BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES • Volume **13**, No 2

Sofia • 2013 Print ISSN: 1311-9702; Online ISSN: 1314-4081
DOI: 10.2478/cait-2013-0014

Some Results on Point Set Domination of Fuzzy Graphs

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Abstract: *Let G be a fuzzy graph. Let $\gamma(G), \gamma_p(G)$ denote respectively the domination number, the point set domination number of a fuzzy graph. A dominating set D of a fuzzy graph is said to be a point set dominating set of a fuzzy graph if for every $S \subseteq V - D$ there exists a node $d \in D$ such that $\langle S \cup \{d\} \rangle$ is a connected fuzzy graph. The minimum cardinality taken over all minimal point set dominating set is called a point set domination number of a fuzzy graph G and it is denoted by $\gamma_p(G)$. In this paper we concentrate on the point set domination number of a fuzzy graph and obtain some bounds using the neighbourhood degree of fuzzy graphs.*

Keywords: *Fuzzy graph, fuzzy star, point set dominating set of a fuzzy graph.*

1. Introduction

A mathematical framework to describe the phenomena of uncertainty in real life situation is first suggested by Zadeh in 1965 and Rosenfeld introduced the notion of a fuzzy graph and several fuzzy analogs of graph theoretic concepts, such as paths, cycles and connectedness. The study of dominating sets in graphs was started by Orge and Berge. The domination number was introduced by Cockayne and Hedetniemi (see [2]). Sampathkumar and Pushpalatha [4] introduced the concept of point set domination in graphs. A. Somasundaram and S. Somasundaram [5] discussed domination in a fuzzy graph using effective edges. Nagoorgani and Chandrasekeran [3] discussed domination in a fuzzy graph using strong arcs. In this paper we introduce the concept of point set domination using effective edges. We concentrate on the point set domination number of a fuzzy graph and obtain some bounds for a new parameter of the fuzzy graphs.

2. Preliminaries

In this section, basic definitions relating to a fuzzy graph are given.

Fuzzy set: Let E be the universal set. A fuzzy set A in E is represented by $A = \{(x,\ \mu_A(x)): \mu_A(x) > 0,\ x \in E\}$, where the function $\mu_A: E \to [0, 1]$ is the membership degree of element x in the fuzzy set A.

Fuzzy graph: A fuzzy graph $G(\sigma,\ \mu)$ is a pair of function $\sigma: V \to [0,\ 1]$ and $\mu: V \times V \to [0,\ 1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

Fuzzy subgraph: The fuzzy graph $H(\tau,\rho)$ is called a fuzzy subgraph of $G(\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u, v) \leq \mu(u, v)$ for all $u, v \in V$.

Spanning fuzzy subgraph: The fuzzy subgraph $H(\tau, \rho)$ is said to be a spanning fuzzy subgraph of $G(\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all $u \in V$ and $\rho(u, v) \leq \mu(u, v)$ for all $u, v \in V$. Let $G(\sigma,\ \mu)$ be a fuzzy graph and τ be any fuzzy subset of σ, i.e., $\tau(u) \leq \sigma(u)$ for all u.

Induced fuzzy subgraph: The fuzzy subgraph of $G(\sigma, \mu)$ induced by τ is the maximal fuzzy subgraph of $G(\sigma, \mu)$ that has a fuzzy node set τ. Evidently, this is just the fuzzy graph (τ, ρ) where $\rho(u, v) = \tau(u) \wedge \tau(v) \wedge \mu(u, v)$ for all $u, v \in V$.

Connected fuzzy graph: Two nodes that are joined by a path are said to be connected. The relation connected is a reflexive, symmetric and transitive, the equivalence classes of nodes under this relation are the connected components of the given fuzzy graph.

Fuzzy cardinality of a set: For any subset S of V and let $S \subseteq V$. Fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Order and size: The order p and size q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $\sum_{xy \in E} \mu(xy)$ respectively.

Degree: Let $G(V, \sigma, \mu)$ be a fuzzy graph. Define the degree of a vertex v to be $d(v) = \sum_{u \neq v} \mu(u, v)$. The minimum degree of G is $\delta(G) = \wedge\{d(v)/v \in V\}$ and the maximum degree of G is $\Delta(G) = \vee\{\ d(v)/v \in V\}$.

Neighbourhood degree: An edge $e = uv$ of a fuzzy graph is called an effective edge if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. $N(u) = \{v \in V\ /\mu(u, v) = \sigma(u) \wedge \sigma(v)\}$ is called the neighbourhood of u and $N[u] = N(u) \cup \{u\}$ is called the closed neighbourhood of u. $d_N(u) = \sum_{v \in N(u)} \sigma(v)$ is called the neighbourhood degree of u. The minimum neighbourhood degree of G is $\delta_N(G) = \wedge\{d_N(v)/v \in V\}$ and the maximum neighbourhood degree of G is $\Delta_N(G) = \vee\{\ d_N(v)/v \in V\}$.

Fuzzy domination number: Let $G(\sigma,\ \mu)$ be a fuzzy graph. A subset D of V is said to be a dominating set of G if for every $v \in V - D$ there exists $u \in D$ such that

$\mu(u,v) = \sigma(u) \wedge \sigma(v)$. A dominating set D of a fuzzy graph G is called the minimal dominating set of G if every node $v \in D, D - \{v\}$ is not a dominating set. The minimum scalar cardinality of D is called a domination number and it is denoted by $\gamma(G)$. Note that the scalar cardinality of a fuzzy subset D of V is $|D|_f = \sum_{v \in D} \sigma(v)$.

Example 2.1.

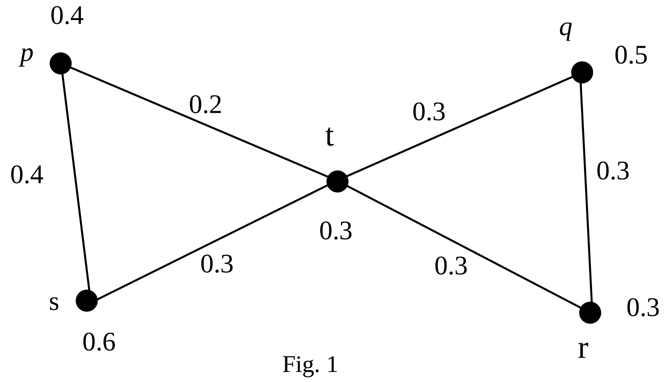

Fig. 1

In Fig. 1 D={s, q}; $\gamma(G)$=1.1.

Connected domination number: A dominating set D is a connected dominating set of a fuzzy graph G if the fuzzy subgraph $\langle D \rangle$ induced by D is connected. The minimum cardinality taken over all minimal connected dominating sets is called a connected domination number of a fuzzy graph G and it is denoted by $\gamma_c(G)$.

Set domination number: A dominating set $D \subseteq V$ of a fuzzy graph is said to be a set dominating set of a fuzzy graph if for every $T \subseteq V - D$ there exists a set $S \subseteq D$ such that $\langle S \cup T \rangle$ is connected fuzzy graph. The minimum cardinality taken over all minimal set dominating set is called a set domination number of a fuzzy graph G and it is denoted by $\gamma_s(G)$.

Point set domination number: A dominating set $D \subseteq V$ of a fuzzy graph is said to be a point set dominating set of a fuzzy graph if for every $S \subseteq V - D$ there exists a node $d \in D$ such that $\langle S \cup \{d\} \rangle$ is a connected fuzzy graph. The minimum cardinality taken over all minimal point set dominating set is called a point set domination number of a fuzzy graph G and it is denoted by $\gamma_p(G)$.

3. Main results

Theorem 3.1. Any point set dominating set of a fuzzy graph is a set dominating set of a fuzzy graph.

Proof: Let D be a point set dominating set of a fuzzy graph, for every set $T \subseteq V - D$ there exists a non empty singleton set $\{v\} \in S \subseteq D$ such that the subgraph $\langle S \cup T \rangle$ is connected. Thus D is a set dominating set of a fuzzy graph.

Note: The converse of the above theorem is not true.

Theorem 3.2. Any set dominating set of a fuzzy graph is a dominating set of a fuzzy graph.

Proof: Let D be a set dominating set of a fuzzy graph, for every set $\{x\} = T \subseteq V - D$ there exists a non empty singleton set $\{v\} = S \subseteq D$ such that the subgraph $\langle S \cup T \rangle$ is connected. Thus D is a dominating set of a fuzzy graph.

Note: The converse of the above theorem is needed not be true.

Theorem 3.3. For any fuzzy graph $G\gamma_p(G) \leq p - \Delta_N(G)$ where $\Delta_N(G)$ is the maximum neighbourhood degree of G.

Proof: Let v be the vertex with a neighbourhood degree $\Delta_N(G)$. Let S_1 be the set of all vertices adjacent to v such that $\mu(u, v) = \sigma(u) \wedge \sigma(v)\ \forall u, v \in V$. Clearly the fuzzy cardinality of $S_1 = \Delta_N(G)$. Let $D = V - S_1$. Since v is adjacent to all the vertices in S_1 for every subset $S \subseteq V - D = S_1$ the subgraph $\langle S \cup \{v\} \rangle$ is connected. Then D is a point set dominating set of a fuzzy graph G. Clearly fuzzy cardinality of $D = p - \Delta_N(G)$.

Theorem 3.4. Let D be a point set dominating set of a fuzzy graph G and $u, v \in V - D$. Then $d(u, v) \leq 2$.

Proof: Suppose u and v are adjacent, then $d(u,\ v) \leq 2$. Let u and v be not adjacent and let $S = \{u, v\}$. Since D is a point set dominating set there exists a vertex x in D such that the subgraph $\langle S \cup \{x\} \rangle$ is connected. That is the vertices u and v are adjacent to x. Therefore $d(u,\ v) \leq 2$.

Theorem 3.5. Let G be a graph with cut vertices. Then $\gamma_p(G) = \min\{p - \Delta_N(G),\ p - k\}$ where $k = \max\left\{\sum_{v \in B} \sigma(v) - \gamma_p(B)\right\}$ the maximum is taken over all the blocks B of G.

Proof: Let D be a γ_p-set of G. We consider two cases.

Case 1. The set $V - D$ contains vertices not belonging to the same block.

Let u and v be vertices in $V - D$ belonging to different blocks. Then both of them are adjacent to a cut vertex $w \in D$. Suppose not since w is a cut vertex, u and v cannot be adjacent to the same vertex other than w. Therefore $d(u, v) > 2$ which is a contradiction since $d(u,\ v) \leq 2$. Therefore u and v are adjacent to w. Further in this case $V - D$ contains only vertices belonging to the blocks at w.

Suppose there exists a vertex x in $V - D$ which belongs to a block at a cut vertex $w_1 \neq w$. Then $d(x, u) > 2$ or $d(x, v) > 2$, which again gives a contradiction to Theorem 3.4 This proves that all the vertices in $V - D$ are adjacent to the cut vertex w. Therefore, the fuzzy cardinality of $V - D \leq d_N(w)$, i.e., $\sum_{v \in D} \sigma(v) \geq p - d_N(w)$ $\therefore \gamma_p \geq p - d_N(w)$. $\therefore \gamma_p \geq p - \Delta_N(G)$. But Theorem 3.3 says that $\gamma_p \leq p - \Delta_N(G)$. Therefore $\gamma_p = p - \Delta_N(G)$.

Case 2. Let all the vertices in $V - D$ belong to a single block B of G.

Then $\sum_{v \in V-D} \sigma(v) = \sum_{v \in B} \sigma(v) - \gamma_p(B)$.

Therefore $\gamma_p = p - \left(\sum_{v\in B} \sigma(v) - \gamma_p(B)\right) \geq p - k$. A set D is the set of all vertices in each block together with the block which contains a minimum point set dominating set of G. Hence $\gamma_p(G) \leq p - k$. Thus $\gamma_p(G) = p - k$. By case 1, $\gamma_p(G) = \min\{p - \Delta_N(G),\ p - k\}$. Hence the proof.

4. Conclusion

In this paper we have introduced the parameter point set domination in fuzzy graphs. Some interesting results related with the above are proved. Further, the authors proposed to introduce new dominating parameters in a fuzzy graph and apply these concepts to the fuzzy graph models. The results obtained can be applied in various areas of engineering, computer science, computer networks, expert systems, and medical diagnosis.

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