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APPLICATIONS OF INTERPOLATION OPERATORS

Alina BABOŞ
alina_babos_24@yahoo.com

"NICOLAE BĂLCESCU" LAND FORCES ACADEMY, SIBIU, ROMANIA

ABSTRACT

We use Lagrange, Hermite and Birkhoff operator that interpolates a function f and certain of its derivatives, defined on a triangle, for construction of some surfaces. We construct this type of surfaces using concrete examples.

Keywords:

Operator, interpolation, blending approximation

1. Introduction

The blending interpolation has many practical applications. Remind that blending interpolation is to interpolate a function at an infinite set of points: segments, curves, surfaces, etc. Thus, if one gives the contour of an object by such elements (segments, curves, surfaces) using a blending interpolation, we can generate a surface that contains the given contour. Hence, we can construct a surface (a blending function interpolant) which matches a given function and certain on its derivatives on the boundary of a plane domain (rectangle, triangle, etc).

Using such a surface fitting technique, we construct some surfaces in given interpolation conditions with concrete examples.

For this, will be used Lagrange, Hermite and Birkhoff operator that interpolates a function f and certain of its derivatives, defined on a given domain $\Omega \subset R^2$.

2. Surfaces Generation

Let Ω be the domain $D_h = \{(x,y) \in R^2 \mid |x| + |y| \le h\}, h > 0$ (Figure no. 1)

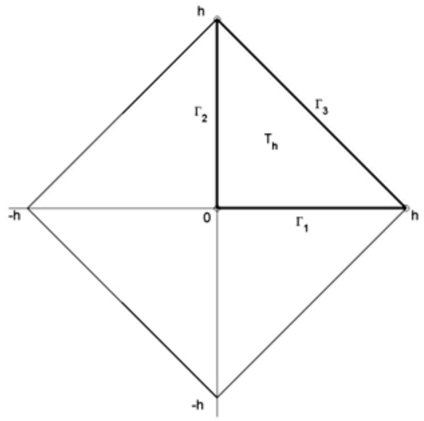

Figure no.1 Domain D_h

Such surface is constructed, first, on the standard triangle

$$T_h = \{(x, y) \in R^2 \mid x \ge 0, y \ge 0, x + y \le h\},$$

after that it is extended, by symmetry, with respect to the coordinate axes, on all domain D_h.

The operators used are: – Lagrange's operators: P_1^x and P_1^y given by

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$$(P_1^x f)(x,y)=\frac{h-x-y}{h-y}f(0,y)+\frac{x}{h-y}f(h-y,y),$$

$$(P_1^y f)(x,y)=\frac{h-x-y}{h-x}f(x,0)+\frac{y}{h-x}f(x,h-x);$$

(1)

— Hermite operators H_1^{xy} defined by

$$(H_1^{xy}f)(x,y)=\frac{y^2(3x+y)}{(x+y)^3}f(0,x+y)+$$
$$+\frac{xy^2}{(x+y)^2}(f^{(1,0)}-f^{(0,1)})(0,x+y)+$$
$$+\frac{x^2(x+3y)}{(x+y)^3}f(x+y,0)-$$
$$-\frac{x^2y}{(x+y)^2}(f^{(1,0)}-f^{(0,1)})(x+y,0);$$

(2)

— Birkhoff operators B_1^x and B_1^y defined by

$$(B_1^x f)(x,y)=f(0,y)+(x+y-h)f^{(1,0)}(h-y,y),$$
$$(B_1^y f)(x,y)=f(x,0)+(x+y-h)f^{(0,1)}(x,h-x).$$

(3)

For the beginning, we construct a scalar interpolating formula generated by the operators P_1^x, P_1^y and B_1^x, B_1^y, H_1^{xy}, using two levels of interpolation.

First, the function f is approximated by the Boolean sum of the operators P_1^x and P_1^y

$$(P_1^x\oplus P_2^y f)(x,y)=\frac{h-x-y}{h-y}f(0,y)+$$
$$+\frac{h-x-y}{h-x}f(x,0)+$$
$$+\frac{y}{h-x}f(x,h-x)-$$
$$-\frac{h-x-y}{h}f(0,0)-$$
$$-\frac{y(h-x-y)}{h(h-y)}f(0,h).$$

(4)

In order to obtain a scalar approximant of f, we use in the second level the following approximations $f(0,y)\approx(B_1^y f)(0,y)$, $f(x,0)\approx(B_1^x f)(x,0)$, $f(x,h-x)\approx(H_3^{xy})(x,h-x)$.

One obtains

$$F(x,y)=\frac{(h-x-y)(h^2-xy)}{h(h-x)(h-y)}f(0,0)+$$
$$+\left[\frac{y(h-x)(2x+h)}{h^3}-\frac{y(h-x-y)}{h(h-y)}\right]f(0,h)+$$
$$+\frac{x^2y(3h-2x)}{(h-x)h^3}f(h,0)-$$
$$-\frac{h^2(h-x-y)+xy(h-x)}{h^2}f^{(0,1)}(0,h)+$$
$$+\frac{x^2y}{h^2}f^{(0,1)}(h,0)-$$
$$-\frac{h^2(h-x-y)+x^2y}{h^2}f^{(1,0)}(h,0)+$$
$$+\frac{xy(h-x)}{h^2}f^{(1,0)}(0,h),$$

(5)

that uses the information

$$\Lambda_F(f)=\{f(0,0),f(0,h),f(h,0),\ f^{(0,1)}(0,h),f^{(0,1)}(h,0),\ f^{(1,0)}(0,h),f^{(1,0)}(h,0)\}.$$

Example no. 1:

For $\Lambda_F(f)=\{4,0,0,0,1,1,0\}$, one obtains

$$F_1(x,y)=\frac{4(h-x-y)(h^2-xy)}{h(h-x)(h-y)}+\frac{x^2y}{h^2}+\frac{xy(h-x)}{h^2}$$

(Figure no. 2)

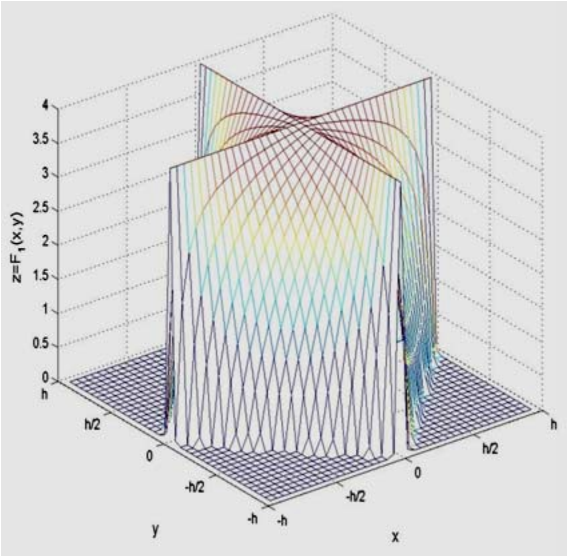

Figure no. 2 $F_1(x,y)$

Example no. 2: The following function

$$F_2(x,y)=\frac{4(h-x-y)(h^2-xy)}{h(h-x)(h-y)}-$$
$$-\frac{x^2y(3h-2x)}{4h^3(h-x)}-\frac{x^2y}{2h^2}-\frac{xy(h-x)}{2h^2}$$

is obtained from (5) for $\Lambda_F(f)=\{4,0,-1/4,0,-1/2,-1/2,0\}$ (Figure no. 3)

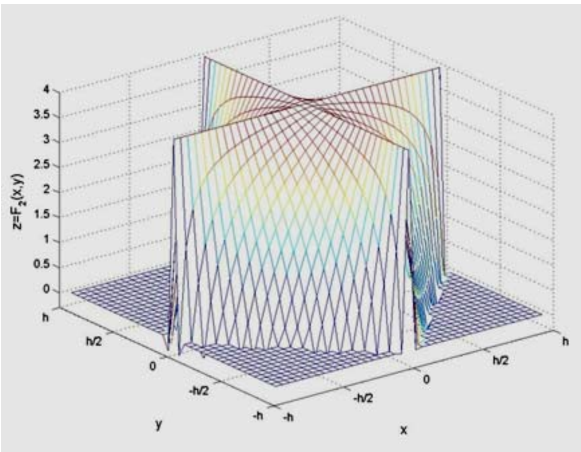

Figure no. 3 $F_2(x,y)$

Now, one supposes that $f\big|_{\partial D_h}=0$, which is equivalent with $f(x,h-x)=0, x\in[0,h]$.

In this conditions (4) becomes

$$L(x,y)=\frac{h-x-y}{h-y}f(0,y)$$
$$+\frac{h-x-y}{h-x}f(x,0)-\frac{h-x-y}{h}f(0,0). \tag{6}$$

Taking $f(0,y)=(B_1^y f)(0,y)$ and $f(x,0)=(B_1^x f)(x,0)$, in the same conditions $f(x,h-x)=0, x\in[0,h]$, one obtains

$$H(x,y)=\frac{(h-x-y)(h^2-xy)}{h(h-x)(h-y)}f(0,0)-$$
$$-(h-x-y)f^{(0,1)}(0,h)-(h-x-y)f^{(1,0)}(h,0) \tag{7}$$

with

$$\Lambda_H(f)=\left\{f(0,0),f^{(0,1)}(0,h),f^{(1,0)}(h,0)\right\}.$$

Example no. 3: For $\Lambda_H(f)=\{4,0,0\}$, one obtains the surface from the Figure no. 4.

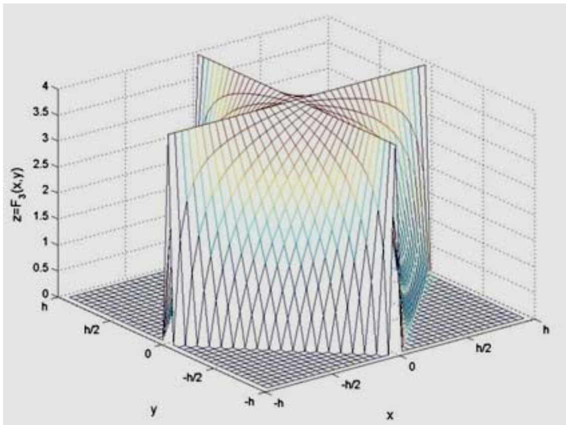

Figure no. 4 $H(x,y)$

3. Conclusions

Interpolation operators have many applications. We used Lagrange, Hermite and Birkhoff operator for construction of some surfaces using blending interpolation. We constructed this type of surfaces using concrete examples.

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