erlang-A model with customer request abandonment), for performance analysis of a call center. A novel method for performance analysis of a call center with balking and abandonment is presented in [33]. In this article, we propose to use a generalization of this formula for customer requests modeling in ELEaaS systems.

## 4. Generalized multi-server queue

The description of the model for a generalized multi-server queue is based on a *M*(*g*)/*M*(*g*)/*n*/*k* queuing system with a generalized Poisson arrival process *M*(*g*), a state-dependent exponentially distributed service time *M*(*g*), *n* servers, and a limited number of waiting positions *k*. This is a birth and death process and as such, the general solution for the stationary probability of having *j* customer requests in the system [36] applies to it:

$$P_j = \frac{\prod_{i=0}^{j-1} \lambda_i / \mu_{i+1}}{1 + \sum_{v=1}^{n+k} \prod_{i=0}^{v-1} \lambda_i / \mu_{i+1}}, \qquad j = 0, 1, 2, \ldots, n+k, \tag{2}$$

where $\lambda_i$ is the arrival intensity when the system is in state *i*; $\mu_i$ is the service intensity when the system is in state *i*.

This generalized queuing system has the following nonlinear state dependence birth and death coefficients:

$$\begin{aligned} &\lambda_i = \lambda (i+1)^{1-1/p} && \text{for} \quad i = 0, 1, 2, \ldots, n+k, \\ &\mu_j = \mu\, j^{2-q} && \text{for} \quad j = 1, 2, 3, \ldots, n, \\ &\mu_j = \mu\, n^{2-q} && \text{for} \quad j = n+1, n+2, \ldots, n+k, \end{aligned} \tag{3}$$

where *p* is the arrival peakedness factor, *q* is the departure peakedness factor.

With these two peakedness factors, one can define the state-dependent arrival and service rates, and generalize the arrival and service processes. A generalized Poisson arrival process is said to be peaked, regular, or smooth, if *p* is respectively greater than, equal to, or less than 1. Similarly, a generalized Bernoulli departure process is considered to be peaked, regular, or smooth according to whether $q > 1$, $q = 1$, or $q < 1$, respectively.

The finite state-transition diagram of the investigated multi-server queue with state-dependent arrival and service rates is shown in Fig. 4.

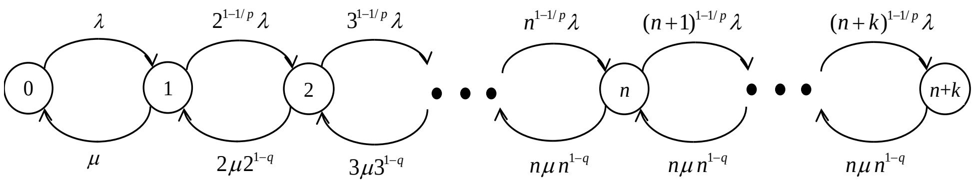


Fig. 4. The state-transition diagram of a *M*(*g*)/*M*(*g*)/*n*/*k* queuing system

By applying the birth and death coefficients to the general solution of the birth and death process and by using traffic intensity ($a = \lambda/\mu$) when the system is empty, one can obtain the steady state probabilities as follows:

(4) $$P_j = a^j \big/ (j!)^{1+1/p-q}\, P_0, \quad 0 \le j \le n,$$

$$P_j = \frac{a^j}{(j!)^{1+1/p-q}} \left(\frac{j!}{n!}\right)^{2-q} \frac{P_0}{n^{j-n}}, \quad n \le j \le n+k,$$

$$P_0 = \left[ \sum_{i=0}^{n} \frac{a^i}{(i!)^{1+1/p-q}} + \sum_{j=n+1}^{n+k} \frac{a^j}{(j!)^{1+1/p-q}} \left(\frac{j!}{n!}\right)^{2-q} \frac{1}{n^{j-n}} \right]^{-1}.$$

The average value of the state-dependent arrival rate is

(5) $$\bar{\lambda} = \sum_{j=0}^{n+k} \lambda_j P_j = \lambda \sum_{j=0}^{n+k} (j+1)^{1-1/p} P_j.$$

The average value of the state-dependent service rate is

(6) $$\bar{\mu} = \sum_{j=1}^{n+k} \mu_j \frac{P_j}{1-P_0} = \frac{\mu}{1-P_0} \left( \sum_{j=1}^{n} j^{1-q} + k\, n^{1-q} \sum_{j=n+1}^{n+k} P_j \right).$$

The offered traffic is calculated by means of the average arrival and service rates as follows:

(7) $$A = \bar{\lambda} \big/ \bar{\mu}.$$

The carried traffic is equivalent to the average number of busy servers, i.e.,

(8) $$A_c = \sum_{j=1}^{n} jP_j + n \sum_{j=n+1}^{n+k} P_j.$$

## 5. Generalized discrete first erlang distribution

In the ideal case for the multi-server system $M(g)/M(g)/S/0/S$ when the number of traffic sources $S$ is equal to the number of servers $n$ (i.e., $S=n$), there will be neither loss nor delay, because all the offered traffic will be carried over. This is the so-called *intended traffic load*.

The stationary probability of having $j$ customer requests in the system has the following generalized discrete first Erlang distribution:

(9) $$P_j{}' = \frac{a^j \big/ (j!)^{1+1/p-q}}{\sum_{i=0}^{S} a^i \big/ (i!)^{1+1/p-q}}, \quad j = 0, 1, 2, \ldots, S.$$

The intended traffic is the mean number of busy servers, i.e.,

(10) $$A_i = \sum_{j=1}^{S} j P_j'.$$

## 6. $M(g)/M(g)/n/k$ performance metrics

*Waiting probability*. The waiting probability is the probability that the waiting time $t_w$ will be greater than 0. For arrival and departure flows with a non-linear state dependence intensity and a limited number of waiting positions, it is defined by the generalized Erlang-C formula

(11) $$P(t_{\mathrm{w}} > 0) = \sum_{j=n}^{n+k-1} P_j = 1 - \sum_{j=0}^{n-1} P_j - P_{n+k}.$$

It allows setting different variances for the *arrival and departure processes* without changing the mean value. When the arrival and departure peakedness factors are equal to 1 (i.e., $p=q=1$) and the queue length is infinite (i.e., $k=\infty$) it turns into the Erlang-C formula (1).

*Blocking probability.* The blocking probability $B_t$ defines the fraction of time when all waiting positions are busy. It could be calculated by using the state probability when the system is busy:

(12) $$B_t = P_{n+k}.$$

*Mean number of customer requests.* The mean number of customer requests present in the system in steady state by definition is

(13) $$L = \sum_{j=1}^{n+k} j\, P_j.$$

*Mean system time.* This could be derived from the Little's formula as follows:

(14) $$W = \mathrm{L}/\bar{\lambda}.$$

*Waiting time distribution*. If a First-In-First-Out (FIFO) discipline is assumed, then from the probability theory the waiting time distribution function $P(t_{\mathrm{w}}>t')$ is defined as the probability of waiting time exceeding the defined time interval $t'$:

(15) $$P(t_{\mathrm{w}} > t') = \sum_{i=n}^{n+k-1} P_i\, Q_i(> t'),$$

where $P_i$ is the probability that an arbitrary customer request will enter the system when $i$ other customer requests are already in the system. Since the service time is exponentially distributed, the probability that $j$ ($j \le i$) customer requests will terminate during the time interval $(0, t']$ becomes a generalized Poisson distribution with a mean value $\mu_i t'$, i.e.,

(16) $$Q_j(t') = \frac{(\mu_i\, t')^j}{j!} e^{-\mu_i t'}.$$

The conditional probability $Q_i(>t')$ that an arbitrary customer request will wait longer than $t'$, given $i$ ($i \ge n$) customer requests in the system, is expressed by the following formula:

(17) $$Q_i(> t') = \sum_{r=0}^{i-n} \frac{\left(n\mu n^{1-q}\, t'\right)^r}{r!} e^{-n\mu n^{1-q} t'}.$$

## 7. Numerical results

In this section, numerical results obtained by means of computer calculations, based on a wide range of variables, are presented and discussed. The results presented in the figures cover a range of different values of the arrival and departure peakedness factors $p$ and $q$. To evaluate the influence of peaked stochastic processes on the multi-server queueing system, we consider the particular case when $p = q$.

Graphs, representing the case when $p=q=1$, consider a regular Poisson arrival process and a Bernoulli departure process. This case could be used for comparison to the classical *M/M/n/k* queuing system.

To obtain numerical results, we first define the intended traffic $A_i$, the number of sources $S$, and the peakedness factors $p$ and $q$, and then calculate the traffic intensity $a$ by a successive-approximations iteration method (9) and (10). After that we calculate the steady state probabilities (4) and the performance metrics of interest.

Fig. 5 shows the state probability distribution in an ideal multi-server queue *M(g)/M(g)/S/0/S* with state-dependent arrival and service rates, for $n$=100 servers, $S$=100 traffic sources, and $A_i$=15 Erl intended traffic. It could be seen from the results that when the peakedness factors increase, the variance of the distribution increases as well.

The rest of the results applies for a generalized multi-server queue *M(g)/M(g)/n/k* with state-dependent arrival and service rates.

Fig. 6 presents the state probability distribution for $n$=15 servers, $k$=35 waiting positions, and $A_i$=12 Erl intended traffic. Unlike other values of $p$ and $q$, the 1.2 value has different influence on the system: after the peak, reached at 11 customer requests, and a slight drop, the state probabilities stay around the 0.01 level.

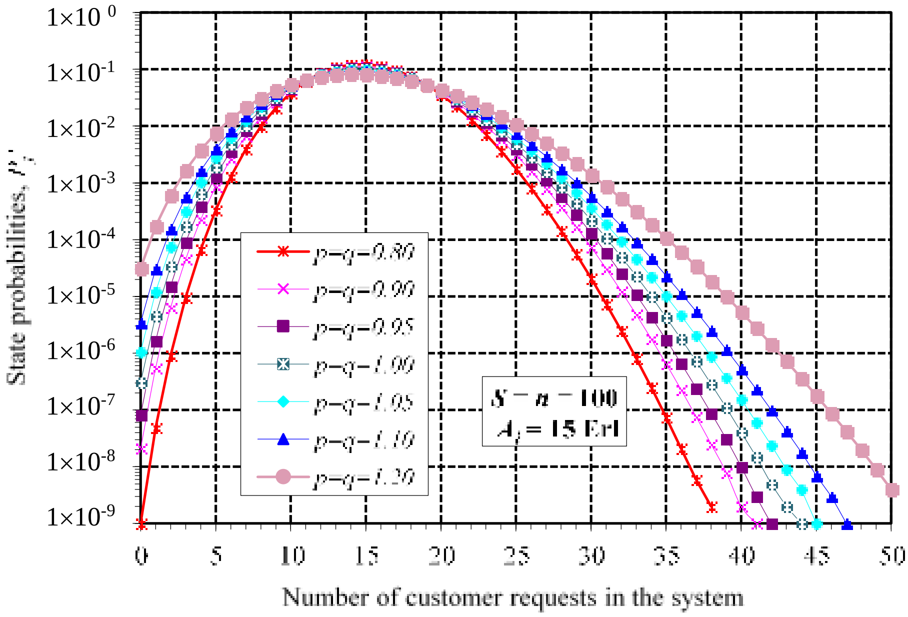


Fig. 5. *M*(*g*)/*M*(*g*)/*S* queue – State probability distribution for different values of peakedness factors $p$ and $q$ ($S$=100, $n$=100, $A_i$=15 Erl)

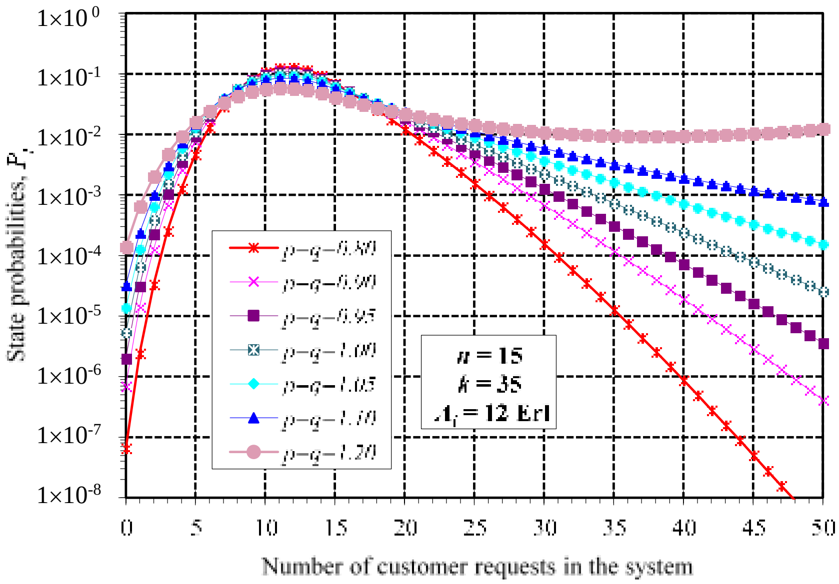


Fig. 6. *M*(*g*)/*M*(*g*)/*n*/*k* queue – State probability distribution for different values of peakedness factors *p* and *q* ($n$=15, $k$=35, $A_i$=12 Erl)

Figs 7-9 illustrate the state probability distribution for $n$=15 servers, $k$=35 waiting positions, different values of the intended traffic $A_i$, and peakedness factors $p$ and $q$ respectively equal to 0.9, 1, and 1.1 ($p$=$q$). It could be seen that in the case of high intended traffic (i.e., $A_i$=15 Erl) and peakedness factors greater than 1 (i.e., peaked traffic, Fig. 9), after the initial peak, the state probabilities start increasing again when 20 customer requests are accommodated in the system. Under the same conditions, in the case of regular intended traffic (Fig. 8), when the number of customer requests reaches 15, the state probabilities stay between the 0.1 and 0.01 levels.

Fig. 10 shows the blocking probability $B_t$ as a function of the buffer size (queue length) for $n$=15 servers, and $A_i$=12 Erl intended traffic. The results show that when the peakedness factors reach the 1.2 value, the influence of the buffer size on the blocking probability is negligible.

For the results presented in Figs 11-13, the offered traffic is calculated (using (5)-(7) after obtaining the state probabilities by iteration method described above.

Fig. 11 presents the blocking probability $B_t$ as a function of the offered traffic for $n$=15 servers and $k$=85 waiting positions. The results show that when the peakedness factors are greater than one, a particular blocking probability value can be guaranteed only by significantly reducing the offered traffic.

Fig. 12 shows the mean number $L$ of customer requests in the system as a function of the offered traffic for $n$=15 servers and $k$=85 waiting positions. Respectively, Fig. 13 presents the normalized mean system time ($W' = W/\tau$) as a function of the offered traffic for $n$=15 servers and $k$=85 waiting positions. It can be seen that, when the peakedness factors are bigger than one, the number of customer requests in the system and the mean system time have big values even when relatively small traffic is offered. This will increase the probability for congestion.

Fig. 14 illustrates the waiting time distribution $P(t_w>t')$ as a function of the defined time $t'$ for $n$=15 servers, $k$=85 waiting positions, and $A_i$=12 Erl intended traffic. The results show that when the peakedness factors increase, the probability of waiting time exceeding the define time interval increases by several orders of magnitude.

The presented numerical results demonstrate that higher peakedness factors (i.e., with values greater than 1) require at least 15-20% more resources to serve customers than the regular case.

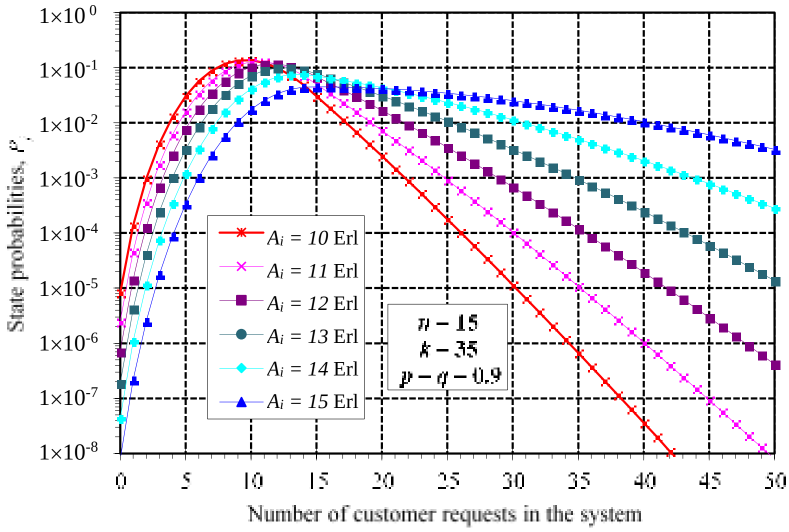


Fig. 7. *M(g)/M(g)/n/k* queue – State probability distribution for different values of smooth intended traffic ($n$=15, $k$=35, $p$=$q$=0.9)

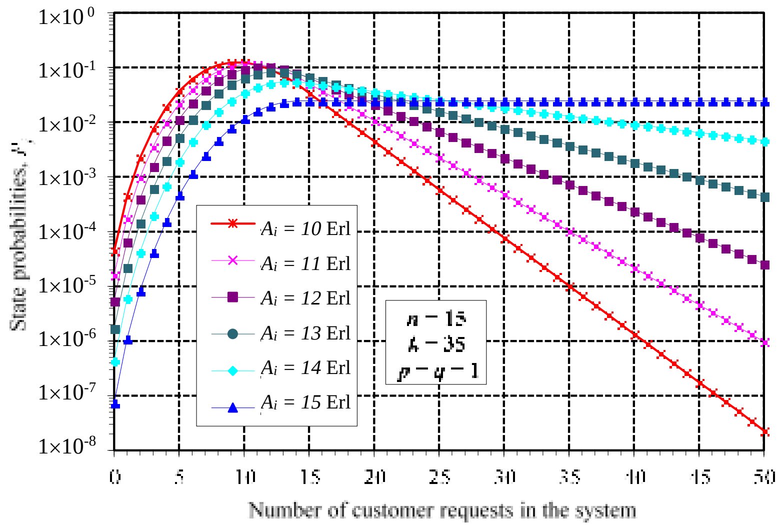


Fig. 8. *M(g)/M(g)/n/k* queue – State probability distribution for different values of regular intended traffic ($n$=15, $k$=35, $p$=$q$=1)

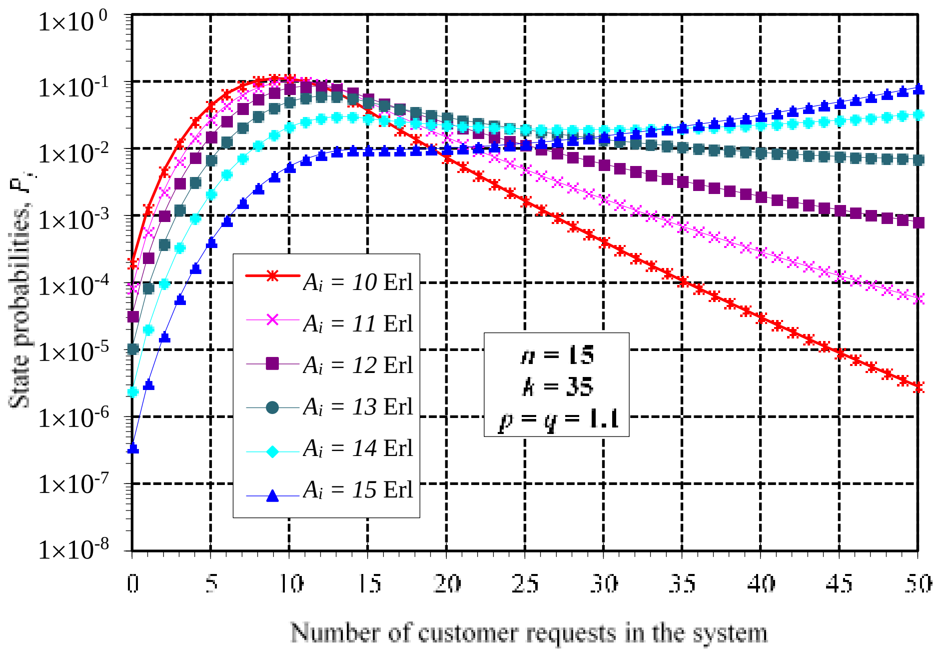


Fig. 9. *M(g)/M(g)/n/k* queue – State probability distribution for different values of peaked intended traffic ($n$=15, $k$=35, $p$=$q$=1.1)

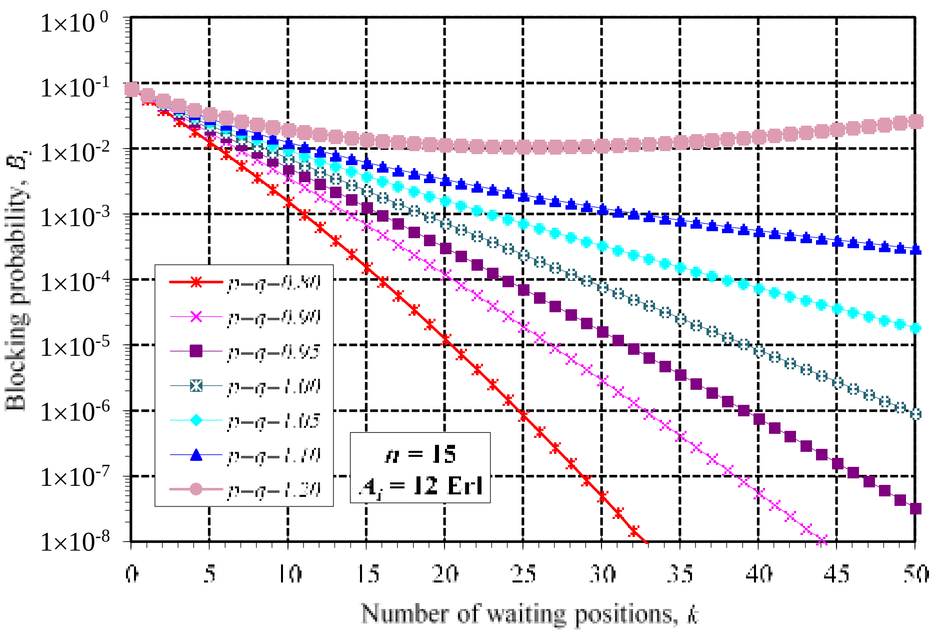


Fig. 10. *M(g)/M(g)/n/k* queue – Blocking probability as a function of the queue length, for different values of arrival and service peakedness factors $p$ and $q$ ($n$=15, $A_i$=12 Erl)

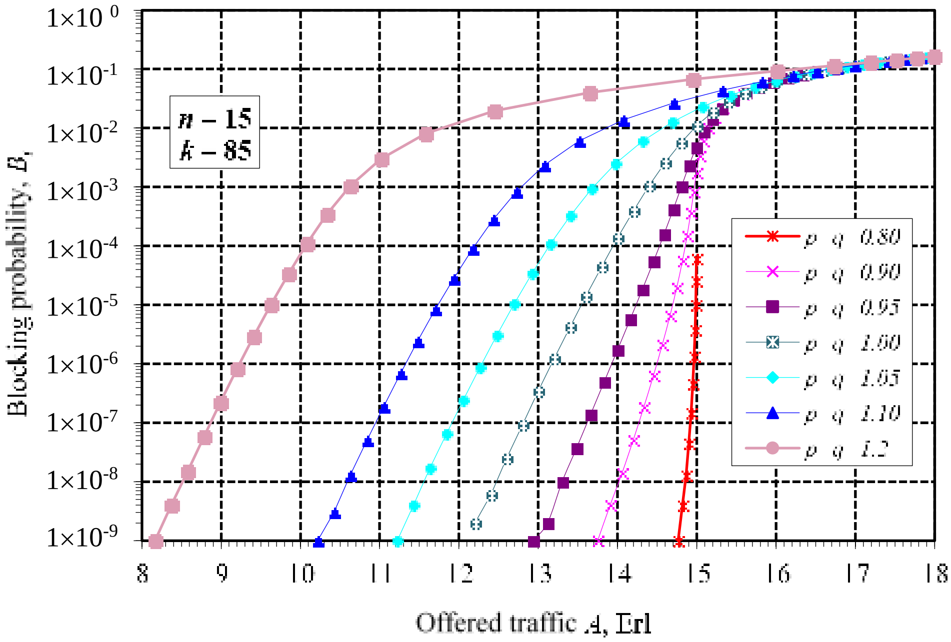


Fig. 11. *M*(*g*)/*M*(*g*)/*n*/*k* queue – Blocking probability as a function of the offered traffic, for different values of arrival and service peakedness factors *p* and *q* (*n*=15, *k*=85)

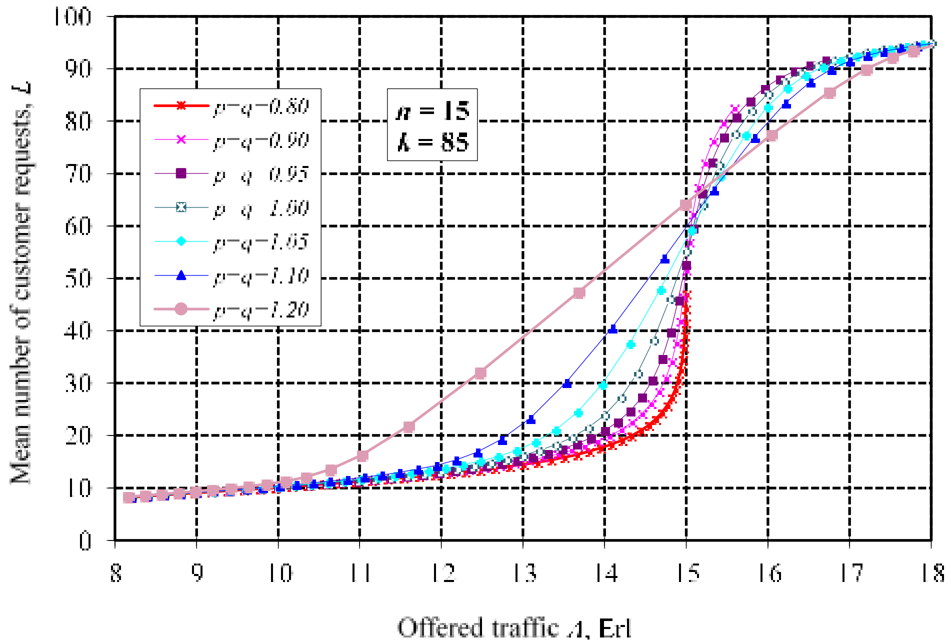


Fig. 12. *M*(*g*)/*M*(*g*)/*n*/*k* – Mean number of customer requests in the system as a function of the offered traffic, for different values of arrival and service peakedness factors *p* and *q* (*n*=15, *k*=85)

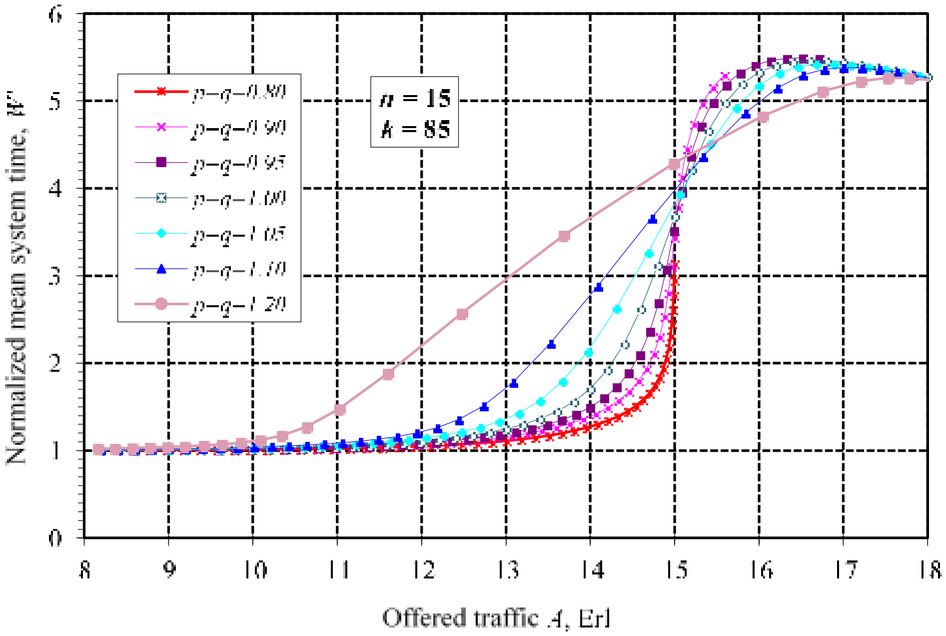


Fig. 13. *M(g)/M(g)/n/k* – Normalized mean system time as a function of the offered traffic, for different values of arrival and service peakedness factors *p* and *q* (*n*=15, *k*=85)

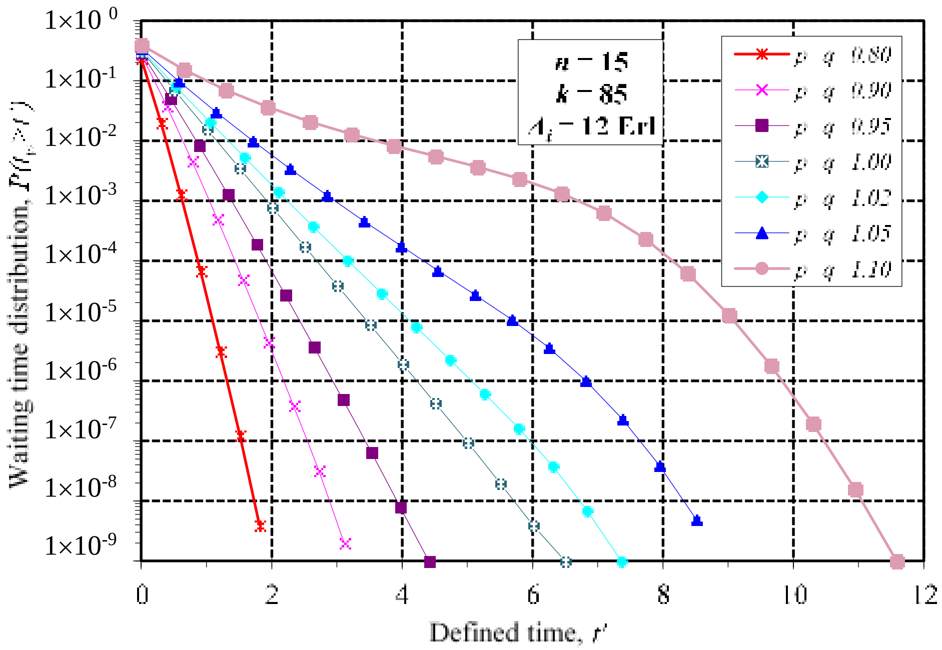


Fig. 14. *M(g)/M(g)/n/k* – Probability of waiting time exceeding the define time interval *t*′, as a function of *t*′, for different values of arrival and service peakedness factors *p* and *q* (*n*=15, *k*=85, $A_i$=12 Erl)

## 8. Conclusion

In this paper, generalized Poisson arrival and Bernoulli departure processes are introduced as a result of the state-dependent arrival and service rates. A generic model for a multi-server delay queue $M(g)/M(g)/n/k$ is examined in greater detail. The proposed model provides a unified framework for studying the peaked, regular and smooth behavior of teletraffic systems. Special attention is paid to the ELEaaS modeling where the traffic sources and data mobility are dynamic and irregular.

By using the suggested generalized discrete first Erlang distribution, it is possible to analyze the behavior of a multi-server delay queue $M(g)/M(g)/n/k$ for a range of values of the intended traffic with different variance. The generalized Erlang-C formula for arrival and departure flows with nonlinear state-dependence intensity and limited waiting positions is suggested for use as it provides the possibility to set a different variance of the *arrival and departure processes* without a need to change the mean value.

The multi-server delay system with state-dependent arrival and service rates could be used as a means for controlling and smoothing the data flows in telecommunications networks. In addition, it could be used to explain the behavior of real traffic in broadband telecommunications networks, evaluate the bandwidth sharing, and study the Internet traffic.

The importance of the teletraffic systems in the case of state-dependent arrival and service rates comes from its ability to describe behavior of today's networks. It is an important feature in designing the telecommunications networks, which makes it quite useful in practice.

Future work will consider the case of virtual machines' migration that will change the number of servers available along service epochs. Active ELEaaS services could also require different processing time that could add to the complexity of the model making it even more dependent on the load and congestions in the cloud.

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