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Compress-and-Forward for Relay Broadcast Channels without Common Messages

Xiaoxia Song, Yong Li

School of Mathematics and Computer Science, Shanxi Datong University, Datong 037009, Shanxi, China
Email: sxxly2002@163.com

Abstract: This paper is connected with compress-and-forward strategy for two-user relay broadcast channels without common messages, where the relay node has private messages from the source, in addition to aiding traditional communication from the source to the destination. For this channel we derive two achievable rate regions based on the compress-and-forward strategy in cases of discrete memoryless channels and Gaussian channels, respectively. The numerical results for Gaussian relay broadcast channel show that the inner bound based on the compress-and-forward strategy improves when all the messages without peeling off any components are compressed and sent to the receiver. It also verifies that the inner bound based on compress-and-forward strategy is better than that based on decode-and-forward strategy, when the relay node is near to the sink node. Moreover, the rate region of the broadcast channel improves considerably when the collaboration between the two receivers is allowed. So the relay node can provide residual resources to help the communication between the source and the sink after its communication rate is satisfied, which gives some insights to select an available relay node in a practical communication system.

Keywords: Relay broadcast channels, relay channels, broadcast channels, achievable rate region, compress-and-forward.

1. Introduction

In 1971 van der Meulen has first introduced the relay channel [1], where the relay node does not transmit its own messages, instead of helping the communication from the sender to the receiver. In [2] the capacities of the physically degraded relay channel, the reversely physically degraded relay channel and the relay channel with a feedback were determined, and an achievable rate based on compress-and-forward strategy for the relay channel was presented. In the relay channel, the relay node has no private messages to communicate with other nodes and all its resources are used to aid the other nodes. However, such an idle relay node is often unavailable in a practical communication system. If some network nodes have residual resources after completing their private communications from the source to the destination, they may be selected as candidate relay nodes [3, 5-11]. In [3] the relay channel with private messages was introduced and its relay node has private messages to communicate with both the source and the destination. But this scenario is rare in practical telecommunication systems. The works in [5-9] adopt cooperative diversity schemes in the uplink of a cellular system, where one user shares another user's resources to improve the transmission rate. In fact, those relay nodes also communicate with the destination, besides aiding the communication of the other user. Another kind of an interesting channel is the Relay Broadcast Channel (RBC) which models the common downlink of a cellular radio system [10, 11]. In their works the capacity bounds of RBC are given, based on decode-and-forward scheme for both the common messages and the private messages of the source. But they do not provide the capacity bounds of RBC via compress-and-forward strategy.

In this paper, for a two-user RBC without common messages, we derive two achievable rate regions based on compress-and-forward strategy in cases of discrete memoryless channels and Gaussian channels. In our scheme the regular block Markov encoding and the backward decoding techniques [12, 4] are used at the source and the destination, respectively. The numerical results for a two-user Gaussian RBC show that the inner bound based on compress-and-forward strategy can be improved when all the messages without peeling off any components are compressed and sent to the receiver. It also verifies that the inner bound based on compress-and-forward strategy is better than that based on decode-and-forward strategy when the relay is near to the sink. More interesting, the rate region of the broadcast channel improves considerably when the collaboration between the two receivers is allowed. So the relay node can provide residual resources to help the communication between the source and the sink after its communication rate is satisfied, which provides a clue to select an available relay node in a practical communication system.

This paper is organized as follows. In Section 2 some related definitions of RBC are given. The achievable rate regions for discrete memoryless channels and Gaussian channels are derived by using the compress-and-forward strategy in Section 3. The numerical results of the Gaussian RBC are also given in this section. The conclusions are given in Section 4.

Before proceeding, we present some notations. Let X and $\mathcal{X}$ denote a random variable and its range, respectively. The deterministic variable or the realization of a random variable and a vector are denoted by the lower case letter x and the bold lower case letter $\mathbf{x}$, respectively. And further define $X_t^i \stackrel{\Delta}{=} (X_{t,1}, X_{t,2}, ..., X_{t,i})$, $C(x) = \frac{1}{2}\log(1+x)$ and $\bar{x} = 1 - x$. Throughout the paper, the logarithmic function is to the base 2.

2. Channel model

Definition 1. A Discrete Memoryless Relay Broadcast Channel (DM-RBC) given in Fig. 1, consists of a channel input alphabet $\mathcal{X}_1$, a relay input alphabet $\mathcal{X}_2$, two channel output alphabets $\mathcal{Y}_2$ and $\mathcal{Y}_3$, and a probability transition function $p(y_2, y_3 | x_1, x_2)$, where x_1, x_2 denote the source and the relay inputs, respectively, while y_2 and y_3 denote the outputs at the relay and the destination, respectively.

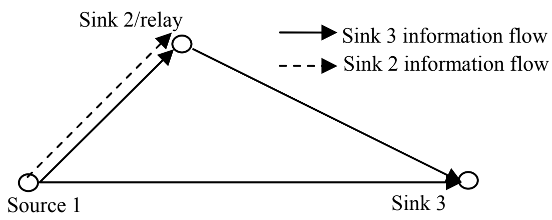

Fig. 1. The model of DM-RBC

Definition 2. A $\left(\left(2^{nR_{13}},\ 2^{nR_{12}}\right),\ n\right)$ code for a DM-RBC consists of the following components:

two index sets $\mathcal{W}_{12} = \left\{1, 2, ..., 2^{nR_{12}}\right\}$ and $\mathcal{W}_{13} = \left\{1, 2, ..., 2^{nR_{13}}\right\}$;

an encoder

$$X_1 : \mathcal{W}_{13} \times \mathcal{W}_{12} \to \mathcal{X}_1^n;$$

a set of relay functions $\{f_i\}_{i=1}^n$, $x_{2,i} = f_i\left(y_{2,1}, ..., y_{2,i-1}\right)$, $1 \le i \le n$.

two decoders

$$d_1 : \mathcal{Y}_2^n \to \mathcal{W}_{12},$$

$$d_2 : \mathcal{Y}_3^n \to \mathcal{W}_{13}.$$

In this paper the relay node is assumed to operate in full duplex and to be causal. The encoding and decoding structure of the messages for DM-RBC is shown in Fig. 2.

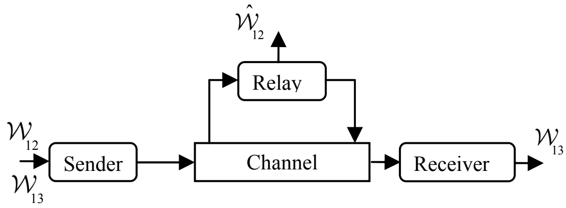

Fig. 2. The encoding and decoding structure for DM-RBC

Definition 3. The RBC is said to be degraded if its transition probability satisfies $p\left(y_2, y_3 | x_1, x_2\right) = p\left(y_2 | x_1, x_2\right) p\left(y_3 | y_2, x_2\right)$, i.e., $X_1 - [Y_2, X_2] - Y_3$ forming a Markov chain.

Definition 4. An Additive White Gaussian Noise RBC (AWGN-RBC) has continuous input and output alphabets and independent additive white Gaussian noise. The channel outputs of the relay and the destination are

(1) $$Y_2 = X_1 + Z_2 ,$$

(2) $$Y_3 = X_1 + X_2 + Z_3 ,$$

where $Z_2 \sim \mathcal{N}(0, N_2)$ and $Z_3 \sim \mathcal{N}(0, N_3)$ are independent Gaussian noises. There are power constraints on the input sequences x_1 and x_2, namely $\varepsilon[X_i^2] < P_i$, i=1, 2.

The message flows in this channel are illustrated in Fig. 3.

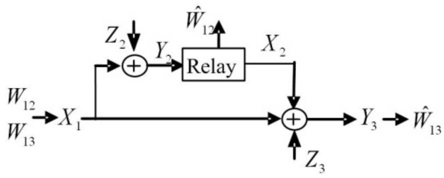

Fig. 3. The message flows of AWGN-RBC

Definition 5. The average probability of the error is defined as the probability that the decoded messages are not equal to the transmitted messages, i.e., $p_{\mathrm{e}}^{(n)} = p(\hat{W}_{12} \neq W_{12} \text{ or } \hat{W}_{13} \neq W_{13})$, where $\hat{W}$ denotes an estimate of W, and (W_{13}, W_{12}) are assumed to be uniformly distributed over $\mathcal{W}_{13} \times \mathcal{W}_{12}$. The probability of the errors at the Relay node and the Destination node are respectively defined as

$$p_{\mathrm{e,R}}^{(n)} = p(\hat{W}_{12} \neq W_{12}) ,$$

$$p_{\mathrm{e,D}}^{(n)} = p(\hat{W}_{13} \neq W_{13}) .$$

According to the union bound we have

$$\max\left\{p_{\mathrm{e,R}}^{(n)}, p_{\mathrm{e,D}}^{(n)}\right\} \leq p_{\mathrm{e}}^{(n)} \leq p_{\mathrm{e,R}}^{(n)} + p_{\mathrm{e,D}}^{(n)} .$$

Hence, if $p_{\mathrm{e}}^{(n)} \to 0$, then $p_{\mathrm{e,R}}^{(n)} \to 0$, $p_{\mathrm{e,D}}^{(n)} \to 0$.

Definition 6. A rate pair (R_{13}, R_{12}) is said to be achievable for RBC, if there exists a sequence of $\left(\left(2^{nR_{13}}, 2^{nR_{12}}\right), n\right)$ codes with $p_{\mathrm{e}}^{(n)} \to 0$ as $n \to \infty$.

3. Main results

3.1. Discrete memoryless channels

In this subsection we obtain an achievable rate region for DM-RBC via compress-and-decode strategy introduced in [2].

Theorem 1. A rate pair (R_{13}, R_{12}) is said to be achievable for DM-RBC if

(3) $$R_{12} < I(U_2; Y_2 | X_2) ,$$

(4) $$R_{13} < I(U_1; \hat{Y}_2, Y_3 | X_2) ,$$

(5) $$R_{12} + R_{13} < I(U_1; \hat{Y}_2, Y_3 | X_2) + I(U_2; Y_2 | X_2) - I(U_2; U_1) ,$$

subject to

(6) $$I(Y_2;\hat{Y}_2|U_2, X_2) \le I(X_2, \hat{Y}_2; Y_3),$$
where the joint distribution of the random variables factors is
$$p(u_1, u_2)p(x_2)p(x_1|u_1, u_2)p(\hat{y}_2|x_2, y_2)p(y_2, y_3|x_1, x_2).$$

P r o o f: The regular encoding and backward decoding technique [4] is used. We consider transmission over B blocks, each of n symbols. A sequence of $B-1$ messages $W_{13}(b)$ and $W_{12}(b)$ will be sent over the channel in nB transmissions, where b denotes the block index, $b=1,2,...,B-1$. The rate pair $\left(R_{13}\frac{B-1}{B}, R_{12}\frac{B-1}{B}\right)$ approaches (R_{13}, R_{12}) as $B\to\infty$. An outline of the proof is given below.

Let $A^{(n)}(U_1)$ and $A^{(n)}(U_2)$ be the set of sequences $\mathbf{u}_1$ and $\mathbf{u}_2$ that are strongly typical in U_1 and U_2, respectively, and $A^{(n)}(U_1, U_2)$ be the set of strongly joint typical sequences. Moreover, define $A^{(n)}(U_2|\mathbf{u}_1)$ as
$$A^{(n)}(U_2|\mathbf{u}_1) = \{\mathbf{u}_2 \in A^{(n)}(U_2) : (\mathbf{u}_1, \mathbf{u}_2) \in A^{(n)}(U_1, U_2)\}.$$

Let $S^{(n)}(U_1)$ denotes the set of all sequences $\mathbf{u}_1 \in A^{(n)}(U_1)$ such that $\left\|A^{(n)}(U_2|\mathbf{u}_1)\right\| > 0$. Similarly, we can define $S^{(n)}(U_2)$.

Fix $p(u_1, u_2)p(x_2)p(x_1|u_1, u_2)p(\hat{y}_2|x_2, y_2)$.

Random codebook construction

1) Generate $2^{nR(U_1)}$ sequences $\mathbf{u}_1$, drawn according to
$$p(\mathbf{u}_1) = \begin{cases} \dfrac{1}{\left\|S^{(n)}(U_1)\right\|} & \text{if } \mathbf{u}_1 \in S^{(n)}(U_1), \\ 0 & \text{otherwise.} \end{cases}$$

2) Generate $2^{nR(U_2)}$ sequences $\mathbf{u}_2$, drawn according to
$$p(\mathbf{u}_2) = \begin{cases} \dfrac{1}{\left\|S^{(n)}(U_2)\right\|} & \text{if } \mathbf{u}_2 \in S^{(n)}(U_2), \\ 0 & \text{otherwise.} \end{cases}$$

3) Randomly assign $\mathbf{u}_1$ one of $2^{nR_{13}}$ bins and $\mathbf{u}_2$ one of $2^{nR_{12}}$ bins.

4) Assign a pair $(\mathbf{u}_1, \mathbf{u}_2) \in A^{(n)}(U_1, U_2)$ for each product bin. For a sufficiently large n, such a pair exists with high probability if the formulation (7) is satisfied [13, 14]

(7) $$R_{13} + R_{12} < R(U_1) + R(U_2) - I(U_1; U_2).$$

5) For each product bin and its corresponding pair $(\mathbf{u}_1, \mathbf{u}_2) \in A^{(n)}(U_1, U_2)$, generate sequences $\mathbf{x}_1(w_{13}, w_{12})$, drawn according to $\prod_{i=1}^{n} p(x_{1,i}|u_{1,i}, u_{2,i})$.

6) Generate $2^{n\tilde{R}}$ sequences $\mathbf{x}_2(z')$, drawn according to $\prod_{i=1}^{n} p(x_{2,i})$.

7) For each $\mathbf{x}_2(z')$, generate $2^{n\hat{R}}$ sequences $\hat{\mathbf{y}}_2(z', z)$, drawn according to $\prod_{i=1}^{n} p(\hat{y}_{2,i}|x_{2,i})$.

Encoding

1) The source node sends the codeword $\mathbf{x}_1(w_{13,b},\ w_{12,b})$ in block b.

2) The relay node has determined z_{b-1}, denoted as z' above, then sends $\mathbf{x}_2(z_{b-1})$ in block b.

So the transmitted codeword pair can be written as:

$$\mathbf{x}_1(w_{13,1}, w_{12,1}), \mathbf{x}_2(1), \quad b=1,$$
$$\mathbf{x}_1(w_{13,b}, w_{12,b}), \mathbf{x}_2(z_{b-1}), \quad b=2,...,B-1,$$
$$\mathbf{x}_1(1,1), \mathbf{x}_2(z_{B-1}), \quad b=B.$$

Decoding

1) Assuming that the relay node has determined z_{b-1} correctly. Then it finds a unique $\hat{w}_{12,b}$ such that $\mathbf{u_2}(\hat{w}_{12,b})$, $\mathbf{x}_2(z_{b-1})$ and $\mathbf{y}_2(b)$ are jointly typical. This can be made reliable if

(8) $$R_{12} < I(U_2;Y_2 | X_2).$$

2) The relay decodes z_b by looking for a unique $\hat{z}_b$ such that $\mathbf{u_2}(w_{12,b})$, $\mathbf{x}_2(z_{b-1})$, $\hat{\mathbf{y}}_2(z_{b-1}, \hat{z}_b)$ and $\mathbf{y}_2(b)$ are jointly typical. The index z_b will be decoded correctly if

(9) $$\hat{R} > I(\hat{Y}_2;Y_2 | U_2, X_2).$$

3) The destination node starts to decode, using backward decoding technique after the transmission of block b is completed. Now it can determine z_b and decode z_{b-1} by looking for a unique $\hat{z}_{b-1}$ such that $\mathbf{x}_2(\hat{z}_{b-1})$, $\hat{\mathbf{y}}_2(\hat{z}_{b-1}, z_b)$ and $\mathbf{y}_3(b)$ are jointly typical. Reliable communication needs

(10) $$\hat{R} < I(X_2, \hat{Y}_2; Y_3).$$

4) After decoding z_b and z_{b-1}, the destination node finds a unique $\hat{w}_{13,b}$ such that $\mathbf{u_1}(\hat{w}_{13,b})$, $\mathbf{x}_2(z_{b-1})$, $\hat{\mathbf{y}}_2(z_{b-1}, z_b)$ and $\mathbf{y}_3(b)$ are jointly typical. The decoding can be made reliable if the formulation (11) is satisfied.

(11) $$R_{13} < I(U_1; \hat{Y}_2, Y_3 | X_2).$$

The achievable rate region in Theorem 1 can be concluded directly from (7), (8) and (11), while the constraint (5) follows from combining (9) and (10).

3.2. Gaussian channels

Characterizing the capacity region of AWGN-RBC is more interesting than that of DM-RBC. For simplicity we assume the input distributions to be Gaussian. By applying the results given in Theorem 1 to AWGN-RBC, we can obtain some numerical results and assess the behavior of the rates in AWGN-RBC.

Corollary 1. An achievable rate region for the AWGN-RBC is the set of all rate pairs (R_{13}, R_{12}) satisfying the next equations:

(12) $$R_{12} < C\left(\frac{P_{U_2} + 2\rho\sqrt{P_{U_1}P_{U_2}}}{P_{U_1} + N_2}\right),$$

(13) $$R_{13} < \mathcal{C}\left(\frac{P_{U_1}P_{U_2}(1-\rho^2)+(N_c+N_2)(P_{U_1}+2\rho\sqrt{P_{U_1}P_{U_2}})+P_{U_1}N_3}{(P_{U_2}+N_3)(N_c+N_2)}\right),$$

(14) $$R_{12}+R_{13} < \mathcal{C}\left(\frac{P_{U_1}P_{U_2}(1-\rho^2)+(N_c+N_2)(P_{U_1}+2\rho\sqrt{P_{U_1}P_{U_2}})+P_{U_1}N_3}{(P_{U_2}+N_3)(N_c+N_2)}\right)+$$

$$+\mathcal{C}\left(\frac{P_{U_2}+2\rho\sqrt{P_{U_1}P_{U_2}}}{P_{U_1}+N_2}\right)+\mathcal{C}\left(-\rho^2\right),$$

subject to

(15) $$N_c \geq \frac{P_{U_1}P_{U_2}(1-\rho^2)+(N_3+P_1)N_2+P_{U_1}N_3}{P_2}.$$

P r o o f: We choose the Gaussian codebook as follows.

$U_1 \sim \mathcal{N}\left(0,P_{U_1}\right)$, $U_2 \sim \mathcal{N}\left(0,P_{U_2}\right)$ and they satisfy $E\left(U_1U_2\right)=\rho\sqrt{P_{U_1}P_{U_2}}$. $X_2 \sim \mathcal{N}\left(0,P_2\right)$, where X_2 is independent on U_1 and U_2.

Let $X_1 = U_1 + U_2$. The random variable representing the compressed channel output of the relay node is chosen as

(16) $$\hat{Y}_2 = U_1 + Z_2 + Z_c,$$

where Z_c denotes the compression noise whose variance N_c is determined by the constraint given in Theorem 1.

From (3) we obtain

(17) $$R_{12} < I(U_2;Y_2|X_2) = h(Y_2|X_2)-h(Y_2|X_2,U_2) = h(X_1+Z_2)-h(U_1+Z_2) =$$
$$=\frac{1}{2}\log(2\pi e)\left(P_1+N_2\right)-\frac{1}{2}\log(2\pi e)\left(P_{U_1}+N_2\right)=\mathcal{C}\left(\frac{P_{U_2}+2\rho\sqrt{P_{U_1}P_{U_2}}}{P_{U_1}+N_2}\right).$$

From (4) we have

(18) $$R_{13} < I(U_1;\hat{Y}_2,Y_3|X_2)=h(\hat{Y}_2,Y_3|X_2)-h(\hat{Y}_2,Y_3|X_2,U_1).$$

For simplicity we derive each term, respectively

(19) $$h(\hat{Y}_2,Y_3|X_2) = h(U_1+Z_2+Z_c,\, X_1+X_2+Z_3|X_2) =$$
$$= h(U_1+Z_2+Z_c, U_1+U_2+Z_3) =$$
$$=\frac{1}{2}\log(2\pi e)^2\begin{vmatrix} P_{U_1}+N_2+N_c & P_{U_1}+\rho\sqrt{P_{U_1}P_{U_2}} \\ P_{U_1}+\rho\sqrt{P_{U_1}P_{U_2}} & P_1+N_3\end{vmatrix}=$$
$$=\frac{1}{2}\log(2\pi e)^2\left(P_{U_1}P_{U_2}(1-\rho^2)+(N_c+N_2)(P_1+N_3)+P_{U_1}N_3\right),$$

where

(20) $$h(\hat{Y}_2,Y_3|X_2,U_1) = h(U_1+Z_2+Z_c,\, X_1+X_2+Z_3|X_2,U_1)=$$
$$= h(Z_2+Z_c, U_2+Z_3) = \frac{1}{2}\log(2\pi e)^2\begin{vmatrix} N_2+N_c & 0 \\ 0 & P_{U_2}+N_3\end{vmatrix}=$$
$$=\frac{1}{2}\log(2\pi e)^2\left(\left(P_{U_2}+N_3\right)\left(N_2+N_c\right)\right).$$

We can obtain (13) in Corollary 1 by combining (18), (19) and (20).

Now we compute the term $I(U_2;U_1)$ in (5).

$$I(U_2;U_1) = h(U_1) + h(U_2) - h(U_1, U_2)$$

$$(21)\quad = \frac{1}{2}\log(2\pi e)\left(P_{U_1}\right) + \frac{1}{2}\log(2\pi e)\left(P_{U_2}\right) - \frac{1}{2}\log(2\pi e)^2 \begin{vmatrix} P_{U_1} & \rho\sqrt{P_{U_1}P_{U_2}} \\ \rho\sqrt{P_{U_1}P_{U_2}} & P_{U_2} \end{vmatrix} =$$

$$= -\mathcal{C}\left(-\rho^2\right).$$

So we obtain (14).

Next, we will compute the constraint on the compression noise variance N_c. From (9) it follows:

$$\hat{R} > I(\hat{Y}_2;Y_2 | U_2, X_2) = h(\hat{Y}_2 | U_2, X_2) - h(\hat{Y}_2 | U_2, X_2, Y_2) = h(U_1 + Z_2 + Z_c) - h(\hat{Y}_2 | Y_2, U_2) =$$

$$= h(U_1 + Z_2 + Z_c) - h(Z_c) = \frac{1}{2}\log(2\pi e)\left(P_{U_1} + N_2 + N_c\right) - \frac{1}{2}\log(2\pi e)\left(N_c\right).$$

Hence,

$$(22)\qquad \hat{R} > \mathcal{C}\left(\frac{P_{U_1} + N_2}{N_c}\right).$$

On the other hand, from (10) it is obtained

$$(23)\qquad \hat{R} < I(X_2, \hat{Y}_2; Y_3) = I(X_2;Y_3) + I(\hat{Y}_2;Y_3 | X_2).$$

and

$$(24)\qquad I(X_2;Y_3) = h(Y_3) - h(Y_3 | X_2) = h(X_1 + X_2 + Z_3) - h(X_1 + X_2 + Z_3 | X_2) =$$

$$= \frac{1}{2}\log\left(\frac{P_1 + P_2 + N_3}{P_1 + N_3}\right),$$

$$(25)\qquad I(\hat{Y}_2;Y_3 | X_2) = h(\hat{Y}_2 | X_2) - h(\hat{Y}_2 | X_2, Y_3) = h(\hat{Y}_2) - h(\hat{Y}_2 | X_2, Y_3) =$$

$$= h(U_1 + Z_2 + Z_c) - h(U_1 + Z_2 + Z_c | X_1 + Z_3).$$

Now

$$(26)\qquad h(U_1 + Z_2 + Z_c) = \frac{1}{2}\log(2\pi e)\left(P_{U_1} + N_2 + N_c\right).$$

where

$$(27)\qquad h(U_1 + Z_2 + Z_c | X_1 + Z_3) = h(U_1 + Z_2 + Z_c, X_1 + Z_3) - h(X_1 + Z_3) =$$

$$= \frac{1}{2}\log(2\pi e)^2 \begin{vmatrix} P_{U_1} + N_2 + N_c & P_{U_1} + \rho\sqrt{P_{U_1}P_{U_2}} \\ P_{U_1} + \rho\sqrt{P_{U_1}P_{U_2}} & P_1 + N_3 \end{vmatrix} - \frac{1}{2}\log(2\pi e)\left(P_1 + N_3\right).$$

Combining (25), (26) and (27) we obtain

$$(28)\qquad I(\hat{Y}_2;Y_3 | X_2) = \frac{1}{2}\log\left(\left(P_1 + N_3\right)\left(P_{U_1} + N_2 + N_c\right)\right) -$$

$$- \frac{1}{2}\log \begin{vmatrix} P_{U_1} + N_2 + N_c & P_{U_1} + \rho\sqrt{P_{U_1}P_{U_2}} \\ P_{U_1} + \rho\sqrt{P_{U_1}P_{U_2}} & P_1 + N_3 \end{vmatrix}.$$

Combining (23), (24) and (28) we have

(29) $$\hat{R} < \frac{1}{2}\log\Big(\big(P_1 + P_2 + N_3\big)\big(P_{U_1} + N_2 + N_c\big)\Big) -$$

$$-\frac{1}{2}\log\begin{vmatrix} P_{U_1} + N_2 + N_c & P_{U_1} + \rho\sqrt{P_{U_1}P_{U_2}} \\ P_{U_1} + \rho\sqrt{P_{U_1}P_{U_2}} & P_1 + N_3 \end{vmatrix}.$$

Combining (22) and (29), we obtain (15). ■

3.3. Remarks

The above results are obtained by compressing the symbols $\mathbf{y}_2$ after peeling off the component of X_1 intended for the relay node which is represented by U_2. Alternatively, the relay does not have to peel off any component from its observation. So, we can obtain the following results.

Theorem 2. A rate pair (R_{13}, R_{12}) is said to be achievable for DM-RBC if

(30) $$R_{12} < \min\left\{I(U_2; Y_2 \,|X_2), I(U_2; \hat{Y}_2, Y_3 \,|X_2)\right\},$$

(31) $$R_{13} < I(U_1; \hat{Y}_2, Y_3 \,|U_2, X_2),$$

(32) $$R_{12} + R_{13} < \min\left\{I(U_2; Y_2 \,|X_2), I(U_2; \hat{Y}_2, Y_3 \,|X_2)\right\} + I(U_1; \hat{Y}_2, Y_3 \,|U_2, X_2) - I(U_2; U_1),$$

subject to

(33) $$I(Y_2; \hat{Y}_2 \,|X_2) \le I(X_2, \hat{Y}_2; Y_3),$$

where the joint distribution of the random variables factors is

$$p(u_1, u_2)p(x_2)p(x_1 \,|u_1, u_2)p(\hat{y}_2 \,|x_2, y_2)p(y_2, y_3 \,|x_1, x_2).$$

Proof: The regular encoding and the backward decoding technique [4] are used. We consider transmission over B blocks and each block has n symbols. A sequence of $B-1$ messages $W_{13}(b)$ and $W_{12}(b)$ will be sent through the channel in nB transmissions, where b denotes the block index, $b = 1, 2, \dots, B-1$. The rate pair $\left(R_{13}\frac{B-1}{B}, R_{12}\frac{B-1}{B}\right)$ approaches (R_{13}, R_{12}), as $B \to \infty$. An outline of the proof is given below.

Let $A^{(n)}(U_1)$ and $A^{(n)}(U_2)$ be the set of sequences $\mathbf{u}_1$ and $\mathbf{u}_2$ that are strongly typical in U_1 and U_2, respectively, and $A^{(n)}(U_1, U_2)$ be the set of strongly joint typical sequences. Moreover, define $A^{(n)}(U_2 \,|\mathbf{u}_1)$ as

$$A^{(n)}(U_2 \,|\mathbf{u}_1) = \left\{\mathbf{u}_2 \in A^{(n)}(U_2) : (\mathbf{u}_1, \mathbf{u}_2) \in A^{(n)}(U_1, U_2)\right\}.$$

Let $S^{(n)}(U_1)$ denotes the set of all sequences $\mathbf{u}_1 \in A^{(n)}(U_1)$, such that $\left\|A^{(n)}(U_2 \,|\mathbf{u}_1)\right\| > 0$. Similarly, we can define $S^{(n)}(U_2)$.

Fix $p(u_1, u_2)p(x_2)p(x_1 \,|u_1, u_2)p(\hat{y}_2 \,|x_2, y_2)$.

Random codebook construction

1) Generate $2^{nR(U_1)}$ sequences $\mathbf{u}_1$, drawn according to

$$p(\mathbf{u}_1)=\begin{cases}\dfrac{1}{\left\|S^{(n)}(U_1)\right\|} & \text{if} \quad \mathbf{u}_1\in S^{(n)}(U_1),\\ 0 & \text{otherwise.}\end{cases}$$

2) Generate $2^{nR(U_2)}$ sequences $\mathbf{u}_2$, drawn according to

$$p(\mathbf{u}_2)=\begin{cases}\dfrac{1}{\left\|S^{(n)}(U_2)\right\|} & \text{if} \quad \mathbf{u}_2\in S^{(n)}(U_2),\\ 0 & \text{otherwise.}\end{cases}$$

3) Randomly assign $\mathbf{u}_1$ one of $2^{nR_{13}}$ bins and $\mathbf{u}_2$ one of $2^{nR_{12}}$ bins.

4) Assign a pair $(\mathbf{u}_1,\mathbf{u}_2)\in A^{(n)}(U_1,U_2)$ for each product bin. For a sufficiently large n, such a pair exists with high probability if

(34) $$R_{13}+R_{12}<R(U_1)+R(U_2)-I(U_1;U_2).$$

5) For each product bin and its corresponding pair $(\mathbf{u}_1,\mathbf{u}_2)\in A^{(n)}(U_1,U_2)$, generate sequences $\mathbf{x}_1(w_{13},w_{12})$, drawn according to $\prod_{i=1}^{n}p(x_{1,i}\,|u_{1,i},u_{2,i})$.

6) Generate $2^{n\hat{R}}$ sequences $\mathbf{x}_2(z')$, drawn according to $\prod_{i=1}^{n}p(x_{2,i})$.

7) For each $\mathbf{x}_2(z')$, generate $2^{n\hat{R}}$ sequences $\hat{\mathbf{y}}_2(z',z)$, drawn according to $\prod_{i=1}^{n}p(\hat{y}_{2,i}\,|x_{2,i})$.

Encoding

1) The source node sends the codeword $\mathbf{x}_1(w_{13,b},w_{12,b})$ in block b.

2) The relay node has determined z_{b-1}, denoted as z' above, then sends $\mathbf{x}_2(z_{b-1})$ in block b.

So, the transmitted codeword pair can be written as:

$$\mathbf{x}_1(w_{13,1},w_{12,1}),\mathbf{x}_2(1),\quad b=1,$$
$$\mathbf{x}_1(w_{13,b},w_{12,b}),\mathbf{x}_2(z_{b-1}),\quad b=2,\dots,B-1$$
$$\mathbf{x}_1(1,1),\mathbf{x}_2(z_{B-1}),\quad b=B.$$

Decoding

1) Assuming that the relay node has determined z_{b-1} correctly, then it finds a unique $\hat{w}_{12,b}$, such that $\mathbf{u}_2(\hat{w}_{12,b})$, $\mathbf{x}_2(z_{b-1})$ and $\mathbf{y}_2(b)$ are jointly typical. This can be made reliable if

(35) $$R_{12}<I(U_2;Y_2\,|X_2).$$

2) The relay decodes z_b by looking for a unique $\hat{z}_b$, such that $\mathbf{x}_2(z_{b-1})$, $\hat{\mathbf{y}}_2(z_{b-1},\hat{z}_b)$, $\mathbf{y}_2(b)$ are jointly typical. The index z_b will be decoded correctly if

(36) $$\hat{R}>I(\hat{Y}_2;Y_2\,|X_2).$$

3) The destination node starts to decode using backward decoding technique after the transmission of block b is completed. Now it can determine z_b and decode z_{b-1} by looking for a unique $\hat{z}_{b-1}$, such that $\mathbf{x}_2(\hat{z}_{b-1})$, $\hat{\mathbf{y}}_2(\hat{z}_{b-1},z_b)$ and $\mathbf{y}_3(b)$ are jointly typical. Reliable communication needs

(37) $$\hat{R} < I(X_2, \hat{Y}_2; Y_3).$$

4) After decoding z_b and z_{b-1}, the destination node finds a unique $\hat{w}_{12,b}$, such that $\mathbf{u}_2(\hat{w}_{12,b})$, $\mathbf{x}_2(z_{b-1})$, $\hat{\mathbf{y}}_2(z_{b-1}, z_b)$ and $\mathbf{y}_3(b)$ are jointly typical. The decoding can be made reliable if

(38) $$R_{12} < I(U_2; \hat{Y}_2, Y_3 | X_2).$$

5) After decoding z_b, z_{b-1} and $w_{12,b}$ the destination node finds a unique $\hat{w}_{13,b}$ such that $\mathbf{u_1}(\hat{w}_{13,b})$, $\mathbf{u}_2(w_{12,b})$ $\mathbf{x}_2(z_{b\text{-}1})$, $\hat{\mathbf{y}}_2(z_{b-1}, z_b)$ and $\mathbf{y}_3(b)$ are jointly typical. The decoding can be made reliable if

(39) $$R_{13} < I(U_1; \hat{Y}_2, Y_3 | U_2, X_2).$$

The achievable rate region in Theorem 2 can be concluded directly from (34), (35), (38) and (39), while the constraint (33) follows from combining (36) and (37). ■

Corollary 2. An achievable rate region for the AWGN-RBC is the set of all the rate pairs (R_{13}, R_{12}) satisfying the next equations (40), (41) and (42), subject to

(40) $$N_c \geq \frac{(N_2 + P_1)N_3 + P_1 N_2}{P_2},$$

(41) $$R_{13} < \mathcal{C}\left(\frac{P_{U_1}\left(N_2 + N_c + N_3\right)}{N_3\left(N_c + N_2\right)}\right),$$

(42) $$R_{12} < \min\left\{\mathcal{C}\left(\frac{P_{U_2} + 2\rho\sqrt{P_{U_1}P_{U_2}}}{P_{U_1} + N_2}\right), \mathcal{C}\left(\frac{(P_{U_2} + 2\rho\sqrt{P_{U_1}P_{U_2}})(N_3 + N_c + N_2)}{P_{U_1}(N_3 + N_c + N_2)N_3(N_c + N_2)}\right)\right\},$$

(43) $$R_{12} + R_{13} < \min\left\{\mathcal{C}\left(\frac{P_{U_2} + 2\rho\sqrt{P_{U_1}P_{U_2}}}{P_{U_1} + N_2}\right), \mathcal{C}\left(\frac{(P_{U_2} + 2\rho\sqrt{P_{U_1}P_{U_2}})(N_3 + N_c + N_2)}{P_{U_1}(N_3 + N_c + N_2)N_3(N_c + N_2)}\right)\right\} +$$
$$+ \mathcal{C}\left(\frac{P_{U_1}\left(N_2 + N_c + N_3\right)}{N_3\left(N_c + N_2\right)}\right) + \mathcal{C}\left(-\rho^2\right),$$

P r o o f: We choose the Gaussian codebook as follows.

$U_1 \sim \mathcal{N}\left(0, P_{U_1}\right), U_2 : N\left(0, P_{U_2}\right)$, and they satisfy $E\left(U_1 U_2\right) = \rho\sqrt{P_{U_1}P_{U_2}}$. $X_2 \sim \mathcal{N}\left(0, P_2\right)$, where X_2 is independent on U_1 and U_2.

Let $X_1 = U_1 + U_2$.The random variable representing the compressed channel output of the relay node is chosen as

(44) $$\hat{Y}_2 = U_1 + U_2 + Z_2 + Z_c,$$

Where Z_c denotes the compression noise whose variance N_c is determined by the constraint given in Theorem 2. Then through computing (30) up to (33), we have the conclusion of Corollary 2.

3.4. Numerical results

In this subsection we give Figs 4 and 5 to compare the inner bounds of AWGN-RBC, AWGN Relay Channel (AWGN-RC) and AWGN Broadcast Channel (AWGN-BC) in the case of $P_1 = P_2 = 5$, $N_2 = 0.5$ or $N_2 = 0.9$ and $N_3 = 1$. The inner bounds of AWGN-RBC include the inner bound based on decode-and-forward strategy [11] denoted by the curve of DF-RBC, the inner bound based on Corollary 1 denoted by the curve of CF1-RBC and the inner bound based on Corollary 2 denoted by the curve of CF2-RBC. The inner bounds of AWGN-RC include the lower bound based on decode-and-forward strategy [2] denoted by a circle point of DF-RC and the lower bound based on compress-and-forward strategy [2] denoted by a circle point of CF-RC.

From the two figures, we can have three results. The first one is that the rate R_{13} from source 1 to sink 3 increases when sink 2 provides some resources to help the communication between source 1 and sink 3. We can use decode-and-forward strategy or compress-and-forward strategy to help the communication from source 1 to sink 3 when the rates from source 1 to sink 2 are less than certain values or the curve BC is lower than the curve RBC. So the rate region of BC improves considerably when the collaboration between the two sinks is allowed. Moreover, the maximum R_{13} is obtained when all the resources of sink 2 are used to help the communication between source 1 and sink 3. The maximum rates R_{13} based on decode-and-forward and compress-and-forward are depicted by small circles denoted by DF-RC and CF-RC on the vertical axis in Figs 4 and 5, respectively. From these figures, we can conclude that the relay channel is a special case of the RBC.

The second one is that the inner bound based on Corollary 1 denoted by the curve CF1-RBC is better than that based on Corollary 2 denoted by the curve CF2-RBC. The numerical results demonstrate that sink 3 can exploit much more information when the relay node uses compress-and-forward messages without peeling off any components. Hence, the inner bound improves.

The third one is that the inner bound based on decode-and-forward strategy is better than that of compress-and-forward strategy when the noise at sink 2 is small or the channel from source 1 to sink 2 is better than that from source 1 to sink 3 or the node sink 2 is near to the node source 1. Otherwise, the inner bound based on compress-and-forward strategy is better than that based on decode-and-forward strategy.

4. Conclusions

In this paper we consider a two-user RBC without common messages and derive two achievable rate regions for DM-RBC and AWGN-RBC via compress-and-forward strategy, respectively. The numerical results for AWGN-RBC show that the inner bound based on compress-and-forward strategy improves when all the messages without peeling off any components are compressed and sent to the receiver. It also verifies that the inner bound based on compress-and-forward strategy is better than that based on decode-and-forward strategy when the relay node is near to the sink node. Moreover, the rate region of the broadcast channel improves significantly when the collaboration between the two receivers is allowed. So the relay node can provide residual resources to help the communication between the source node and the sink node after its private communication rate is satisfied, which provides some insights to select an available relay node in a practical communication system.

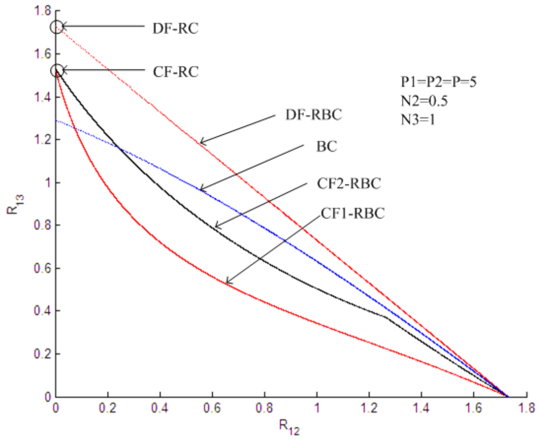

Fig. 4. Comparisons of several inner bounds under N_2=0.5

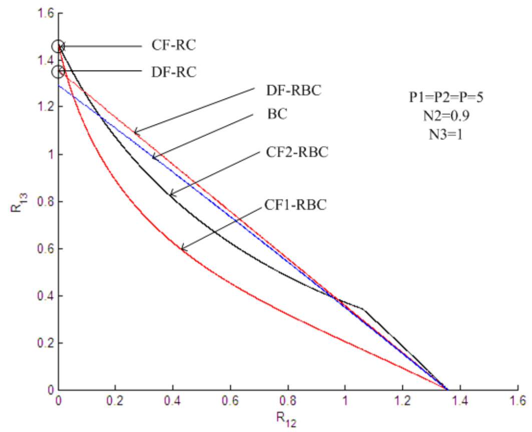

Fig. 5. Comparisons of several inner bounds under N_2=0.9

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