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Adaptive Fuzzy H^{∞} Robust Tracking Control for Nonlinear MIMO Systems

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Abstract: *In this paper an adaptive fuzzy $H\infty$ robust tracking control scheme is developed for a class of uncertain nonlinear Multi-Input and Multi-Output (MIMO) systems. Firstly, fuzzy logic systems are introduced to approximate the unknown nonlinear function of the system by an adaptive algorithm. Next, a $H\infty$ robust compensator controller is employed to eliminate the effect of the approximation error and external disturbances. Consequently, a fuzzy adaptive robust controller is proposed, such that the tracking error of the resulting closed-loop system converges to zero and the tracking robustness performance can be guaranteed. The simulation results performed on a two-link robotic manipulator demonstrate the validity of the proposed control scheme.*

Keywords: *Nonlinear MIMO system, adaptive fuzzy control, H^{∞} control.*

1. Introduction

Considering a class of uncertain nonlinear MIMO systems, controller design has attracted great attention and has achieved significant development during the recent decades. Many control algorithms have been proposed, such as the feedback linearization methods [1], the variable structure control [2, 3], the adaptive control [4, 5], the fuzzy control and others. The feedback linearization methods have lead to great success in the development of controllers for nonlinear systems. However, these control schemes can only be applied to nonlinear systems whose dynamics are well known. Unfortunately, some kinds of uncertainties, such as uncertain plant parameters, modelling errors and unknown external disturbances, always exist in the actual application system, which will significantly degrade the control performance of the feedback linearization control method. An adaptive control algorithm can be applied to deal with the unknown dynamics of a nonlinear system. But in this control method, the unknown parameters must be of a linear structure and are assumed to be constant and slowly varying. For a highly uncertain nonlinear system, the performance of the adaptive control scheme may not be guaranteed. The variable structure control is widely accepted as a powerful control tool for solving the control problem of uncertain nonlinear systems. This approach is one of the efficient robust control methods to compensate the uncertainties, but it will cause the undesired phenomenon of chattering due to the switching signal.

Fuzzy logic control [6-8], as one of the important intelligent techniques, provides an efficient approach to handle a nonlinear system with unmolded dynamics and unknown disturbances. The universal approximation property shows that any nonlinear function over a compact set can be approximated by a fuzzy system with arbitrary accuracy, which makes fuzzy control so widely employed to modelling or controlling uncertain nonlinear systems. But usually fuzzy control systems would be affected by uncertainties, such as various parameters and unknown external disturbances, which may deteriorate the control performance or even cause the closed-loop system instability. And just H^{∞} robust compensate controller is an efficient control method to reject these uncertainties, which have been widely discussed for the robustness and its capability of disturbance attenuation in nonlinear control systems. Thus in this paper an adaptive fuzzy H^{∞} tracking control scheme, comprised by an adaptive fuzzy controller and an additional H^{∞} compensation controller is proposed for the uncertain MIMO nonlinear systems.

The structure of the paper is organized as follows. The problem formulation and some theoretical preliminaries are discussed in Section 2. The description of a fuzzy system is given in Section 3. The design procedure of the control scheme and the stability analysis are given in Section 4. Simulation results, performed on a two-link robot manipulator validate the efficiency of the proposed control method in Section 5. Finally, some conclusions are considered in Section 6.

2. Problem formulation

Consider a class of n-th order MIMO nonlinear system described by [9]

(1) $$x^{(n)} = F(X) + G(X)u + d,$$

where: $X = [x^{\mathrm{T}}, \dot{x}^{\mathrm{T}}, \cdots, x^{(n-1)\mathrm{T}}]^{\mathrm{T}} \in R^{mn}$ denotes the state vector of the system, which is assumed to be available; $x = [x_1, \cdots, x_m]^{\mathrm{T}} \in R^m$, $u = [u_1, \cdots, u_m]^{\mathrm{T}} \in R^m$ are the system outputs and control inputs, respectively; m is the number of the system inputs and outputs; $F(X) \in R^m$ and $G(X) \in R^{m\times m}$ are nonlinear uncertainty functions; d is the unknown external disturbance.

Control objectives. The control object in this paper is to design a suitable control scheme u for the nonlinear system given by (1), such that the state X can track any given bounded desired trajectory $X_d = [x_d^{\mathrm{T}}, \dot{x}_d^{\mathrm{T}}, \cdots, x_d^{(n-1)\mathrm{T}}]^{\mathrm{T}} \in R^{mn}$ in the presence of uncertainties and external disturbances, with all the signals in the resulting closed-loop switched system remaining bounded.

Assumption 1. The unknown external disturbance of system (1), d is assumed to belong to $L_2[0,\infty]$, and the upper bound is assumed as $\bar{d}$, i.e., $|d| \le \bar{d}$.

Assumption 2. The matrix $G(X)$, as previously defined is nonsingular, i.e., $G^{-1}(X)$ exists.

Assumption 3. The desired trajectories x_d and their time derivatives up to the n-th order are continuous and bounded.

Define the following tracking error

(2) $$e = x - x_d.$$

If the knowledge of the system dynamics is complete and assuming that the external disturbances are ignored, i.e., the nonlinear functions $F(X)$ and $G(X)$ are both known and the external disturbance is $d=0$, according to the feedback linearization techniques, the following control law can be obtained:

(3) $$u = G^{-1}(X)[x_d^{(n)} - k_n e^{(n-1)} - \cdots - k_1 e - F(X)],$$

where the coefficients $k_1, \cdots, k_n$ must be chosen so that $h(S) = S^n + k_n S^{n-1} + \cdots + k_2 S + k_1$ is a Hurwitz polynomial, all roots being in the open left-half of the S-plane, in which S is a complex Laplace transform variable.

Substituting (3) into (1) leads to

(4) $$x^{(n)} = x_d^{(n)} - \sum_{i=0}^{n-1} k_{i+1} e^{(i)}.$$

That equation can be written as

(5) $$e^{(n)} + \sum_{i=0}^{n-1} k_{i+1} e^{(i)} = e^{(n)} + k_n e^{(n-1)} + \cdots + k_1 e = 0.$$

So $\lim_{t\to\infty} e = 0$, the system is globally asymptotically stable if the coefficients $k_1, \cdots, k_n$ in (5) are symmetric and positive definite constant matrices. The control objective is achieved.

However, the precise values of the nonlinear functions $F(X)$ and $G(X)$ are difficult to be acquired, due to the parameters measurement errors and time varying uncertainties. The external disturbance is also inevitable in actual practical engineering .Therefore, the unmolded dynamics and the external disturbances cannot be ignored. Namely, $F(X)$ and $G(X)$ are usually both unknown and the disturbance vector $d \ne 0$.Considering the universal approximation ability of fuzzy control, in the next step we will use an adaptive fuzzy system as a tool for modelling nonlinear functions up to a small error tolerance.

3. Design of an adaptive fuzzy H^{∞} robust tracking controller

Denote

$$F(x) = [f_1(x), \cdots, f_n(x)]^{\mathrm{T}},$$

$$G(x) = \begin{bmatrix} g_{11}(x) & \cdots & g_{1n}(x) \\ \vdots & \ddots & \vdots \\ g_{n1}(x) & \cdots & g_{nn}(x) \end{bmatrix}.$$

Since $f_i(x)$ and $g_{ij}(x)$ are supposed as unknown, the objective in this section is to approximate $f_i(x)$ and $g_{ij}(x)$ by using a fuzzy system which inputs are $x_1, \cdots, x_n$.

The used Fuzzy Logic System (FLS) performs a mapping from an input vector $x \in \Omega_{\omega} \in R^n$ to a scale output $y \in \Omega_r \in R$, where $\Omega_{\omega} = \Omega_{\omega 1} \times \Omega_{\omega 2} \times \cdots \times \Omega_{\omega n}, \Omega_{\omega i} \in R$.

The fuzzy rule base comprises a collection of the following IF-THEN rule:

$R^{(k)}$: If x_1 is A_1^k, and x_2 is A_2^k, $\cdots$, and x_n is A_n^k then y is B^k, $k = 1, \cdots, N$,

where $A_i^k \in \left\{A_i^1, \cdots, A_i^n\right\}$, $i = 1, \cdots, n,$ and B^k denote the linguistic variables of the input and output of the fuzzy sets, defined respectively for x_i and y. Besides, $N = \prod_{i=1}^{n} n_i$ is the total number of rules.

Through singleton fuzzification, product inference engine and center average defuzzification, the final output of the fuzzy logic system can be expressed as follows:

$$y = \hat{f}(x, \theta) = \xi^{\mathrm{T}}(x)\theta, \tag{6}$$

where $\theta = [\theta^1, \theta^2, \cdots, \theta^N]^{\mathrm{T}}$ is called a parameter vector and $\xi(x) = [\xi_1(x), \cdots, \xi_N(x)]^{\mathrm{T}}$ is a set of fuzzy basis functions defined as

$$\xi_k(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^k}(x_i)}{\sum_{k=1}^{N} \left(\prod_{i=1}^{n} \mu_{A_i^k}(x_i)\right)}, \qquad k = 1, \cdots, N, \tag{7}$$

where $\mu_{A_i^k}(x_i)$ is the membership function of the linguistic variables x_i and it represents the fuzzy meaning of the symbol A_i^k; θ^k corresponds to the value of a singleton which is the fuzzy meaning of B^k.

Lemma 1. FLS in (6) is a universal function approximator, i.e., for any given real continuous function $f(x)$ on a compact set D and an arbitrary small constant $\varepsilon_f > 0$ there exists a FLC system such that $\sup_{\omega \in D} \left| f(x) - \hat{f}(x, \theta) \right| < \varepsilon_f$, where $D \subset R^n$ is an approximation region.

Then the fuzzy logic system is used to approximate the unknown nonlinear functions $f_i(x)$ and $g_{ij}(x)$ in the following form:

$$\hat{f}_i(x, \theta_{fi}) = \xi_{fi}^T(x)\theta_{fi}, \quad i = 1, \cdots, n, \tag{8}$$

(9) $$\hat{g}_{ij}(x, \theta_{gij}) = \xi_{gij}^{\mathrm{T}}(x)\theta_{gij}, \quad j = 1, \cdots, n,$$

where ξ_{fi} and ξ_{gij} are fuzzy basis vectors from (7); θ_{fi} and θ_{gij} are the corresponding adjustable parameter vectors of the fuzzy system which is tuned on-line.

Let us define the following variables:

(10) $$\theta_{fi}^{*} = \arg\min_{\theta_{fi}}\left\{\sup_{\omega\in D_{\omega}}\left|f_i(x) - \hat{f}_i(x, \theta_{fi})\right|\right\},$$

(11) $$\theta_{gij}^{*} = \arg\min_{\theta_{gij}}\left\{\sup_{\omega\in D_{\omega}}\left|g_{ij}(x) - \hat{g}_{ij}(x, \theta_{gij})\right|\right\},$$

(12) $$\tilde{\theta}_{fi} = \theta_{fi}^{*} - \theta_{fi},$$

(13) $$\tilde{\theta}_{gij} = \theta_{gij}^{*} - \theta_{gij},$$

(14) $$\varepsilon_{fi}(x) = f_i(x) - \hat{f}_i(x, \theta_{fi}^{*}),$$

(15) $$\varepsilon_{gij}(x) = g_{ij}(x) - \hat{g}_{ij}(x, \theta_{gij}^{*}),$$

where θ_{fi}^{*} and θ_{gij}^{*} are the optimal approximation parameters of θ_{fi} and θ_{gij}; $\tilde{\theta}_{fi}$ and $\tilde{\theta}_{gij}$ are the parameters approximation errors; $\varepsilon_{fi}(x)$ and $\varepsilon_{gij}(x)$ denote the minimum approximation errors, which correspond to the approximation errors obtained when optimal parameters are used.

Assumption 4. We assume that the approximation errors $\varepsilon_{fi}(x)$ and $\varepsilon_{gij}(x)$ are bounded, i.e., $\left|\varepsilon_{fi}(x)\right| \le \bar{\varepsilon}_{fi}$, $\left|\varepsilon_{gij}(x)\right| \le \bar{\varepsilon}_{gij}$, where $\bar{\varepsilon}_{fi}$ and $\bar{\varepsilon}_{gij}$ are both positive known constants.

Denote

$$\varepsilon_f = [\varepsilon_{f1}(x), \cdots, \varepsilon_{fn}(x)]^{\mathrm{T}},$$

$$\varepsilon_g(x) = \begin{bmatrix} \varepsilon_{g11}(x) & \cdots & \varepsilon_{g1n}(x) \\ \vdots & \ddots & \vdots \\ \varepsilon_{gn1}(x) & \cdots & \varepsilon_{gnn}(x) \end{bmatrix}.$$

Then, from the above analysis we have

(16) $$F(x) - \hat{F}(x, \theta_f) = \hat{F}(x, \theta_f^{*}) - \hat{F}(x, \theta_f) + \varepsilon_f(x),$$

(17) $$G(x) - \hat{G}(x, \theta_g) = \hat{G}(x, \theta_g^{*}) - \hat{G}(x, \theta_g) + \varepsilon_g(x).$$

So, based on the above fuzzy control system, the uncertain functions $F(X)$ and $G(X)$ are estimated by $\hat{F}(x)$ and $\hat{G}(x)$ respectively. Then the feedback linearization control term of (3) can be rewritten as

(18) $$u_0 = \hat{G}^{-1}(X)[x_d^{(n)} - k_n e^{(n-1)} - \cdots - k_1 e - \hat{F}(X)].$$

The parameter adaptive control laws are given by

(19) $$\dot{\theta}_{fi} = -\eta_{fi}\xi_{fi}s_i,$$

(20) $$\dot{\theta}_{gij} = -\eta_{gij}\xi_{gij}s_i u_{0_j},$$

where $s = \Lambda E$, $E = [e^{\mathrm{T}}, \dot{e}^{\mathrm{T}}, e^{(n-1)\mathrm{T}}]^{\mathrm{T}} \in R^{mn}$, the adaptive gains η_{fi}, η_{gij} are both positive constants.

Furthermore, considering the presence of external perturbation, the controller can be chosen as follows:

(21) $$u = u_0 + u_h,$$

where u_0 is the adaptive feedback linearization term (18), u_h is a H^∞ robust control term, which is introduced to compensate the effect of the fuzzy system approximation error and of the external disturbance, thereby it enhances the tracking error attenuation quality and satisfies the H^∞ tracking error performance

The design procedure of the H^∞ robust control term is as follows:

Substituting (18) and (21) in (1), we obtain

(22) $$e^{(n)} + k_n e^{(n-1)} + \cdots + k_1 e = (\hat{F}(x) - F(x)) + (\hat{G}(x) - G(x))u_0 - \hat{G}(x)u_h - d.$$

The derivative of the tracking error is defined as

(23) $$E = [e^{\mathrm{T}}, \dot{e}^{\mathrm{T}}, e^{(n-1)\mathrm{T}}]^{\mathrm{T}} \in R^{mn}.$$

Then the output tracking error dynamic equation of the uncertain nonlinear system (1) can be described by

(24) $$\dot{E} = AE + B[(\hat{F}(x) - F(x)) + (\hat{G}(x) - G(x))u_0 + u_h - d] = AE + Bu_h - Bw',$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -k_1 & -k_2 & \cdots & -k_{n-1} & -k_n \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

Suppose that there exist matrices $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$ for the output tracking error dynamic equation (24), such that

(25) $$A^{\mathrm{T}}P + PA + Q + PB(\gamma^{-2} - 2R^{-1})B^{\mathrm{T}}P = 0,$$

where the given positive constant γ denotes the attenuation level, R is a positive gain matrix, satisfying $2\gamma^2 \geq R$.

Then the robust H^∞ controller can be chosen as

(26) $$u_h = -R^{-1}B^{\mathrm{T}}PE,$$

which satisfies the H^∞ control performance.

Theorem 1. Consider the nonlinear MIMO system (1), satisfying Assumptions 1-4, the proposed hybrid controllers (21), the feedback linearization control term (18), the nonlinear functions $f_i(x)$ and $g_{ij}(x)$ estimated by (8), (9), the adaptive parameters updated laws (19), (20), and H^∞ robust compensate control law (26) can guarantee that the tracking error closed-loop system is robustly stable, i.e., the tracking error satisfies $\lim_{t\to\infty} e = 0$, and all the variables of the closed-loop system are bounded.

P r o o f: Consider a Lyapunov function candidate as follows:

(27) $$V=\frac{1}{2}s^{\mathrm{T}}s+\frac{1}{2}\frac{1}{\eta_f}\tilde{\theta}_f^{\mathrm{T}}\tilde{\theta}_f+\frac{1}{2}\frac{1}{\eta_g}\tilde{\theta}_g^{\mathrm{T}}\tilde{\theta}_g.$$

Then the time derivative of Lyapunov function is given by

(28) $$\begin{aligned}\dot{V}&=s\dot{s}+\frac{1}{\eta_f}\tilde{\theta}_f^{\mathrm{T}}\dot{\tilde{\theta}}_f+\frac{1}{\eta_g}\tilde{\theta}_g^{\mathrm{T}}\dot{\tilde{\theta}}_g=\\&=s[\sum_{n=1}^{n}k_n e^{(n-1)}+x_d^{(n)}-F(x)-G(x)u-d]+\frac{1}{\eta_f}\tilde{\theta}_f^{\mathrm{T}}\dot{\tilde{\theta}}_f+\frac{1}{\eta_g}\tilde{\theta}_g^{\mathrm{T}}\dot{\tilde{\theta}}_g=\\&=s[\hat{F}(x,\theta_f^*)+\hat{G}(x,\theta_g^*)u_0-F(x)-G(x)u-d]+\frac{1}{\eta_f}\tilde{\theta}_f^{\mathrm{T}}\dot{\tilde{\theta}}_f+\frac{1}{\eta_g}\tilde{\theta}_g^{\mathrm{T}}\dot{\tilde{\theta}}_g=\\&=\tilde{\theta}_f^{\mathrm{T}}(\frac{1}{\eta_f}\tilde{\theta}_f-s\xi_f(x))+\tilde{\theta}_g^{\mathrm{T}}(\frac{1}{\eta_g}\tilde{\theta}_g-s\xi_g(x))-s[\varepsilon_f(x)+\varepsilon_g(x)u_0+G(x)u_h+d].\end{aligned}$$

After that from the parameter update law of the fuzzy system (19), (20), we can obtain

(29) $$\dot{V}=-s[\varepsilon_f(x)+\varepsilon_g(x)u_0+G(x)u_h+d]\le 0.$$

Therefore, the resulting closed-loop system is stable.

The overall adaptive fuzzy H^{∞} robust control scheme is shown in Fig. 1.

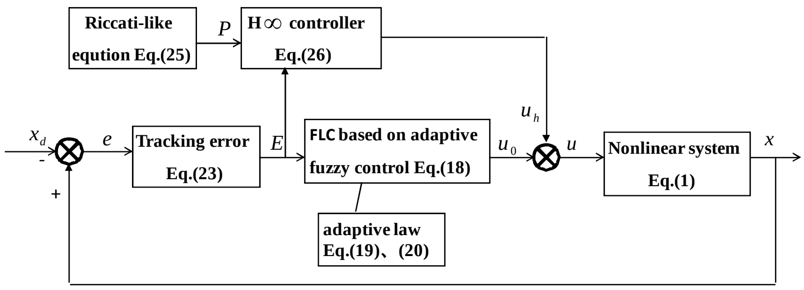

Fig. 1. Architecture of the proposed adaptive fuzzy robust control

4. Simulation results and analysis

4.1. Simulation results

In this section, we present a two-link robot manipulator as an example to verify the efficiency of the proposed hybrid control scheme. The two-link robot manipulator is described by [10]

$$\ddot{x}=-M^{-1}(x)[C(x,\dot{x})\dot{x}+G'(x)]+M^{-1}(x)u+M^{-1}(x)w=F(x)+G(x)u+d,$$

where

$$F(x)=-M^{-1}(x)[C(x,\dot{x})\dot{x}+G'(x)],\ G(x)=M^{-1}(x)u,\ d=M^{-1}(x)w,$$

$$M(x)=\begin{bmatrix} m_1l_1^2+m_2(l_1^2+l_2^2+2l_1l_2\cos(x_2) & m_2l_2^2+m_2l_1l_2\cos(x_2) \\ m_2l_2^2+m_2l_1l_2\cos(x_2) & m_2l_2^2\end{bmatrix},$$

$$C(x,\dot{x})=\begin{bmatrix}-2m_2l_1l_2\sin(x_2)\dot{x}_2 & m_2l_1l_2\sin(x_2)\dot{x}_2\\ m_2l_1l_2\sin(x_2)\dot{x}_2 & 0\end{bmatrix},$$

$$G'(x)=\begin{bmatrix} m_2 l_2 g\cos(x_1+x_2)+(m_1+m_2)l_1 g\cos(x_1) \\ m_2 l_2 g\cos(x_1+x_2) \end{bmatrix};$$

$x=[x_1, x_2]^{\mathrm{T}}$ denotes the joint position vector: $X=[X, \dot{X}^{\mathrm{T}}]^{\mathrm{T}}$; the torque vector $u=[u_1, u_2]^{\mathrm{T}}$ is the control input vector; $M(x)$ denotes the symmetric and positive definite inertia matrix; $C(x, \dot{x})$ denotes Coriolis and centrifugal force vector; $G'(x)$ is the gravity vector; w denotes the unknown external disturbances; m_1 and m_2 are the masses of link 1 and link 2 respectively; l_1 and l_2 are the lengths of link 1 and link 2 respectively; and the acceleration of gravity $g=9.8\,\mathrm{m/s^2}$. The simulation parameters values of this two-link robot manipulator are given as $m_1=0.5$, $m_2=0.5$, $l_1=1$, $l_2=0.8$.

The design procedure of the proposed controller parameters is in two steps.

(1) First constructing of the fuzzy logic systems $\hat{F}(x)$ and $\hat{G}(x)$ to approximate the nonlinear functions $F(X)$ and $G(X)$ of the feedback linearization controller in (18). Three membership functions of x_i are chosen:

$$\mu F_i^1(x_i)=\exp\left(-\frac{1}{2}\left(\frac{x_i+1.25}{0.6}\right)^2\right),\ \mu F_i^2(x_i)=\exp\left(-\frac{1}{2}\left(\frac{x_i}{0.6}\right)^2\right),$$

$$\mu F_i^3(x_i)=\exp\left(-\frac{1}{2}\left(\frac{x_i-1.25}{0.6}\right)^2\right),\ i=1,\cdots,4.$$

The adaptive gains η_{fi}, η_{gij} of the fuzzy system parameter adjusted law in (18)-(19) are both chosen as $\eta_{fi}=0.5$, $\eta_{gij}=0.5$.

The other parameter values used in FLC controller are chosen as $\Lambda=\begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$, then $s=\dot{e}+\Lambda e=\dot{e}+25e$, $k_1=k_2=\begin{bmatrix} 15 & 0 \\ 0 & 15 \end{bmatrix}=15I_2$.

(2) Second, designing the H^∞ robust compensating controller of (26). Let Hurwitz matrix $A=\begin{bmatrix} 0_2 & I_2 \\ -15I_2 & -15I_2 \end{bmatrix}$; choosing parameters $Q=30I_4$, $R=I_2$, $\gamma=0.2$ and obtain the H^∞ gain matrix P from the Riccati-like (25)

$$P=\begin{bmatrix} 30.9985I_2 & 0.9986I_2 \\ 0.9986I_2 & 1.0649I_2 \end{bmatrix}.$$

The control objective of this section is to force the system actual output to track the desired trajectories, which are given as $x_{d1}=\sin(t)+\sin(2t)$, $x_{d2}=0.5\sin(t)+\cos(2t)$ respectively, with the initial conditions $x_1(0)=x_2(0)=0.1\,\mathrm{rad}$, $\dot{x}_1(0)=\dot{x}_2(0)=0\,\mathrm{rad/s}$. The external disturbance is chosen as $w=1.5+2e+5\dot{e}$.

The simulation results are shown in Fig. 2 and Fig. 3. Fig. 2 shows the Membership function degree of the fuzzy logic systems $\hat{F}(X)$ and $\hat{G}(X)$. With the proposed adaptive fuzzy robust control scheme, Fig. 3 denotes the tracking performance of link 1 and link 2 respectively, Fig. 3a presents the position tracking of link 1, Fig. 3b is the position tracking of link 2, Fig. 3c and d denote the speed tracking performance of link 1 and link 2 respectively, Fig. 3e and f show the control input of link 1 and link 2 respectively.

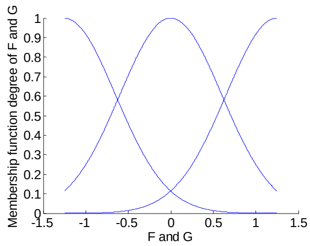

Fig. 2. Membership function degree of fuzzy logic systems $\hat{F}(X)$ and $\hat{G}(X)$

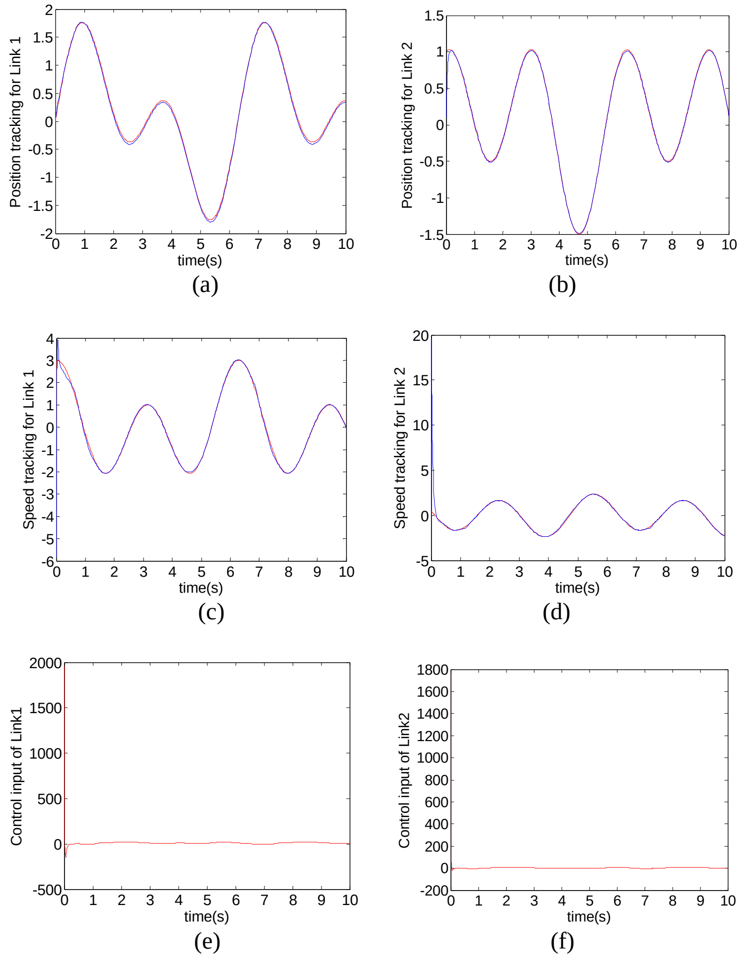

Fig. 3. Simulation results of the proposed adaptive fuzzy robust control: Position tracking of link 1 (a); position tracking of link 2 (b); velocity tracking of link 1 (c); velocity tracking of link 2 (d); control input of link 1 (e); control input of link 2 (f)

4.2. Analysis and discussion

From the above simulation results it is observed that the actual output signal can track the reference trajectory with good position and velocity tracking performance, the tracking errors going to a small value after some transient. The effect of uncertainties and disturbances is also successfully compensated by H^{∞} robust controller. The simulation results demonstrate that the proposed adaptive fuzzy H^{∞} robust controller can deal efficiently with a nonlinear system with uncertainties.

In order to better demonstrate the superiority of the proposed control method, simulation results based on variable structure control are also conducted, shown in Fig. 4.

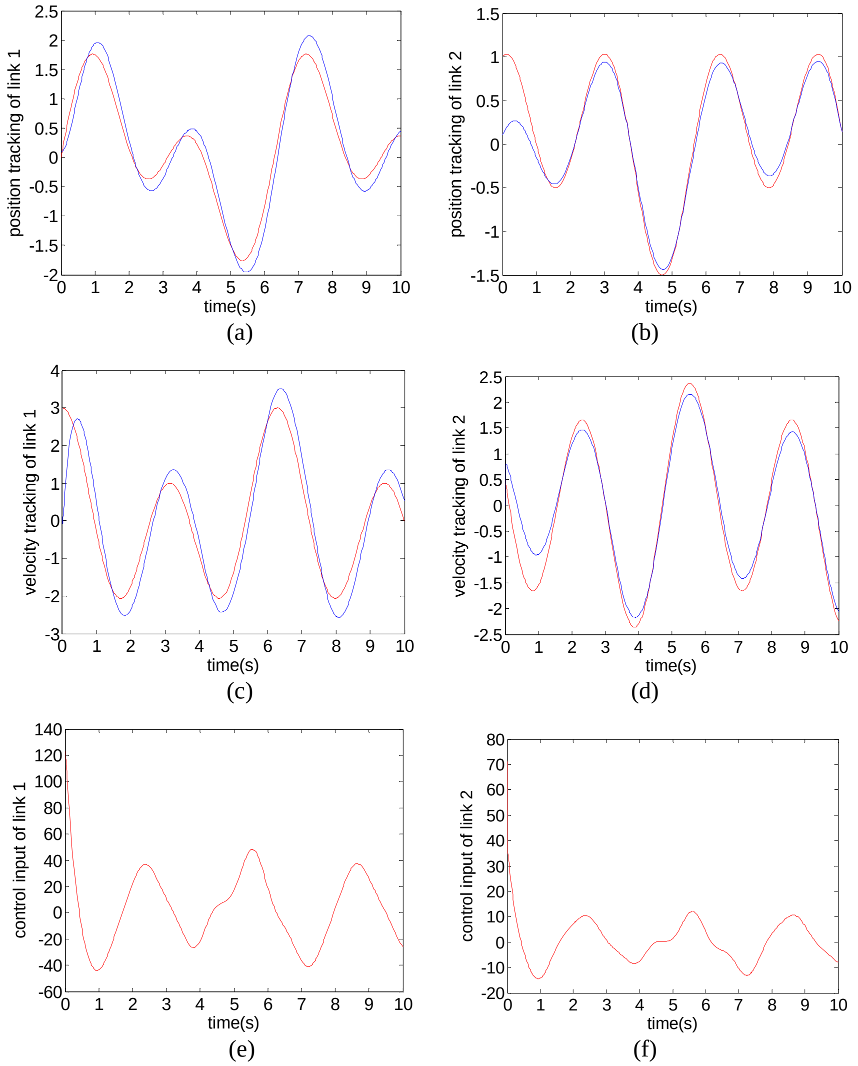

Fig. 4. Simulation results of the variable structure control: Position tracking of link 1 (a); position tracking of link 2 (b); velocity tracking of link 1 (c); velocity tracking of link 2 (d); control input of link 1 (e); control input of link 2 (f)

After comparison of the simulation results between the proposed adaptive fuzzy robust control and the variable structure control, it can be easily concluded that the tracking performance of the proposed controller is superior to the variable structure control. Under control of the variable structure control, the system has obvious position and velocity tracking errors, and the control input signal has a significant buffeting. So for uncertain robot manipulator systems, using just variable structure control is not enough, it also needs another control theory to further eliminate buffeting.

5. Conclusion and discussion

This paper presents an adaptive fuzzy H^{∞} robust tracking control scheme for uncertain nonlinear MIMO systems. The control idea is comprised by a feedback linearization controller based on adaptive fuzzy logic control and a H^{∞} robust compensate controller. The feedback linearization controller acts as the main controller; fuzzy logic systems are introduced to approximate the unknown nonlinear function of the system by an adaptive algorithm. The H^{∞} robust controller is used to eliminate the effect of the approximation error and external disturbances and improve the tracking performance. The simulation results performed on a two-link robotic manipulator demonstrate the robustness and efficient tracking control performance of the proposed control scheme. Finally, by comparing with the control performance of variable structure control, we highlight the advantages and superiority of the proposed control method.

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