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Derivation of Some Results on the Generalized Relative Orders of Meromorphic Functions

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Abstract. In this paper we intend to find out relative order (relative lower order) of a meromorphic function f with respect to another entire function g when generalized relative order (generalized relative lower order) of f and generalized relative order (generalized relative lower order) of g with respect to another entire function h are given.

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1 Introduction, Definitions and Notations

Let f be an entire function defined in the finite complex plane $\mathbb{C}$. The maximum modulus function corresponding to entire f is defined as $M_f(r) = \max\{|f(z)| : |z| = r\}$. While f is meromorphic, one may define a different function $T_f(r)$ famous as Nevanlinna's Characteristic function of f, playing same role as maximum modulus function in the following manner:

$$T_f(r) = N_f(r) + m_f(r),$$

where the function $N_f(r,a)$ $\left(\bar{N}_f(r,a)\right)$ known as counting function of a-points (distinct a-points) of meromorphic f is defined as

$$N_f(r,a) = \int_0^r \frac{n_f(t,a) - n_f(0,a)}{t} dt + n_f(0,a)\log r$$

$$\left(\bar{N}_f(r,a) = \int_0^r \frac{\bar{n}_f(t,a) - \bar{n}_f(0,a)}{t} dt + \bar{n}_f(0,a)\log r\right),$$

moreover we denote by $n_f(r,a)$ $\left(\bar{n}_f(r,a)\right)$ the number of a-points (distinct a-points) of f in $|z| \leq r$ and an ∞ -point is a pole of f. In many occasions $N_f(r,\infty)$ and $\bar{N}_f(r,\infty)$ are denoted by $N_f(r)$ and $\bar{N}_f(r)$ respectively.

Also the function $m_f(r,\infty)$ alternatively denoted by $m_f(r)$ known as the proximity function of f is defined as follows:

$$m_f(r) = \frac{1}{2\pi}\int_0^{2\pi} \log^+ \left|f\left(re^{i\theta}\right)\right| d\theta, \quad \text{where}$$

$$\log^+ x = \max(\log x, 0) \text{ for all } x \geqslant 0 .$$

Also we may denote $m\left(r, \frac{1}{f-a}\right)$ by $m_f(r,a)$.

If f is entire function, then the Nevanlinna's Characteristic function $T_f(r)$ of f is defined as

$$T_f(r) = m_f(r) .$$

However, the study of comparative growth properties of entire and meromorphic functions which is one of the prominent branch of the value distribution theory of entire and meromorphic functions is the prime concern of the paper. We do not explain the standard definitions and notations in the theory of entire and meromorphic functions as those are available in [11] and [14]. In the sequel the following two notations are used:

$$\begin{aligned} \log^{[k]} x &= \log\left(\log^{[k-1]} x\right) \text{ for } k = 1,2,3,\cdots; \\ \log^{[0]} x &= x \end{aligned}$$

and

$$\begin{aligned} \exp^{[k]} x &= \exp\left(\exp^{[k-1]} x\right) \text{ for } k = 1,2,3,\cdots; \\ \exp^{[0]} x &= x. \end{aligned}$$

Taking this into account the *generalized order* (respectively, *generalized lower order*) of an entire function f as introduced by Sato [13] is given by:

$$\rho_f^{[l]} = \limsup_{r\to\infty} \frac{\log^{[l]} M_f(r)}{\log\log M_{\exp z}(r)} = \limsup_{r\to\infty} \frac{\log^{[l]} M_f(r)}{\log r}$$

$$\left(\text{respectively } \lambda_f^{[l]} = \liminf_{r\to\infty} \frac{\log^{[l]} M_f(r)}{\log\log M_{\exp z}(r)} = \liminf_{r\to\infty} \frac{\log^{[l]} M_f(r)}{\log r} \right)$$

where $l \geq 1$.

When f is meromorphic function, one can easily verify that

$$\rho_f^{[l]} = \limsup_{r\to\infty} \frac{\log^{[l-1]} T_f(r)}{\log T_{\exp z}(r)} = \limsup_{r\to\infty} \frac{\log^{[l-1]} T_f(r)}{\log\left(\frac{r}{\pi}\right)} = \limsup_{r\to\infty} \frac{\log^{[l-1]} T_f(r)}{\log r + O(1)}$$

$$\left(\text{respectively } \lambda_f^{[l]} = \liminf_{r\to\infty} \frac{\log^{[l-1]} T_f(r)}{\log T_{\exp z}(r)} = \liminf_{r\to\infty} \frac{\log^{[l-1]} T_f(r)}{\log r + O(1)} \right)$$

where $l \geq 1$.

These definitions extend the definitions of *order* ρ_f and *lower order* λ_f of an entire or meromorphic function f since for $l = 2$, these correspond to the particular case $\rho_f^{[2]} = \rho_f$ and $\lambda_f^{[2]} = \lambda_f$.

Given a non-constant entire function g defined in the open complex plane $\mathbb{C}$, its Nevanlinna's Characteristic function $T_g(r)$ is strictly increasing and continuous functions of r. Also the inverse $T_g^{-1} : (T_g(0), \infty) \to (0, \infty)$ exists and is such that $\lim_{s\to\infty} T_g^{-1}(s) = \infty$.

Extending the idea of relative order of entire functions as established by Bernal $\{[1], [2]\}$, Lahiri and Banerjee [12] introduced the definition of relative order of a meromorphic function f with respect to another entire function g, denoted by $\rho_g(f)$ to avoid comparing growth just with $\exp z$ as follows:

$$\begin{aligned} \rho_g(f) &= \inf\{\mu > 0 : T_f(r) < T_g(r^\mu) \text{ for all sufficiently large } r\} \\ &= \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} . \end{aligned}$$

The definition coincides with the classical one if $g(z) = \exp z$ {cf. [12] }.

Likewise, one can define the relative lower order of a meromorphic function f with respect to an entire function g denoted by $\lambda_g(f)$ as follows :

$$\lambda_g(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} .$$

Debnath et. al. [3] gave a more generalized concept of relative order a meromorphic function with respect to an entire function in the following way :

Definition 1.1. [3] *Let f be any meromorphic function and g be any entire function with index-pairs (m_1, q) and (m_2, p) respectively where $m_1 = m_2 = m$ and p, q, m are all positive integers such that $m \geq p$ and $m \geq q$. Then the relative (p, q) th order of f with respect to g is defined as*

$$\rho_g^{(p,q)}(f) == \limsup_{r\to\infty} \frac{\log^{[p]} T_g^{-1} T_f(r)}{\log^{[q]} r} .$$

For details about index-pair of meromorphic function, one may see [3].

When $p = l \geq 1$ and $q = 1$, the above definition reduces to the definition of generalized relative order *of a meromorphic function f with respect to an entire function g, denoted by $\rho_g^{[l]}(f)$ which is as follows*

$$\rho_g^{[l]}(f) = \limsup_{r\to\infty} \frac{\log^{[l]} T_g^{-1} T_f(r)}{\log r} .$$

Likewise one can define the generalized relative lower order *of a meromorphic function f with respect to an entire function g denoted by $\lambda_g^{[l]}(f)$ as*

$$\lambda_g^{[l]}(f) = \liminf_{r\to\infty} \frac{\log^{[l]} T_g^{-1} T_f(r)}{\log r} .$$

For entire and meromrophic functions, the notions of their growth indicators such as *order* is classical in complex analysis and during the past decades, several researchers have already been exploring their studies in the area of comparative growth properties of composite entire and meromorphic functions in different directions using the classical growth indicators. But at that time, the concepts of *relative orders* and consequently the *generalized relative orders* of entire and meromorphic functions with respect to another entire function and as well as their technical advantages of not comparing with the growths of $\exp z$ are not at all known to the researchers of this area. Therefore the growth of composite entire and meromorphic functions needs to be modified on the basis of their *relative order* some of which has been explored in [4], [5], [6], [7], [8], [9] and [10]. In this paper we establish some newly developed results related to the growth rates of entire and meromorphic functions on the basis of their *relative orders (* respectively *relative lower orders)* and *generalized relative orders (* respectively *generalized relative lower orders).*

2 Theorem

In this section we present the main results of the paper.

Theorem 2.1. *Let f be a meromorphic function and g and h be any two entire functions such that $0 < \lambda_h^{[l]}(f) \leq \rho_h^{[l]}(f) < \infty$ and $0 < \lambda_h^{[l]}(g) \leq \rho_h^{[l]}(g) < \infty$ where $l \geq 1$. Then*

$$\frac{\lambda_h^{[l]}(f)}{\rho_h^{[l]}(g)} \leq \lambda_g(f) \leq \min\left\{\frac{\lambda_h^{[l]}(f)}{\lambda_h^{[l]}(g)}, \frac{\rho_h^{[l]}(f)}{\rho_h^{[l]}(g)}\right\}$$
$$\leq \max\left\{\frac{\lambda_h^{[l]}(f)}{\lambda_h^{[l]}(g)}, \frac{\rho_h^{[l]}(f)}{\rho_h^{[l]}(g)}\right\} \leq \rho_g(f) \leq \frac{\rho_h^{[l]}(f)}{\lambda_h^{[l]}(g)}.$$

Proof. From the definitions of $\rho_h^{[l]}(f)$ and $\lambda_h^{[l]}(f)$, we have for all sufficiently large values of r that

$$T_f(r) \leq T_h\left[\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)+\varepsilon\right)\log r\right\}\right], \tag{2.1}$$
$$T_f(r) \geq T_h\left[\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)-\varepsilon\right)\log r\right\}\right] \tag{2.2}$$

and also for a sequence of values of r tending to infinity, we get that

$$T_f(r) \geq T_h\left[\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)-\varepsilon\right)\log r\right\}\right], \tag{2.3}$$
$$T_f(r) \leq T_h\left[\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)+\varepsilon\right)\log r\right\}\right]. \tag{2.4}$$

Similarly from the definitions of $\rho_h^{[l]}(g)$ and $\lambda_h^{[l]}(g)$, it follows for all sufficiently large values of r that

$$\begin{aligned} T_h^{-1}T_g(r) &\leq \exp^{[l]}\left\{\left(\rho_h^{[l]}(g)+\varepsilon\right)\log r\right\} \\ \text{i.e., } T_g(r) &\leq T_h\left[\exp^{[l]}\left\{\left(\rho_h^{[l]}(g)+\varepsilon\right)\log r\right\}\right] \\ \text{i.e., } T_h(r) &\geq T_g\left[\exp\left\{\frac{\log^{[l]} r}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}\right\}\right], \end{aligned} \tag{2.5}$$

$$\begin{aligned} T_h^{-1}T_g(r) &\geq \exp^{[l]}\left\{\left(\lambda_h^{[l]}(g)-\varepsilon\right)\log r\right\} \\ \text{i.e., } T_h(r) &\leq T_g\left[\exp\left\{\frac{\log^{[l]} r}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)}\right\}\right] \end{aligned} \tag{2.6}$$

and for a sequence of values of r tending to infinity we obtain that

$$
\begin{aligned}
T_h^{-1}T_g(r) &\geq \exp^{[l]}\left\{\left(\rho_h^{[l]}(g)-\varepsilon\right)\log r\right\}\\
i.e.\ T_h(r) &\leq T_g\left[\exp\left\{\frac{\log^{[l]} r}{\left(\rho_h^{[l]}(g)-\varepsilon\right)}\right\}\right], \qquad (2.7)
\end{aligned}
$$

$$
\begin{aligned}
T_h^{-1}T_g(r) &\leq \exp^{[l]}\left\{\left(\lambda_h^{[l]}(g)+\varepsilon\right)\log r\right\}\\
i.e.,\ T_h(r) &\geq T_g\left[\exp\left\{\frac{\log^{[l]} r}{\left(\lambda_h^{[l]}(g)+\varepsilon\right)}\right\}\right]. \qquad (2.8)
\end{aligned}
$$

Now from (2.3) and in view of (2.5), we get for a sequence of values of r tending to infinity that

$$T_g^{-1}T_f(r) \geq T_g^{-1}T_h\left[\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)-\varepsilon\right)\log r\right\}\right]$$

$$
\begin{aligned}
i.e.,\ T_g^{-1}T_f(r) &\geq T_g^{-1}T_g\left[\exp\left\{\frac{\log^{[l]}\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)-\varepsilon\right)\log r\right\}}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}\right\}\right]\\
&= \exp\left\{\frac{\left(\rho_h^{[l]}(f)-\varepsilon\right)}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}\log r\right\}
\end{aligned}
$$

$$i.e.,\ \frac{\log T_g^{-1}T_f(r)}{\log r} \geq \frac{\left(\rho_h^{[l]}(f)-\varepsilon\right)}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}.$$

As $\varepsilon > 0$ is arbitrary, it follows that

$$\rho_g(f) \geq \frac{\rho_h^{[l]}(f)}{\rho_h^{[l]}(g)}. \qquad (2.9)$$

Analogously from (2.2) and in view of (2.8), it follows for a sequence of values of r tending to infinity that

$$T_g^{-1}T_f(r) \geq T_g^{-1}T_h\left[\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)-\varepsilon\right)\log r\right\}\right]$$

$$
\begin{aligned}
i.e., \ T_g^{-1}T_f(r) &\geq T_g^{-1}T_g\left[\exp\left\{\frac{\log^{[l]}\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)-\varepsilon\right)\log r\right\}}{\left(\lambda_h^{[l]}(g)+\varepsilon\right)}\right\}\right] \\
&= \exp\left\{\frac{\left(\lambda_h^{[l]}(f)-\varepsilon\right)}{\left(\lambda_h^{[l]}(g)+\varepsilon\right)}\log r\right\}
\end{aligned}
$$

$$
i.e., \ \frac{\log T_g^{-1}T_f(r)}{\log r} \geq \frac{\left(\lambda_h^{[l]}(f)-\varepsilon\right)}{\left(\lambda_h^{[l]}(g)+\varepsilon\right)} .
$$

Since $\varepsilon\,(>0)$ is arbitrary, we get from above that

$$
\rho_g(f) \geq \frac{\lambda_h^{[l]}(f)}{\lambda_h^{[l]}(g)} . \tag{2.10}
$$

Again in view of (2.6), we have from (2.1), for all sufficiently large values of r that

$$
T_g^{-1}T_f(r) \leq T_g^{-1}T_h\left[\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)+\varepsilon\right)\log r\right\}\right]
$$

$$
\begin{aligned}
i.e., \ T_g^{-1}T_f(r) &\leq T_g^{-1}T_g\left[\exp\left\{\frac{\log^{[l]}\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)+\varepsilon\right)\log r\right\}}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)}\right\}\right] \\
&= \exp\left\{\frac{\left(\rho_h^{[l]}(f)+\varepsilon\right)}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)}\log r\right\}
\end{aligned}
$$

$$
i.e., \ \frac{\log T_g^{-1}T_f(r)}{\log r} \leq \frac{\left(\rho_h^{[l]}(f)+\varepsilon\right)}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)} .
$$

Since $\varepsilon\,(>0)$ is arbitrary, we obtain that

$$
\rho_g(f) \leq \frac{\rho_h^{[l]}(f)}{\lambda_h^{[l]}(g)} . \tag{2.11}
$$

Again from (2.2) and in view of (2.5), it follows for all sufficiently large values of r that

$$
T_g^{-1}T_f(r) \geq T_g^{-1}T_h\left[\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)-\varepsilon\right)\log r\right\}\right]
$$

$$
\begin{aligned}
i.e.,\ T_g^{-1}T_f(r) &\geq T_g^{-1}T_g\left[\exp\left\{\frac{\log^{[l]}\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)-\varepsilon\right)\log r\right\}}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}\right\}\right]\\
&= \exp\left\{\frac{\left(\lambda_h^{[l]}(f)-\varepsilon\right)}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}\log r\right\}
\end{aligned}
$$

$$
i.e.,\ \frac{\log T_g^{-1}T_f(r)}{\log r} \geq \frac{\left(\lambda_h^{[l]}(f)-\varepsilon\right)}{\left(\rho_h^{[l]}(g)+\varepsilon\right)}\ .
$$

Since $\varepsilon\,(>0)$ is arbitrary, we get from above that

$$
\lambda_g(f) \geq \frac{\lambda_h^{[l]}(f)}{\rho_h^{[l]}(g)}\ . \tag{2.12}
$$

Also in view of (2.7), we get from (2.1) for a sequence of values of r tending to infinity that

$$
T_g^{-1}T_f(r) \leq T_g^{-1}T_h\left[\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)+\varepsilon\right)\log r\right\}\right]
$$

$$
\begin{aligned}
i.e.,\ T_g^{-1}T_f(r) &\leq T_g^{-1}T_g\left[\exp\left\{\frac{\log^{[l]}\exp^{[l]}\left\{\left(\rho_h^{[l]}(f)+\varepsilon\right)\log r\right\}}{\left(\rho_h^{[l]}(g)-\varepsilon\right)}\right\}\right]\\
&= \exp\left\{\frac{\left(\rho_h^{[l]}(f)+\varepsilon\right)}{\left(\rho_h^{[l]}(g)-\varepsilon\right)}\log r\right\}
\end{aligned}
$$

$$
i.e.,\ \frac{\log T_g^{-1}T_f(r)}{\log r} \leq \frac{\left(\rho_h^{[l]}(f)+\varepsilon\right)}{\left(\rho_h^{[l]}(g)-\varepsilon\right)}\ .
$$

Since $\varepsilon\,(>0)$ is arbitrary, we get from above that

$$
\lambda_g(f) \leq \frac{\rho_h^{[l]}(f)}{\rho_h^{[l]}(g)}\ . \tag{2.13}
$$

Similarly from (2.4) and in view of (2.6), it follows for a sequence of values of r tending to infinity that

$$
T_g^{-1}T_f(r) \leq T_g^{-1}T_h\left[\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)+\varepsilon\right)\log r\right\}\right]
$$

$$i.e.,\ T_g^{-1}T_f(r) \leq T_g^{-1}T_g\left[\exp\left\{\frac{\log^{[l]}\exp^{[l]}\left\{\left(\lambda_h^{[l]}(f)+\varepsilon\right)\log r\right\}}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)}\right\}\right]$$
$$= \exp\left\{\frac{\left(\lambda_h^{[l]}(f)+\varepsilon\right)}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)}\log r\right\}$$

$$i.e.,\ \frac{\log T_g^{-1}T_f(r)}{\log r} \leq \frac{\left(\lambda_h^{[l]}(f)+\varepsilon\right)}{\left(\lambda_h^{[l]}(g)-\varepsilon\right)}\ .$$

As $\varepsilon\,(>0)$ is arbitrary, we obtain from above that

$$\lambda_g(f) \leq \frac{\lambda_h^{[l]}(f)}{\lambda_h^{[l]}(g)}\ . \tag{2.14}$$

The theorem follows from (2.9), (2.10), (2.11), (2.12), (2.13) and (2.14). □

Corollary 2.2. *Let f be a meromorphic function and g and h be any two entire functions such that $0 < \lambda_h^{[l]}(f) \leq \rho_h^{[l]}(f) < \infty$ and $0 < \lambda_h^{[l]}(g) = \rho_h^{[l]}(g) < \infty$ where $l \geq 1$. Then*

$$\lambda_g(f) = \frac{\lambda_h^{[l]}(f)}{\rho_h^{[l]}(g)} \quad \textit{and} \quad \rho_g(f) = \frac{\rho_h^{[l]}(f)}{\rho_h^{[l]}(g)}\ .$$

In addition, if $\rho_h^{[l]}(f) = \rho_h^{[l]}(g)$, then

$$\rho_g(f) = 1\ .$$

Corollary 2.3. *Let f be a meromorphic function and g and h be any two entire functions such that $0 < \lambda_h^{[l]}(f) = \rho_h^{[l]}(f) < \infty$ and $0 < \lambda_h^{[l]}(g) = \rho_h^{[l]}(g) < \infty$ where $l \geq 1$. Then*

$$\lambda_g(f) = \rho_g(f) = \frac{\rho_h^{[l]}(f)}{\rho_h^{[l]}(g)}\ .$$

Corollary 2.4. *Let f be a meromorphic function and g and h be any two entire functions such that $0 < \lambda_h^{[l]}(f) = \rho_h^{[l]}(f) = \lambda_h^{[l]}(g) = \rho_h^{[l]}(g) < \infty$ where $l \geq 1$. Then*

$$\lambda_g(f) = \rho_g(f) = 1\ .$$

Corollary 2.5. *Let f be a meromorphic function and g and h be any two entire functions such that $0 < \lambda_h^{[l]}(f) < \rho_h^{[l]}(f) < \infty$ where $l \geq 1$. Then*

$$\begin{aligned} (i)\ \lambda_g(f) &= \infty \text{ when } \rho_h^{[l]}(g) = 0\ , \\ (ii)\ \rho_g(f) &= \infty \text{ when } \lambda_h^{[l]}(g) = 0\ , \\ (iii)\ \lambda_g(f) &= 0 \text{ when } \rho_h^{[l]}(g) = \infty \end{aligned}$$

and

$$(iv)\ \rho_g(f) = 0 \text{ when } \lambda_h^{[l]}(g) = \infty\ .$$

Corollary 2.6. *Let f be a meromorphic function and g and h be any two entire functions such that $0 < \lambda_h^{[l]}(g) = \rho_h^{[l]}(g) < \infty$ where $l \geq 1$. Then*

$$\begin{aligned} (i)\ \rho_g(f) &= 0 \text{ when } \rho_h^{[l]}(f) = 0\ , \\ (ii)\ \lambda_g(f) &= 0 \text{ when } \lambda_h^{[l]}(f) = 0\ , \\ (iii)\ \rho_g(f) &= \infty \text{ when } \rho_h^{[l]}(f) = \infty \end{aligned}$$

and

$$(iv)\ \lambda_g(f) = \infty \text{ when } \lambda_h^{[l]}(f) = \infty\ .$$

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