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Report of Meeting

16th International Conference on Functional Equations and Inequalities, Będlewo, Poland, May 17-23, 2015

Contents

The **16th International Conference on Functional Equations and Inequalities** (16th ICFEI) was held in Będlewo (Poland) in the Mathematical Research and Conference Center (MRCC), on May 17–23, 2015. It was organized by the Department of Mathematics of the Pedagogical University of Cracow, in a cooperation with Faculty of Applied Mathematics of AGH University of Science and Technology and with organizational and financial supports of MRCC, Stefan Banach International Mathematical Center and Warsaw Center of Mathematics and Computer Science.

The Scientific Committee of the 16th ICFEI consisted of Professors: Nicole Brillouët-Belluot (France), Dobiesław Brydak (Poland) – honorary chairman, Janusz Brzdęk (Poland) – chairman, Jacek Chmieliński (Poland), Roman Ger (Poland), Zsolt Páles (Hungary), Ekaterina Shulman (Russia), László Székelyhidi (Hungary), Marek Cezary Zdun (Poland).

The Organizing Committee consisted of Janusz Brzdęk (chairman), Krzysztof Ciepliński (co-chairman), Anna Bahyrycz (vice-chairman), Magdalena Piszczek (vice-chairman), Zbigniew Leśniak (scientific secretary), Jolanta Olko (scientific secretary), Paweł Solarz (technical support), Janina Wiercioch.

The 77 participants came from 19 countries: Austria (4 participants), China (2), Croatia (1), Denmark (1), Finland (1), France (1), Germany (1), Japan (1), Lithuania (1), Hungary (5), Iran (2), Romania (4), Serbia (1), Slovakia (1), Spain (1), Sweden (1), Thailand (4), United Kingdom (1) and Poland (44).

The conference was opened on Monday, May 18, by Professor Janusz Brzdęk, the Chairman of the Scientific and Organizing Committees, who welcomed the participants on behalf of the Organizing Committee. The opening address was given by Professor Krzysztof Ciepliński, co-Chairman of the Organizing Committee.

During 24 scientific sessions, 69 talks were presented; three of them were plenary lectures (50-minutes long) delivered by Professors Adam Ostaszewski, László Székelyhidi and Pavol Zlatoš. The talks were devoted mainly to functional equations and inequalities, their applications in various branches of mathematics and in other scientific disciplines, and numerous related topics. In particular, differential, integral and integro-differential equations (also fractional) were discussed. The presented talks concerned, among others, classical functional equations (e.g., of Cauchy, Jensen, d'Alembert, Gołąb-Schinzel), Ulam's type stability, means, conditional and alternative equations, difference equations, equations and inclusions for set-valued functions, dynamical systems, iteration theory (in particular, iterative roots), different kinds of convexity and various applications in science and technology. During special Problems and Remarks sessions, several open problems and remarks were presented. On Tuesday, May 19, the Ulam's Type Stability Day was organized on the occasion of the 75th anniversary of Ulam's Problem.

On Tuesday night a picnic was organized. On the next day afternoon the participants visited Poznań and the Poznań Croissant Museum. In the evening the piano recitals were performed by Professors Marek Czerni and László Székelyhidi. On Thursday, May 21, a banquet was held.

At the end of the conference, Professors Krzysztof Ciepliński, Dorian Popa and Henrik Stetkær were invited to become members of the Scientific Committee and they accepted the invitation.

The conference was closed on Saturday, May 23, by Professor Janusz Brzdęk.

Abstracts of Talks

Shoshana Abramovich *Quasiconvexity and the Jensen gap*

For some classes of functions φ we consider the Jensen gap

$$J(\varphi,\mu,f)=\int_\Omega \varphi(f(s))\,d\mu(s)-\varphi\biggl(\int_\Omega f(s)\,d\mu(s)\biggr),$$

where (μ,Ω) is a probability measure space. We prove several equalities and inequalities which help us to get new estimates of the upper and lower bounds of the Jensen gap and to compare them with other results of this type. We treat also discrete results of the Jensen gap.

Marcin Adam *Drygas functional equation in the class of differentiable functions*

Let Y be a real Banach space. Denote by $C^n(\mathbb{R}, Y)$ the class of n-times continuously differentiable functions $f\colon \mathbb{R} \to Y$. Applying some results on the quadratic and Cauchy differences [1, 2] we prove that if the Drygas difference $Df(x,y) := f(x+y) + f(x-y) - 2f(x) - f(y) - f(-y) \in C^n(\mathbb{R} \times \mathbb{R}, Y)$, then there exist a unique additive function $A\colon \mathbb{R} \to Y$ and a unique quadratic function $Q\colon \mathbb{R} \to Y$ such that $f - A - Q \in C^n(\mathbb{R}, Y)$. Moreover, some stability result for the Drygas functional equation in the class of differentiable functions is also presented.

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Nutefe Kwami Agbeko *The validity of Carathéodory's Theorem for some lattice-valued homomorphisms*

Replacing in the definition of outer measure the addition with the supremum operation we obtained a new set function called *retuo measure* (cf. [2]). We showed that *a real-valued set function defined on a power set is a retuo measure if and only if it is a σ-maxitive measure on the power set.* (A σ-maxitive measure is a real-valued homomorphism Φ defined on a σ-algebra which satisfies the following axioms: $\Phi(\emptyset) = 0$, $\Phi(A \cup B) = \sup\{\Phi(A); \Phi(B)\}$ and Φ is continuous from below.) We could construct retuo measure the same way outer measure can be constructed. Constructed retuo measure never exceeds its corresponding constructed outer measure. Consequently, we proved that *if Φ is a σ-maxitive measure defined on a σ-algebra $\mathcal{F}$, then $\mathcal{F}$ contains some sub-σ-algebra on which Φ is continuous from above.* As theoretical application, we provided some corresponding Hausdorff measure and dimension. (For the precise definition – or its weaker form – and further developments of σ-maxitive measures see [1, 3, 4, 5], say.)

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Pekka Alestalo *On the sharpness of the Hyers-Ulam theorem for bounded sets in finite dimensions*

Let $A \subset \mathbb{R}^n$ be a bounded set and let $f\colon A \to \mathbb{R}^n$ satisfy

$$\big| \|f(x) - f(y)\| - \|x - y\| \big| \le \varepsilon$$

for all $x, y \in A$. In many cases, there is an isometry T satisfying

$$\sup\{\|T(x) - f(x)\| \mid \ x \in A\} \le c\varepsilon;$$

cf. the references. I will discuss the question, how the constant c depends on the dimension n in different cases.

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Giedrius Alkauskas *The projective translation equation*

The general "affine" translation equation is a functional equation of the type [1]

$$F(\mathbf{x}, s+t) = F(F(\mathbf{x}, s), t), \qquad \mathbf{x} \in \mathbb{C}^n,\ s, t \in \mathbb{C}.$$

In this talk we describe the theory of the newly found projective version of this equation. Namely, *the projective translation equation* is the equation of the type

$$(1-z)\phi(\mathbf{x}) = \phi\Big(\phi(\mathbf{x}z)\frac{1-z}{z}\Big). \tag{1}$$

So far, we tackled this equation in the 2-dimensional case. Thus, in the equation (1), $\mathbf{x} = x \bullet y$, and $\phi(x, y) = u(x, y) \bullet v(x, y)$ is a pair of functions in two real or complex variables (here and below $a \bullet b$, stands for (a, b)).

This equation can be investigated from the point of view of topology of higher dimensional spheres and other compact surfaces [2], birational geometry [3], algebraic geometry, finite fields, group representations, special hypergeometric, elliptic or abelian functions, rational function fields [4]. All the results are of the classification type. We define the requirements for the solutions - for example, they should be continuous functions on a single point compactification of the sphere, rational functions, unramified flows, highly symmetric flows, and so on, and give the result which classifies all such flows. Thus, in the case of birational geometry, the results reads as follows.

Theorem
Let $\phi(x, y) = u(x, y) \bullet v(x, y)$ be a pair of rational functions in $\mathbb{R}(x, y)$ which satisfies the functional equation (1) *and the boundary condition*

$$\lim_{z \to 0} \frac{\phi(\mathbf{x}z)}{z} = \mathbf{x}.$$

Assume that $\phi(\mathbf{x}) \neq \phi_{\mathrm{id}}(\mathbf{x}) := x \bullet y$*. Then there exists an integer* $N \geq 0$*, which is the basic invariant of the flow, called the level. Such a flow* $\phi(x, y)$ *can be given by*

$$\phi(x, y) = \ell^{-1} \circ \phi_N \circ \ell(x, y),$$

where ℓ *is a 1-homogenic birational plane transformation, and* ϕ_N *is the canonical solution of level* N *given by*

$$\phi_N(x, y) = x(y + 1)^{N-1} \bullet \frac{y}{y + 1}.$$

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Jose María Almira *On the closure of translation-dilation invariant linear spaces of polynomials*
(joint work with **László Székelyhidi**)

In the Forty-ninth International Symposium on Functional Equations, and, later on, in the 14th International Conference on Functional Equations and Inequalities, Székelyhidi proposed the following problem: Assume that V is a linear space of complex polynomials of several variables, which is translation invariant. Assume that $\{p_n\} \subseteq V$ converges pointwise to a polynomial p. Is it true that p is in V? This question is still open. In this address we solve the problem in the positive for the special case when V is a translation-dilation invariant linear space of polynomials. We also investigate how far is our solution from being an answer to the original problem.

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Michał Baczyński *On the Jensen equation extended to the infinities (Part 2)*
(joint work with **Wanda Niemyska**)

Recently in some considerations connected with the distributivity laws of fuzzy implications over triangular norms and conorms, the following functional equation appeared $f(\min(x+y,a)) = \min(f(x)+f(y),b)$, where a, b are finite or infinite nonnegative constants (see [1]). In [2] we have considered a generalized version of this equation in the case when both a and b are finite, namely the equation $f(m_1(x+y)) = m_2(f(x)+f(y))$, where m_1, m_2 are functions defined on some finite intervals of $\mathbb{R}$ satisfying additional assumptions. Now we consider the above equation when m_1, m_2 are defined on some finite or infinite sets and satisfy only one additional assumption: m_2 is injective. To obtain the solutions in this case we need to solve the Jensen equation, i.e. $f(x)+f(y) = 2f(\frac{x+y}{2})$, where domain and codomain of function f are different subsets of $\mathbb{R}$ extended to infinities.

The second part of our contribution will be devoted to the application of Jensen equation to the distributivity of implications in multi-valued logic.

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Roman Badora *Approximately multiplicative functions in function algebras*

For approximately multiplicative function f defined on a semigroup with values in a normed algebra we are looking for a decomposition of f on a sum $f_s + f_b$, where f_s is a multiplicative function, a function f_b is bounded and $f_s \cdot f_b = 0$. In particular, we show that such a representation have approximately multiplicative functions with values in algebras with orthogonal Schauder basis.

Anna Bahyrycz *Criteria for hyperstability of general linear functional equation* (joint work with **Jolanta Olko**)

Our purpose is to investigate criteria for hyperstability of linear type functional equations. We prove that a function satisfying the equation approximately in some sense, must be a solution of it. We give some conditions on coefficients of the functional equation and a control function which guarantee hyperstability. Moreover, we show how our outcomes may be used to check whether the particular funcional equation is hyperstable.

Karol Baron *On the continuous dependence of solutions to orthogonal additivity problem on given functions*

Considering Tychonoff topology we show that the solution to the orthogonal additivity problem in real inner product spaces depends continuously on the given function. As a corollary we get that the set of all additive functions is closed and nowhere dense in the space of all orthogonally additive functions mapping a real inner product space of dimension at least 2 into an abelian topological Hausdorff group, provided there exists a non-trivial additive function from reals to the group.

Janusz Brzdęk *Ulam's stability of some delayed fractional differential equations* (joint work with **Nasrin Eghbali**)

Let $C_J(\mathbb{R})$ denote the family of all continuous functions mapping a real interval J into $\mathbb{R}$, $\alpha \in (0,1)$, t_0 and $h > 0$ be fixed real numbers, I be a real interval of one of the forms: $[t_0, \infty)$, $[t_0, a)$, $[t_0, a]$ (with some $a > t_0$), $I_h := I \cup [t_0 - h, t_0]$, and $\phi\colon [t_0 - h, t_0] \to \mathbb{R}$ and $f\colon I \times C_{[-h,0]}(\mathbb{R}) \to \mathbb{R}$ be fixed continuous functions ($C_{[-h,0]}(\mathbb{R})$ is endowed with the supremum norm). Given $y \in C_{I_h}(\mathbb{R})$, we define $y_t \in C_{[-h,0]}(\mathbb{R})$ for $t \in I$ by $y_t(\theta) := y(t+\theta)$ for $\theta \in [-h, 0]$.

Some results on Ulam's type stability of the delayed fractional differential equation

$$D^\alpha[y(t)g(t)] = f(t, y_t), \quad t \in I, \qquad y(t) = \phi(t), \quad t \in [t_0 - h, t_0],$$

are presented for $y\colon I_h \to \mathbb{R}$, where $g\colon I \to \mathbb{R}$ is a given suitable function and D^α stands for the Caputo fractional derivative, defined by

$$D^\kappa h(t) = \frac{1}{\Gamma(n-\kappa)} \int_{t_0}^{t} (t-s)^{n-\kappa-1} h^{(n)}(s)\, ds, \qquad t \in I,$$

with $n = [\kappa] + 1$ ($[\kappa]$ denotes the integer part of $\kappa \in \mathbb{R}$). An auxiliary outcome on Ulam's stability of a Volterra integral equation is provided, as well.

Liviu Cădariu *Fixed points and generalized Ulam-Hyers stability of the monomial functional equation*
(joint work with **Ioan Goleţ**)

A new trend in the field of Ulam-Hyers stability focuses on general methods allowing to obtain stability results for large classes of functional, differential and integral equations, in various spaces. For example, some fixed points theorems for operators (not necessarily linear) satisfying suitable very general properties have been proved recently. After that, these outcomes were used to obtain properties of stability, hyperstability, superstability, etc, for different classes of functional equations.

The aim of this talk is to present an application of such a fixed point theorem for proving generalized Ulam-Hyers stability properties of the monomial functional equation.

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Jacek Chudziak *On a functional equation stemming from utility theory and psychophysics*

In the paper [1] two functional equations stemming from some problems in utility theory and psychophysics have been considered. One of them has the form

$$f(\sigma(y)x + (1-\sigma(y))y) = \tau(y)f(x) + (1-\tau(y))f(y) \tag{1}$$

for $x, y \in [0, \infty)$, $x \geq y$, where $f\colon [0,\infty) \to \mathbb{R}$ and $\sigma, \tau\colon [0,\infty) \to [0,1]$ are unknown functions.

In [1] the solutions of (1) under relatively strong regularity assumptions on f have been determined. In the talk we deal with the solutions of (1) satisfying significantly weaker assumptions. Moreover, we investigate equation (1) in a more general setting.

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Marek Czerni *Asymptotic properties ensuring the existence of a one-parameter family of continuous solutions of a linear functional inequality of second order*

In this talk we shall give sufficient conditions for the existence of a one-parameter family of continuous solutions with asymptotic property of linear non-homogeneous functional equation

$$\varphi[f(x)] = g(x)\varphi(x) + h(x)$$

with the unknown function φ. W also adopt this conditions to determine a one-parameter family of continuous solutions with asymptotic property of functional inequality of the second order

$$\psi[f^2(x)] \leq (p(x) + g[f(x)])\psi[f(x)] - p(x)g(x)\psi(x)$$

with the unknown function ψ.

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Joachim Domsta *On conjugacy by regularly varying functions*

Two selfmappings f and g of the open interval $I = (0, \infty)$ are said to be *regularly conjugate* with respect to the distinguished end-point $\xi = 0$ whenever for some uniformly regularly varying at ξ bijective selfmapping ψ we have $g \circ \psi = \psi \circ f$. Let $\mathcal{S}_0$ denote the (Szekeres-Lundberg) family of fixed point free bijections f of I, strongly attracting to ξ, with derivative at ξ satisfying $0 < D_0 f < 1$, which additionally possess continuous (weakly) increasing principal function $\varphi_f(x|y) := \lim_{n\to\infty} f^n(x)/f^n(y) > 0$ for $x, y \in I$. All regular conjugacy classes of elements of $\mathcal{S}_0$, the non-measurable included, are constructed in terms of the principal functions. Few representatives of different regular conjugacy classes among S-L diffeomorphisms are constructed, too.

El-Sayed El-Hady *On a functional equation arising from a switch*
(joint work with **Wolfgang Förg-Rob** and **Janusz Brzdęk**)

During the last few decades, a certain class of two-place functional equations arose from many interesting applications like e.g. communications. A special case of such general class of functional equations is given by

$$(xy - \phi(x,y))f(x,y) = (x-1)(y-1)\phi(x,y)\Big(\frac{f(x,0)}{(x-1)} + \frac{f(0,y)}{(y-1)} + f(0,0)\Big),$$

where $\phi(x,y)$ is some given function in two complex variables. The unknown functions $f(x,y)$, $f(x,0)$, and $f(0,y)$ are generating functions defined as follows

$$f(x,y) = \sum_{m,n=0}^{\infty} p_{m,n}x^m y^n, \qquad |x| < 1,\ |y| < 1$$

for some sequence of interest $p_{m,n}$, see [2] and [3] e.g.. In this talk I will introduce a solution of the given functional equation, such equation arose from a queueing model of an asymmetric 2×2 switch. The solution is obtained by assuming full symmetry on the system parameters and by using the theory of boundary value problems.

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Roman Ger *Mazur's type problem for convexity of higher orders*

It is widely known that the regularity behaviour of Jensen-convex functionals is, in general, very similar to that of additive ones. Therefore, it seems natural to ask after the following generalization of Mazur's problem 24 from the famous *Scottish Book*:

Assume that we are given a nonempty open and convex subdomain D of a real Banach space $(E, \|\cdot\|)$ endowed with a cone C of positive elements, and that F is a higher order Jensen-convex functional on D, i.e. F is a solution to the functional inequality

$$y - x \in C \implies \sum_{j=0}^{n+1} (-1)^{n+1-j} \binom{n+1}{j} F\Big(\Big(1 - \frac{j}{n+1}\Big)x + \frac{j}{n+1}y\Big) \geq 0, \quad x, y \in D.$$

Suppose that the superposition $F \circ x$ is Lebesgue measurable for every continuous map $x\colon [0,1] \to D$. Does it force F to be continuous?

This will be discussed and reported on.

Attila Gilányi *On higher-order convex functions with a modulus*
(joint work with **Nelson Merentes, Kazimierz Nikodem** and **Zsolt Páles**)

Related to the investigations of strongly convex functions of higher order, we consider $(t_1, \dots, t_n)$-Wright-convex functions with a modulus c, i.e. real valued functions defined on an interval I satisfying the inequality

$$\Delta_{t_1 h} \dots \Delta_{t_n h} f(x) \geq c n! (t_1 h) \dots (t_n h)$$

for all $x \in I$, $h > 0$ such that $x + (t_1 + \dots + t_n)h \in I$, where n is a positive integer, c is an arbitrary and $t_1, \dots, t_n$ are positive real numbers. We give some notes on such type of functions, we characterize them via generalized derivatives and we prove that the property above is localizable. As consequences of our results, we obtain characterizations of various higher order (strong) convexity concepts and we get that those convexity properties are also localizable.

Dorota Głazowska *Subcommuting real homographic functions*
(joint work with **Janusz Matkowski**)

If the difference of two real homographic functions is nonnegative, then it is constant. Motivated by this property we determine all pairs of subcommuting (supercommuting) real homographic functions. Moreover, we show that simple modification of subcommuting (supercommuting) functions transforms them into commuting ones.

Xiaobing Gong *Convex solutions of the polynomial-like iterative equation in Banach spaces*
(joint work with **Weinian Zhang**)

Although convex (concave) solutions were investigated for the polynomial-like iterative equation on a compact interval of $\mathbb{R}$, there are much more difficulties in discussion on convexity of solutions in Banach spaces. In this talk we consider a partial order in Banach spaces, which is defined by an order cone, and discuss monotonicity and convexity of operators under iteration in Banach spaces. Then we give the existence of monotone solutions in the ordered real Banach spaces and further obtain conditions under which the solutions are convex or concave in the order. Moreover, the uniqueness and stability of those solutions are also discussed.

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Grzegorz Guzik *On a simple criterion of convergence of Markov process induced by random iterations*

Assume that $\{w_\lambda\colon X \to X :\ \lambda \in \Lambda\}$ is a family of transformations of a Polish space X into itself indexed by elements of a measurable space (Λ, Σ). We consider a homogenous Markov chain induced by a difference equation of the form

$$x_{n+1} = w_{\lambda_n}(x_n),$$

where transformations $w_{\lambda_0}, w_{\lambda_1}, \ldots, w_{\lambda_n}$ are randomly chosen with some common distribution p.

We show that under quite weak assumptions on w_λ, that is: all w_λ are supposed to be non-expansive and at least one of them is a continuous Matkowski's contraction with some additional regularity property, the considered Markov chain converges to a unique stationary Borel probability measure on X. The convergence does not depend on p and moreover the support of an attractive probability measure for every p is the same.

The corollary on asymptotic properties for discrete random dynamical systems can be easily established too.

Mohammad Hadi Hooshmand *A new approach to the solution of some functional equations on algebraic structures*

In this talk, I establish several new approaches for study of some functional equations on algebraic structures. By using them, we classify and make reasonable titles for many basic and important functional equations on magmas (groupoids), semigroups, quasigroups and other group-like structures. In this way we show that all of the main equations (such as $f(f(x)y) = f(xf(y))$, $f(f(x)y) = f(x)f(y) = f(xf(y))$, etc.) are special cases of the decomposer equation $f(f^*(x)f(y)) = f(f(y)f_*(x)) = f(y)$, introduced by the author on 2007 (where $f^*(x)f(x) = f(x)f_*(x) = x$). A section of the talk is about the multiplicative symmetric equation $f(f(x)y) = f(xf(y))$ (studied by J.G. Dhombres and others) and we construct a vast class of its solutions in semigroups and then pose a very probable conjecture about its general solution in arbitrary groups (regrading to the related open problem). It is worth noting that the first ideas and motivation of the topics come from b-parts real functions and their generalizations, introduced by M.H. Hooshmand, and the methods of the paper could be used for studying such equations on more general algebraic structures.

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Hideaki Izumi *General terms of algebraic recurrence relations*

We develop methods of obtaining general terms of the algebraic recurrence relation

$$P(a_n, a_{n+1}, a_{n+2}, \dots, a_{n+k}) = 0,$$

where P denotes a polynomial in $(k+1)$ indeterminates. As an application, we observe asymptotic behavior of the sequence satisfying an algebraic recurrence relation. Moreover, we apply these methods to iterative functional equations.

Eliza Jabłońska *Fixed points almost everywhere and Hyers-Ulam stability* (joint work with **Janusz Brzdęk, Anna Bahyrycz** and **Jolanta Olko**)

In 2011 J. Brzdęk, J. Chudziak, Z. Páles [1] and J. Brzdęk, K. Ciepliński [2] proved fixed point theorems for some operators and derive from it several results on the stability of a very wide class of functional equations in single variable. In 2012 L. Cădariu, L. Găvruţa, P. Găvruţa [3] generalized these results.

Here we generalized results from [3]; more precisely, we prove a fixed point theorem almost everywhere and apply it to obtain some stability results.

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Justyna Jarczyk *Around a problem of Zoltán Daróczy*

The results presented in the talk have been obtained jointly with Zoltán Daróczy and Witold Jarczyk. We answer in negative the following problem posed by Zoltán Daróczy during the previous International Symposium on Functional Equations in Innsbruck, in connection with a theorem of Roman Ger and Tomasz Kochanek.

Let M be a mean on an interval I. Is it true that if M is not quasi-arithmetic, then every solution $f\colon I \to \mathbb{R}$ of the equation

$$f(M(x,y)) = \frac{f(x)+f(y)}{2}$$

is constant?

Next we answer (again in negative) some modification of the problem and finally we formulate a new its version.

Witold Jarczyk *On some properties of strictly convex functions*

This is a preliminary report on a joint work with Kazimierz Nikodem. Let D be a convex subset of a normed linear space. Given a positive number c a function $f\colon D \to \mathbb{R}$ is called *strongly convex with modulus c* if

$$f(tx+(1-t)y) \le tf(x) + (1-t)f(y) - ct(1-t)\|x-y\|^2$$

for all different $x, y \in D$ and every $t \in (0,1)$. Obviously any strongly convex function f is *strictly convex*

$$f(tx+(1-t)y) < tf(x) + (1-t)f(y)$$

for all $x, y \in D, x \neq y$ and $t \in (0,1)$. The converse is not true.

However, we prove that some careless formal modification of the definition of strong convexity leads to a condition which is equivalent to that one of strict convexity.

Bartosz Kołodziejek *Fundamental equation of information on matrices*

Let Ω_+ denote the cone of positive definite real symmetric matrices of rank r and let $\mathcal{D} = \{\mathbf{x} \in \Omega_+ :\ I - \mathbf{x} \in \Omega_+\}$ be the analogue of $(0,1)$ interval in Ω_+. Consider following generalization of fundamental equation of information, when unknown functions are defined on matrices

$$f(\mathbf{x}) + g\big((I-\mathbf{x})^{-\frac{1}{2}}\mathbf{y}(I-\mathbf{x})^{-\frac{1}{2}}\big) = h(\mathbf{y}) + k\big((I-\mathbf{y})^{-\frac{1}{2}}\mathbf{x}(I-\mathbf{y})^{-\frac{1}{2}}\big), \qquad (1)$$

where $\mathbf{x}, \mathbf{y}, \mathbf{x}+\mathbf{y} \in \mathcal{D}$ and $\mathbf{a}^{\frac{1}{2}}$ is the unique positive definite square root of $\mathbf{a} = \mathbf{a}^{\frac{1}{2}}\mathbf{a}^{\frac{1}{2}}$.

Take $\mathbf{x} = \mathbf{u}\,\mathrm{diag}(x)\,\mathbf{u}^T$ and $\mathbf{y} = \mathbf{u}\,\mathrm{diag}(y)\,\mathbf{u}^T$, where $\mathbf{u}$ is a fixed orthogonal matrix and vectors $x, y \in (0,1)^r$. In this case $\mathbf{x}$ and $\mathbf{y}$ commute and (1) gets the form

$$f_{\mathbf{u}}(x) + g_{\mathbf{u}}\Big(\frac{y}{\underline{1}-x}\Big) = h_{\mathbf{u}}(y) + k_{\mathbf{u}}\Big(\frac{x}{\underline{1}-y}\Big) \qquad (2)$$

for $x_i, y_i, x_i + y_i \in (0,1)$, where division of vectors is performed component-wise, $f_{\mathbf{u}}(x) = f(\mathbf{u}\,\mathrm{diag}(x)\,\mathbf{u}^T)$ and so on. This means that (1) is a generalization of fundamental equation of information to a wider domain, since (2) is satisfied for any orthogonal matrix $\mathbf{u}$ and when one takes non-commutative $\mathbf{x}$ and $\mathbf{y}$, the situation is far more complicated, because the operations are not performed component-wise. It also justifies the name “fundamental equation of information on Ω_+”, despite its lack of clear connection to the developed information theory.

In the talk we will give all continuous solution to (1) and its generalizations. This solutions are used in the proof of characterization of matrix variate beta probability distribution.

Vichian Laohakosol *Solutions of some particular pexiderized digital filtering functional equations*
(joint work with **Charinthip Hengkrawit** and **Khanithar Naenudorn**)

Consider the pexiderized digital filtering functional equation

$$f_1(x+t,y+t)+f_2(x-t,y)+f_3(x,y-t) = f_4(x-t,y-t)+f_5(x+t,y)+f_6(x,y+t).$$

We determine three kinds of solutions, namely, biadditive, symmetric and skew-symmetric solution functions, subject to different sets of conditions on the functions involved.

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Zbigniew Leśniak *On the topological equivalence of flows of Brouwer homeomorphisms*

We study the problem of topological equivalence of flows of Brouwer homeomorphisms. We prove that the set of all regular points of such a flow is invariant under topological equivalence. We also show that a similar result holds for the first prolongational limit set. Moreover, we describe the completed separatrix configurations of considered flows by means of trajectories which consist of irregular and regular points.

Gyula Maksa *On local mean values*
(joint work with **Zsolt Páles**)

Let $I \subset \mathbb{R}$ (the reals) be an interval of positive length, $1 < n$ be a fixed integer, $f_1, \dots, f_n \colon I \to \mathbb{R}$ be continuous and strictly increasing functions. The function $A_{f_1,\dots,f_n} \colon I^n \to \mathbb{R}$ defined by

$$A_{f_1,\dots,f_n}(x_1,\dots,x_n) = (f_1 + \dots + f_n)^{-1}(f_1(x_1) + \dots + f_n(x_n))$$

is called Matkowski mean, which is a generalization of the weighted quasi-arithmetic mean.

In this talk, we present results which show that some local properties of these means characterize them.

Renata Malejki *On stability of a functional equation stemming from a characterization of inner product spaces*

We present stability and hyperstability results for the functional equation

$$A_1f(x+y+z)+A_2f(x)+A_3f(y)+A_4f(z) = A_5f(x+y)+A_6f(x+z)+A_7f(y+z),$$

in the class of functions mapping a linear space X over a field $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ into a Banach space Y, where $A_1, \ldots, A_7 \in \mathbb{K}$ are constants. The results generalize those of [1].

In the proof of the main theorem we use the fixed point theorem given in [2]. One of possible application of the main result is a sufficient condition for a normed space to be an inner product space.

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Tomasz Małolepszy *The application of the Schröder equation in the theory of blow-up solutions for Volterra integral equations*

We consider a class of Volterra integral equations with the convolution kernel

$$u(t) = \int_0^t k(t-s)g(u(s))\,\mathrm{d}s, \qquad t \geq 0, \tag{1}$$

where, in particular, the following conditions about nonlinearity g are valid:

$$\begin{aligned} g(0) &= 0, \\ t/g(t) &\to 0 \quad \text{as } t \to 0^+, \\ t/g(t) &\to 0 \quad \text{as } t \to \infty. \end{aligned}$$

Our main goal is to estimate the blow-up time of so-called blow-up solutions [1, 2] of (1). It turns out that this task can be accomplished with the help of the Schröder functional equation. We illustrate our results with an example of equation (1) related to the model of the formation of shear bands in steel [3].

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Janusz Matkowski *Directional convexity and a characterization of the Beta function*
(joint with **Martin Himmel**)

The restictions of the Beta functions to rays parallel to the main diagonal are logarithmically convex. This fact and the functional equation

$$\psi(x+1) = \frac{x(x+k)}{(2x+k+1)(2x+k)}\psi(x), \qquad x > 0$$

allow to get a characterizations of the Beta function. This result and a notion of the beta-type function lead to a new characterization of the Gamma function.

Sukrawan Mavecha *Iterative roots of an increasing function with no fixed point*
(joint work with **Boonrod Yuttanan** and **Vichian Laohakosol**)

For a strictly increasing function with no fixed point $g(n)$, and an integer $q \geq 2$, the iterative functional equation $f^q(n) = g(n)$, where f^q denotes the q^{th} composite of f, is solved for functions whose domain and range are the set of nonnegative integers.

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Janusz Morawiec *Integrable solutions of inhomogeneous refinement type equations on intervals*
(joint work with **Rafał Kapica**)

Given a probability measure P on a σ-algebra of subsets of a set Ω, an interval $I \subset \mathbb{R}$, $g \in L^1(I)$ and a function $\varphi\colon I \times \Omega \to I$ fulfilling some conditions we obtain results on the existence of solutions $f \in L^1(I)$ of the inhomogeneous refinement type equation

$$f(x) = \int_\Omega \left|\varphi'_x(x,\omega)\right| f(\varphi(x,\omega))\, dP(\omega) + g(x).$$

Mohammad Sal Moslehian *Chebyshev and Grüss type operator inequalities*

The Grüss inequality is a complement of the Chebyshev inequality. In this talk, we give a general Grüss inequality for unital completely positive maps and unitarily invariant norms (see [1, 3]). We also establish a Grüss operator inequality in the setting of C^*-algebras and apply it to inequalities involving continuous fields of operators. We study some operator extensions of the Chebyshev inequality and apply states on C^*-algebras to obtain some versions related to synchronous functions. Finally, we give some Chebyshev type inequalities concerning the singular values of positive matrices (see also [2]).

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Wanda Niemyska *On the Jensen equation extended to the enfinities (Part 1)* (joint work with **Michał Baczyński**)

Recently in some considerations connected with the distributivity laws of fuzzy implications over triangular norms and conorms, the following functional equation appeared $f(\min(x+y,a)) = \min(f(x)+f(y),b)$, where a, b are finite or infinite nonnegative constants (see [1]). In [2] we have considered a generalized version of this equation in the case when both a and b are finite, namely the equation $f(m_1(x+y)) = m_2(f(x)+f(y))$, where m_1, m_2 are functions defined on some finite intervals of $\mathbb{R}$ satisfying additional assumptions. Now we consider the above equation when m_1, m_2 are defined on some finite or infinite sets and satisfy only one additional assumption: m_2 is injective. To obtain the solutions in this case we need to solve the Jensen equation, i.e. $f(x)+f(y) = 2f(\frac{x+y}{2})$, where domain and codomain of function f are different subsets of $\mathbb{R}$ extended to infinities.

In the first part of the talk we want to present new results pertaining to the Jensen equation.

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Andrzej Olbryś *On separation theorem for delta-subadditive and delta-superadditive mappings*

Let $(X,\cdot)$ be a semigroup, and let $(Y,\|\cdot\|)$ be a real Banach space. Motivated by the dissertation of L. Veselý and L. Zajiček [3] R. Ger in [2] considered the following functional inequality

$$\|F(x) + F(y) - F(x \cdot y)\| \le f(x) + f(y) - f(x \cdot y), \qquad x, y \in X.$$

If a pair (F, f) satisfies the above inequality, then we say that a map $F\colon X \to Y$ is delta-subadditive with a control function $f\colon X \to \mathbb{R}$. If a pair $(-G, -g)$ satisfies the above inequality, then we say that G is delta-superadditive with a control function g. Inspired by methods contained in [1] we generalize the well known separation theorem for subadditive and superadditive functionals to the case of delta-subadditive and delta-superadditive mappings. We also consider the problem of supporting delta-subadditive maps by additive ones. As a consequence of these theorems we obtain the stability result for Cauchy's equation.

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Jolanta Olko *On measurable microperiodic multifunctions*

Let X be a topological group, $Y \neq \emptyset$ and let $F\colon X \to 2^Y$ be a set-valued function (multifunction for brevity). We call F microperiodic if there exists a dense set $P \subset X$ such that

$$F(px) \subset F(x), \qquad x \in X,\, p \in P.$$

The notion is a generalization of a concept of microperiodic single-valued function. It is known that a real Lebesgue measurable microperiodic fuction is constant almost everywhere (see [6, 4, 7]), an analogous result for Baire measurable functions can be found in [8]. Afterwards M. Kuczma in [5] proved that a microperiodic function defined on a topological group with values in a separable metric space which is Haar or Baire measurable must be constant almost everywhere. Further generalizations of the above mentioned outcomes are due to J. Brzdęk [1, 2, 3].

We present a counterpart to the above mentioned results in the multivalued case. Moreover, we show that a measurable multifunction satisfying the above inclusion approximately, in some sense, is close to a constant one almost everywhere.

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Mohsen Erfanian Omidvar *Numerical radius and operator norm inequalities of operators*

Let $\mathcal{B}(\mathcal{H})$ denote the C^*-algebra of all bounded linear operator on a complex Hilbert space $\mathcal{H}$ with inner product $\langle\cdot,\cdot\rangle$. For $A\in\mathcal{B}(\mathcal{H})$ let $\omega(A)=\sup\{|\langle x,Ax\rangle| : \|x\|=1\}$, $\|A\|=\sup\{\|Ax\| : \ \|x\|=1\}$ and $|A|=(A^*A)^{1/2}$ denote the numerical radius, the usual operator norm of A and the absolute value of A, respectively. It is well know that $\omega(\cdot)$ is a norm on $\mathcal{B}(\mathcal{H})$, and that for all $A\in\mathcal{B}(\mathcal{H})$,

$$\frac{1}{2}\|A\|\le\omega(A)\le\|A\|.$$

It is shown that if $A_j\in\mathcal{B}(\mathcal{H})$, then

$$\omega^r\left(\sum_{j=1}^n A_j\right)\le(2n)^{r-1}\sum_{j=1}^n\left\||B_j|^{2r}+|C_j|^{2r}\right\|^{\frac{1}{2}},$$

where $A_j\in\mathcal{B}(\mathcal{H})$ and $A_j=B_j+iC_j$, $j=1,2,\dots,n$ and $r\ge1$.

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Zsolt Páles *Asymptotic stability of the Cauchy and the Jensen functional equations*
(joint work with **Anna Bahyrycz** and **Magdalena Piszczek**)

Given two normed spaces X and Y, we consider fuctions $f\colon X\to Y$ satisfying the asymptotic stability properties

$$\limsup_{\min(\|x\|,\|y\|)\to\infty}\|f(x+y)-f(x)-f(y)\|\le\varepsilon$$

and
$$\limsup_{\min(\|x\|,\|y\|)\to\infty} \left\| f\left(\frac{1}{2}x + \frac{1}{2}y\right) - \frac{1}{2}f(x) - \frac{1}{2}f(y) \right\| \le \varepsilon.$$
Our main results show that these inequalities imply the following ordinary stability properties
$$\|f(x+y) - f(x) - f(y)\| \le 5\varepsilon, \qquad x, y \in X$$
and
$$\left\| f\left(\frac{1}{2}x + \frac{1}{2}y\right) - \frac{1}{2}f(x) - \frac{1}{2}f(y) \right\| \le 3\varepsilon, \qquad x, y \in X,$$
respectively.

Paweł Pasteczka *Limit properties in a family of quasi-arithmetic means*

It is known that the family of power means tends to maximum pointwise if we pass argument to infinity. We will give some necessary and sufficient condition for the family of quasi-arithmetic means generated by a functions satisfying certain smoothness conditions to have analogous property.

Zlatko Pavić *Improvements of the Hermite-Hadamard inequality for multivariate convex functions*

This work presents a generalization of the Hermite-Hadamard inequality for convex functions on the simplex in the Euclidean space. The generalization refers to points of the subsimplex whose vertices are barycenters of the given simplex facets. The improved inequality applies to all points of the inscribed subsimplex, with the exception of its vertices.

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Magdalena Piszczek *Stability results for a general linear inclusion*

We present some results on selections of set-valued function $F\colon X \to ccl(Y)$ satisfying the following general linear inclusion
$$AF(px+ry) + BF(qx+sy) \subset CF(x) + DF(y), \qquad x, y \in X,$$
where X, Y are normed spaces, $p, r, q, s, \in \mathbb{R}$, $A, B, C, D \ge 0$.

Dorian Popa *On the stability of linear operators with respect to gauges* (joint work with **Ioan Raşa**)

A characterization of the Hyers-Ulam stability of linear operators acting between two Banach spaces is given in [3]. In [2] the authors obtain a characterization of Hyers-Ulam stability for the linear differential operator of order n with constant coefficients.

We consider linear operators $T\colon A \to B$, where A, B are linear spaces endowed with gauges, and obtain a characterization of their Hyers-Ulam stability. As a consequence follows some results on the stability of the linear differential operator.

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Teresa Rajba *On H-Wright-convex functions*

For a fixed number $h \in \mathbb{R}$ the difference operator Δ_h, acting on a real function $f\colon \mathbb{R} \to \mathbb{R}$, is defined by $\Delta_h f(x) = f(x+h) - f(x)$ $(x \in \mathbb{R})$.

Given $h > 0$, we say that the function f is h-Wright-convex ([1]) if

$$\Delta_t \Delta_h f(x) \geqslant 0 \quad \text{for all } t > 0 \text{ and } x \in \mathbb{R}.$$

In other words, f is h-Wright-convex if the function $\Delta_h f$ is non-decreasing. Let $H \subset (0, \infty)$ be a set. We say that f is H-Wright-convex if f is h-Wright-convex for all $h \in H$. Then f is $(0, \infty)$-Wright-convex if and only if it is Wright-convex. We say that f is h-Wright-concave (H-Wright-concave) if $-f$ is h-Wright-convex (H-Wright-convex).

We study the sets

$$A_f = \{h > 0 : \ f \text{ is } h\text{-Wright-convex}\}$$

and

$$B_f = \{h > 0 : \ f \text{ is } h\text{-Wright-concave}\}.$$

The idea of study of H-Wright-convex functions was motivated by the problem posed by T. Szostok (Aequationes Math. 73:172-200, 2007) [2].

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Ioan Raşa *Hyers-Ulam stability of generalized Laplace equations*
(joint work with **Dorian Popa**)

Let $(T(t))_{t\geq 0}$ be a strongly continuous semigroup of operators on a Banach space X, having a strong limit $\lim_{t\to\infty} T(t)$. Let A be the infinitesimal generator of this semigroup. We investigate the Hyers-Ulam stability of the equation $Au = 0$. Examples are presented, involving the Laplace operator Δ and other generators studied in the monograph [1].

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Jens Schwaiger *Construction methods for the field of reals connected with functional equations and stability thereof*

There are many methods which can be utilized to construct the field of reals when starting from the field of rationals. It is less well known that it is also possible to start directly from the ring of integers and to arrive at the field of reals without using the rationals as an intermediate step.

Two methods are compared and also their connections to the stability investigations for certain functional equations.

Yong-Guo Shi *Topological conjugacy of piecewise monotonic functions of nonmonotonicity height* ≥ 1
(joint work with **Zbigniew Leśniak**)

The conjugacy problem is an important topic in the theory of dynamical systems and functional equations. In this talk, we investigate a class of piecewise monotone and continuous maps with nonmonotonicity height ≥ 1. We give a sufficient and necessary condition under which any two of these maps are topologically conjugate, and construct a topological conjugacy with an extension method if such a conjugacy exists.

References

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Ekaterina Shulman *On approximate solutions of the Levi-Civita equation*

We consider a measurable function f on an amenable group G which "almost satisfies" a Levi-Civita functional equation

$$f(gh) = \sum_{j=1}^{N} u_j(g)v_j(h), \qquad g, h \in G$$

with some $\{u_i\}$ and $\{v_i\}$. We try to prove that f is close to a proper solution of such an equation. For bounded functions that problem was solved in [1] by means of the techniques of covariant widths of convex compacts in Banach G-spaces. We will show that the general case can be studied by estimation of norms of operators that realize 1-cocycles of representations of G on Banach spaces.

Reference

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Justyna Sikorska *Set-valued vs. single-valued approximately orthogonally additive mappings*

A set-valued orthogonally additive function from an orthogonality space into a family of all nonempty compact and convex subsets of a Fréchet space is of the form $a + Q$, where a is single-valued additive and Q is set-valued quadratic [1].

We will study approximately orthogonally additive set-valued mappings and compare the results with the single-valued case.

Reference

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Slavko Simić *Improvements of some moment inequalities*

For an arbitrary probability law with support on $(0, \infty)$, define

$$\lambda(s) = \lambda_s(X) := \begin{cases} (EX^s - (EX)^s)/s(s-1), & s \neq 0, 1, \\ \log(EX) - E(\log X), & s = 0, \\ E(X \log X) - (EX)\log(EX), & s = 1. \end{cases}$$

By Jensen's inequality it follows that $\lambda(s) \geq 0$, $s \in \mathbb{R}$.

It is also known ([1]) that $\lambda(s)$ is log-convex for $s \in \mathbb{R}$ that is,

$$\xi(s,t) := \lambda(s)\lambda(t) - \lambda^2\Big(\frac{s+t}{2}\Big) \geq 0, \qquad s, t \in \mathbb{R}, \tag{1}$$

providing that the corresponding moments exist.

A possible refinement of the inequality (1) is given by

$$\mu(s,t;u,v) := \xi(s,t)\xi(u,v) - \left(\xi\left(\frac{s+u}{2},\frac{t+v}{2}\right) - \xi\left(\frac{s+v}{2},\frac{t+u}{2}\right)\right)^2 \geq 0$$

for any $s,t,u,v \in \mathbb{R}$, where corresponding moments exist.

The above inequality is very precise but, unfortunately, we are able to prove it just in case $s = u$ (or $s = v$).

References

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Wutiphol Sintunavarat *On new type of stability for radical quadratic functional equations approach by Brzdęk's fixed point theorem*

In 2013, Brzdęk [1] gave the fixed point results for some nonlinear mappings in metric spaces. By using this result, Brzdęk improved Hyers-Ulam-Rassias stability of Rassias [2]. In this work, we show that the Brzdęk's fixed point result allow to investigate new type of stability for radical quadratic functional equations

$$f\left(\sqrt{x^2+y^2}\right) = f(x) + f(y),$$

where f is self mapping on the set of real numbers. Our results generalize, extend and complement some results concerning the Hyers-Ulam-Rassias stability for radical quadratic functional equations.

References

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Dorota Śliwińska *Symmetrization and convexity*

In the papers [1, 2] the results concerning Hermite-Haddamard type inequalities for convex and Wright-convex functions were proved by using a symmetrization method. We develop it to prove the Hermite-Hadamard type inequalities for strongly convex and strongly Wright-convex functions of several variables defined on convex polytopes.

References

[1] A. Guessab, F. Guessab, *Symmetrization, convexity and applications*, Appl. Math. Comput. **240** (2014), 149–160.

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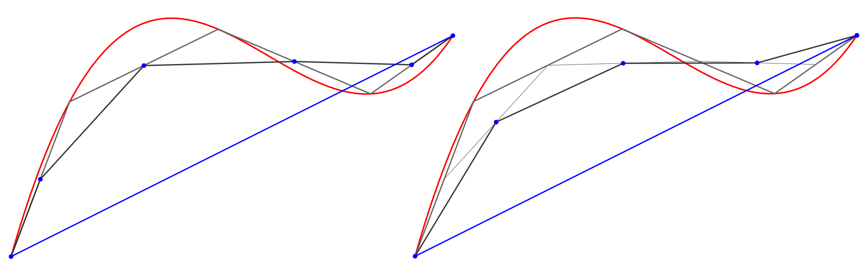
Peter Stadler *Curve shortening by short rulers*

We look at homomorphisms $h\colon (\mathbb{R},+) \to (G,\circ)$ on a Lie group G:

$$h(s+t) = h(s) \circ h(t), \quad h(0) = e, \quad h(1) = g.$$

The restriction of h to the interval $[0,1]$ is a geodesic.

On Riemannian manifolds geodesics are locally shortest lines. The problem is to construct long geodesics. But any curve connecting starting point and end point can be shortened by using a ruler which allows to construct short geodesics:



In normed vector spaces, the curve converges to the straight line if it's shortened iterative. This result can be generalized to some Riemannian manifolds.

Henrik Stetkær *On Wilson's functional equations*
(joint work with **Bruce R. Ebanks**)

Let G be a group, not necessarily abelian. We report on progress in the study of the solutions $f, g\colon G \to \mathbb{C}$ of Wilson's functional equation

$$f(xy) + f(xy^{-1}) = 2f(x)g(y), \quad \forall x, y \in G, \tag{1}$$

and of the following similar functional equation

$$f(xy) + f(y^{-1}x) = 2f(x)g(y), \quad \forall x, y \in G.$$

Part of our discussion is concerned with a recent result: If (f,g) is a solution of (1) such that $f \neq 0$, then g satisfies d'Alembert's functional equation

$$g(xy) + g(xy^{-1}) = 2g(x)g(y), \quad \forall x, y \in G.$$

References

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László Székelyhidi *Functional equations and stability problems on hypergroups*

This is a survey talk about functional equations on hypergroups. We exhibit the structure of hypergroups and also some of the fundamental types of hypergroups. We show how some fundamental functional equations can be treated on some classes of hypergroups. It turns out that in some cases the classical solution methods cannot simply be adapted. We also present stability and superstability results using Hyers' sequences, invariant means and other tools.

Tomasz Szostok *Error of quadrature rules and functional equations*

In some previous papers functional equations stemming from quadrature rules were considered. However in all these results it was assumed that a quadrature rule in question is satisfied exact. Therefore, functional equations of the form

$$F(y) - F(x) = (y - x)\sum_{i=1}^{n} a_i f(\alpha_i x + \beta_i y)$$

were considered. In this talk we assume that a quadrature rule is satisfied with some error. For example we shall consider the following functional equation

$$F(y) - F(x) = (y - x)\sum_{i=1}^{n} a_i f(\alpha_i x + \beta_i y) + (y - x)^k g(\alpha x + \beta y).$$

It may be shown that F, f and g which satisfy this equation are polynomial functions. Thus the main problem is that of continuity of these functions.

Jacek Tabor *A gentle introduction to iterated reweighted least squares*

In this talk we discuss IRLS (=iterated reweighted least squares): one of the well-known methods of finding the local (and sometimes global) minima of the function of the type

$$\mathcal{L}\colon x \longmapsto \sum_i L(x - m_i),$$

where L is a fixed function which usually belongs to some convexity class. The idea is based on using the classical least squares method, but with the changing weights. By applying this approach one can for example efficiently calculate the generalized median in the vector case.

We show, see [1, Appendix], that although if L is not convex one cannot in general guarantee the uniqueness of the minimum, under some additional weak assumptions the IRLS will be guaranteed to decrease the value of $\mathcal{L}$.

Reference

[1] P. Spurek, J. Tabor, *The memory center*, Inform. Sci. **252** (2013), 132–143.

Józef Tabor *Conditionally δ-midconvex functions*
(joint work with **Jacek Chudziak** and **Jacek Tabor**)

Let X be a real linear space, V be a nonempty subset of X and δ be a nonnegative real number. A function $f\colon V \to \mathbb{R}$ is said to be conditionally δ-midconvex provided $f(\frac{x+y}{2}) \leq \frac{f(x)+f(y)}{2} + \delta$ for every $x, y \in V$ such that $\frac{x+y}{2} \in V$. We show that if V satisfies some reasonable assumptions, then for every bounded from above conditionally δ-midconvex function $f\colon V \to \mathbb{R}$ the following estimation holds $\sup f(V) \leq \sup f(\operatorname{ext} V) + k(V)\delta$, where $\operatorname{ext} V$ denotes the set of all extremal points of V and $k(V)$ is a respective constant depending on V.

Reference

[1] J. Chudziak, Ja. Tabor, J. Tabor, *Conditionally δ-midconvex functions*, Aequationes Math. DOI: 10.1007/s00010-014-0304-4.

Pinthira Tangsupphathawat *Fifth order linear recurrence sequences and their positivity*

A fifth order linear recurrence sequence is a sequence of integers $(u_n)_{n\geq 0}$ satisfying

$$u_n = a_1u_{n-1} + a_2u_{n-2} + a_3u_{n-3} + a_4u_{n-4} + a_5u_{n-5}, \qquad n \geq 5,$$

where $a_1, a_2, a_3, a_4, a_5 (\neq 0)$ are given integers. The Positivity Problem associated with this recurrence asks whether it is possible to decide whether $u_n \geq 0$ for all $n \geq 0$? The Positivity Problem for sequences satisfying a second order linear recurrence relation was shown to be decidable by Halava-Harju-Hirvensalo, [1], in 2006. The Positivity Problem for sequences satisfying a third or a fourth order linear recurrence relation was shown to be decidable in [2], [3], [4] and [5]. We show here that the same conclusion holds for a fifth order linear recurrence sequence.

References

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Szymon Wąsowicz *Convex multifunctions and local affine selections*

It is known that a convex multifunction defined on a real interval with compact and convex values on the real line admits the affine selection. In general, for a convex multifunction defined on a convex subset of a vector space with (necessarily convex) values in another vector space, this statement need not be true. Nevertheless, if a domain is a convex subset of $\mathbb{R}^n$, then any convex multifunction admits a local affine selection. We consider a problem of whether it is true in the general setting.

Fabian Wirth *Functional equations and stability of interconnected dynamical systems*

In this talk we consider a particular class of functional equations that arises in the study of so-called small-gain conditions for the stability of interconnected dynamical systems. We will briefly outline the way in which the problem appears and why a deeper understanding of the problem would be of interest.

In the main part of the talk we discuss the problem itself: We consider the problem of solving functional equations in $\mathcal{K}_\infty$, the space of homeomorphisms of $[0,\infty)$. Within this class several problems with existence and uniqueness of solutions of our equations appear. The problem is thus simplified and treated for the subclass of continuous functions that are defined as affine functions on a countable partition of $[0,\infty)$ and for which the partition has no accumulation of intervals from the right. Within this class existence and uniqueness of solutions can be shown. The proof is constructive and can be implemented numerically. Some properties of solutions are discussed.

Alfred Witkowski *Explicit solutions of the invariance equation for means* (joint work with **Janusz Matkowski** and **Monika Nowicka**)

Given two symmetric, strict, bivariate means K, L it is relatively easy to construct a mean M that is (K,L)-invariant, i.e. satisfies

$$M(K(x,y),L(x,y)) = M(x,y) \quad \text{for all } x,y. \tag{1}$$

It is rather obvious, that the reverse problem: given M find K, L satisfying (1) has usually infinitely many solutions. But finding their explicit form is usually a matter of luck.

We show that the explicit solutions can be found in the form

$$K = C^t N^{1-t}, \qquad L = D^t N^{1-t},$$

where C, D are means, $0 < t < 1$ and N is a function that can be expressed in terms of C, D and M.

Paweł Wójcik *Orthogonality preserving property with two linear operators* (joint work with **Jacek Chmieliński**)

Let $\mathcal{H}$ be a Hilbert spaces. For $\varepsilon \in [0,1)$, we define *approximate orthogonality* of vectors x and y

$$x \perp^{\varepsilon} y \;:\Longleftrightarrow\; |\langle x|y\rangle| \leqslant \varepsilon\|x\|\cdot\|y\|.$$

We say that $A, B \in \mathcal{B}(\mathcal{H})$ are *approximately orthogonality preserving* iff

$$\forall_{x,y\in\mathcal{H}} \;\; x\perp y \implies Ax \perp^{\varepsilon} By$$

with some $\varepsilon \in [0,1)$. We show some general properties of such mappings. We will discuss the case of two linear isometries.

Sebastian Wójcik *Scale invariance of the mean-value premium principle*

One of the frequently applied methods of pricing insurance contracts is the so-called *mean-value premium principle.* Various interesting properties of the premium under Cumulative Prospect Theory have been recently studied by M. Kałuszka and M. Krzeszowiec [1]. In this talk, inspired by the results of [1], we consider one of such properties, namely the scale invariance. More precisely, we deal with the case where the scale invariance holds just for two values of the scaling parameter. This problem leads to some functinal equations on restrited domain. The similar problem for others premium principles has been studied by A. Reich [2] under the Expected Utility Theory.

References

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Marek Cezary Zdun *Iterative convexity of diffeomorphisms*

Let I be a closed interval. A function $f\colon I \to I$ is said to be *iteratively convex* if f possesses a convex iteration semigroup that is a family of functions $\{f^t\colon I \to I\}$ such that $f^t \circ f^s = f^{t+s}$, $t, s \geq 0$, $f^1 = f$ and all f^t are convex.

Assume that (H) $f\colon [0,1] \to [0,1]$ is strictly increasing convex, differentiable, $0 < f(x) < x$ for $x \in (0,1)$ and $f'(0) \neq 0$. Every function f satisfying (H) is iteratively convex if and only if f possesses convex iterative roots of all order.

If f of class C^3 satisfies (H) and $f''(0) \neq 0$ then f is iteratively convex in a neighbourhood of 0. If f is iteratively convex on $[0,a]$ then for every $b \in (a,1)$ there exists a unique iteratively convex functions $\widetilde{f}$ on $[0,1]$ such that $f|_{[0,a]} = \tilde{f}|_{[0,a]}$ and $\tilde{f}|_{[b,1]}$ is affine.

Let $0 < \lambda_0 < 1 < \lambda_1$ and denote by $Conv(\lambda_0,\lambda_1)$ the set of all convex functions f of class C^2 satisfying (H) such that $f(1) = 1$, $f'(0) = \lambda_0$ and $f'(1) = \lambda_1$. The set of all iteratively convex functions is of the first category in the space $Conv(\lambda_0,\lambda_1)$ endowed with the classical metric in $C^2[0,1]$ space.

If $f \in Conv(\lambda_0, \lambda_1)$ is iteratively convex then for every $p \in (0,1)$ there exists a neighbourhood U of p such that every convex function F for which $F|_{[0,1]\setminus U} = f|_{[0,1]\setminus U}$ and $F|_U > f|_U$ is not iteratively convex.

Pavol Zlatoš *Approximate extension of partial ε-characters with application to integral point lattices*

Let G be an abelian group, $S \subseteq G$ be a finite set, and $\mathbb{T}$ denote the multiplicative group of complex units with the invariant arc metric $|\arg(a/b)|$. We will show that for a mapping $f\colon S \to \mathbb{T}$ to be ε-close on S to a genuine character $\varphi\colon G \to \mathbb{T}$ it is enough that f be extendable to a mapping $\bar{f}\colon \left(S \cup \{1\} \cup S^{-1}\right)^n \to \mathbb{T}$, where n is big enough and $\bar{f}$ violates the homomorphy condition at most up to an arbitrary $\delta < \min(\varepsilon, \pi/2)$. Moreover, n can be chosen uniformly, independently of G and both f and $\bar{f}$, depending just on δ, ε and the number of elements of S.

The proof is non-constructive, using the ultraproduct construction and Pontryagin-van Kampen duality, hence yielding no estimate on the actual size of n. As one of the applications we show that, for a vector $u \in \mathbb{R}^q$ to be ε-close to some vector from the dual lattice $H^\star$ of a full rank integral point lattice $H \subseteq \mathbb{Z}^q$, it is enough for the inner product ux to be δ-close (with $\delta < 1/3$) to an integer for all vectors $x \in H$ satisfying $\sum_i |x_i| \leq n$, where n depends on δ, ε and q only.

Both results can be interpreted as a kind of stability theorems. The generalization of the former to arbitrary locally compact abelian groups will also be presented along with its nonstandard (and intuitively more clear) formulation.

References

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[2] P. Zlatoš, *Gordon's conjectures: Pontryagin-van Kampen duality and Fourier transform in hyperfinite ambience*, arXiv:1409.6128 [math.CA].

Marek Żołdak *On some kind of approximately convex functions*

Functions $f\colon D \to \mathbb{R}$ defined on an open convex subset of $\mathbb{R}^n$ satisfying the approximate type convexity condition with bound of the form $\varepsilon\sqrt{t(1-t)}\|x-y\|$ are considered.

We discuss properties concerning of such functions characteristic for convex functions.

Problem and Remarks

1. Problem.

Let $f\colon S \to G$ be a function, where $(S,+)$ is a semigroup and $(G,+)$ is an Abelian group. It is well known that the Cauchy difference defined by the formula

$$Cf(x,y) := f(x+y) - f(x) - f(y), \qquad x, y \in S$$

satisfies the cocycle equation

$$Cf(x+y,z)+Cf(x,y)=Cf(x,y+z)+Cf(y,z), \qquad x,y,z\in S.$$

Let $f\colon \mathbb{R}\to\mathbb{R}$ be any function and let us consider the Davison functional equation

$$f(xy)+f(x+y)=f(xy+x)+f(y), \qquad x,y\in\mathbb{R}.$$

R. Girgensohn and K. Lajkó [1] have obtained the general solution of the Davison functional equation without any regularity assumptions on f. They proved that every solution $f\colon\mathbb{R}\to\mathbb{R}$ has the form $f(x)=A(x)+b$, where $A\colon\mathbb{R}\to\mathbb{R}$ is an additive function and b is an arbitrary constant.

We define the Davison difference by the following formula

$$Df(x,y):=f(xy)+f(x+y)-f(xy+x)-f(y), \qquad x,y\in\mathbb{R}.$$

It is easily to check that

$$Df(x,y)=Cf(x,y)-Cf(x,xy), \qquad x,y\in\mathbb{R}.$$

Moreover, denoting by $\partial_k(Df)(x,y)$, $k=1,2$, the partial derivative of the Davison difference Df with respect to the k-th variable at the point (x,y), one can also prove the following result.

Lemma

Let $f\colon\mathbb{R}\to\mathbb{R}$ be a differentiable function. Then we have

$$\begin{aligned}\partial_1(Df)(x,y)+(y+1)\partial_1(Df)(0,xy+x)&\\ =\partial_1(Df)(0,x+y)+y\partial_1(Df)(0,xy),&\\ \partial_2(Df)(x,y)+\partial_1(Df)(0,y)+x\partial_1(Df)(0,xy+x)&\\ =\partial_1(Df)(0,x+y)+x\partial_1(Df)(0,xy)&\end{aligned}$$

for all $x,y\in\mathbb{R}$.

The open problem is to prove that these equalities hold true under weaker assumption that the Davison difference Df is differentiable. In other words, as in the case of Cauchy difference and cocycle equation, we would like to find some equation(s) consisting of several Davison differences $Df(\cdot,\cdot)$, i.e. equation(s) which will be satisfied by the Davison difference Df.

Reference

[1] R. Girgensohn, K. Lajkó, *A functional equation of Davison and its generalization*, Aequationes Math. **60** (2000), no. 3, 219–224.

Marcin Adam

2. Problem. *On the closure of translation-dilation invariant linear spaces of polynomials*

F.B. Jones [5] proved in 1942, in a famous paper, the existence of additive discontinuous functions $f\colon \mathbb{R} \to \mathbb{R}$ whose graph $G(f) = \{(x, f(x)) : \ x \in \mathbb{R}\}$ is connected, and characterized them. These functions are extraordinary since their graphs are dense connected subsets of the plane, containing exactly one point in each vertical line $\{x\} \times \mathbb{R}$ [8]. In his paper the author also stated, without proof, that the graph of a discontinuous additive function must be connected or totally disconnected. For this result he just referenced another famous paper, by Hamel [6], but the proof is not there. Indeed, up to our knowledge, a proof of this dichotomy result has waited for long time, and the first ones are still under consideration [1, 9]. Indeed, quite recently, we proved that a dichotomy result of this type holds true for monomial functions $f\colon \mathbb{R} \to \mathbb{R}$ [1]. Concretely, in that paper, we characterize the discontinuous monomial functions $f\colon \mathbb{R} \to \mathbb{R}$ with connected graph and we use this characterization to prove that the graph of a discontinuous monomial is either connected or totally disconnected. On the other hand, we also prove that dichotomy is transformed into $(d+1)$-chotomy result for additive functions defined over $\mathbb{R}^d$.

Just to motivate our open problem, let us include the precise statements of the mentioned results, and the easiest proof we know of Jones's original statement:

Theorem 1 (Dichotomy, for additive functions $f\colon \mathbb{R} \to \mathbb{R}$)
Let $f\colon \mathbb{R} \to \mathbb{R}$ be an additive function. Then $G(f)$ is connected or totally disconnected. Furthermore, there exists discontinuous additive functions $f\colon \mathbb{R} \to \mathbb{R}$ with connected graph $G(f)$.

Proof. [by Székelyhidi, 2014] Let $\pi\colon \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ denote the canonical projection $\pi(x, y) = x$, and let $W = \pi(G)$ be the projection of the connected component of $G(f)$ which contains the zero element. Then $\pi(G) = \{0\}$ or $\pi(G) = \mathbb{R}$, since the only connected subgroups of the real line are $\{0\}$ and $\mathbb{R}$. Thus, if $G(f)$ is not totally disconnected, then $\pi(G) = \mathbb{R}$, which implies $G = G(f)$ and hence, $G(f)$ is connected. Note we are using that all connected components of a topological group are homeomorphic.

Theorem 2 ($(d+2)$-chotomy property of additive functions)
If $f\colon \mathbb{R}^d \to \mathbb{R}$ an additive function, then

(a) *There exists $s \in \{0, 1, \dots, d+1\}$ such that every connected component of $G(f)$ is a dense subgroup of an s-dimensional affine subspace of $\mathbb{R}^{d+1}$.*

(b) *All cases described in* (a) *are attained by concrete examples.*

Note that Theorem 2 includes the case when f is continuous. This, for $d = 1$, produces the case $s = 1$.

Theorem 3 (Dichotomy, for monomial functions $f\colon \mathbb{R} \to \mathbb{R}$)
Let $f\colon \mathbb{R} \to \mathbb{R}$ be a discontinuous n-monomial function and let $\Gamma_f = \overline{G(f)}^{\mathbb{R}^2}$, and $\Omega_f = \mathrm{Int}(\Gamma_f)$. Then

(i) *$G(f)$ is connected if and only if $G(f)$ intersects all continuum $K \subseteq \Omega_f$ which touches two distinct vertical lines.*

(ii) *$G(f)$ is either connected or totally disconnected. Furthermore, both cases are attained by concrete examples of discontinuous n-monomials $f\colon \mathbb{R} \to \mathbb{R}$, for every n.*

For the proof of Theorem 3 it was necessary to use a full description of the closures of the graphs of monomial functions $f\colon \mathbb{R} \to \mathbb{R}$. This result was proved earlier by the author in [2].

Now we can state our open problem: Does the dichotomy property (connectedness/totally disconnectedness) holds true for:

- Polynomial functions $f\colon \mathbb{R} \to \mathbb{R}$?
- Exponential monomials $f\colon \mathbb{R} \to \mathbb{R}$?
- Exponential polynomials $f\colon \mathbb{R} \to \mathbb{R}$?

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Jose María Almira

3. Remark.

This remark is an answer to a question of J. Schwaiger concerning solutions $f\colon \mathbb{R} \to \mathbb{R}$ of the functional equation

$$f\big(\sqrt{x^2+y^2}\big) = f(x) + f(y). \tag{1}$$

It has been asked in connection with the talk of W. Sintunavarat, in which Ulam's stability of the equation has been discussed.

It is very easy to show that $f\colon \mathbb{R} \to \mathbb{R}$ is a solution to (1) if and only if

$$f(x) = A(x^2), \qquad x \in \mathbb{R}, \tag{2}$$

with some additive $A\colon \mathbb{R} \to \mathbb{R}$.

Namely, one can check without any problem that if $f\colon \mathbb{R} \to \mathbb{R}$ has form (2), then it is a solution of (1).

On the other hand, if $f\colon \mathbb{R} \to \mathbb{R}$ is a solution to (1) and we write $A_0(x) = f(\sqrt{x})$ for $x \in [0,\infty)$, then from (1) we get that $A_0(x+y) = A_0(x) + A_0(y)$ for $x, y \in [0,\infty)$. Next, with $y = 0$ in (1) we deduce that f is even. So, $f(-x) = f(x) = A_0(x^2)$ for $x \in [0,\infty)$. Now, it is enough to observe that there is an additive $A\colon \mathbb{R} \to \mathbb{R}$ with $A_0(x) = A(x)$ for $x \in [0,\infty)$. This completes the proof.

Janusz Brzdęk

4. Remark. *Remark to the talks by Anna Bahyrycz and Zsolt Páles*

In connection with the talks by Anna Bahyrycz and Zsolt Páles presented at this conference, we are able prove the following result.

THEOREM
Let Y be a linear normed space, n be a positive integer and let A and B real numbers. If a function $f\colon \mathbb{R} \to Y$ satisfies

$$\Delta_y^n f(x) = 0, \qquad x \le A,\ y \ge B,$$

then f is a polynomial function of degree $n-1$, that is, it fulfils

$$\Delta_y^n f(x) = 0, \qquad x, y \in \mathbb{R}.$$

In the proof, the following Lemma plays an important role.

LEMMA ([1], [2])
Let $n \ge 1$ and $\lambda \ge 2$ be integers and consider the matrix

$$A = \begin{pmatrix} \alpha_0^{(0)} & \dots & \alpha_0^{(\lambda n)} \\ \vdots & \ddots & \vdots \\ \alpha_{(\lambda-1)n}^{(0)} & \dots & \alpha_{(\lambda-1)n}^{(\lambda n)} \end{pmatrix}$$

with elements

$$\alpha_i^{(i+j)} = \begin{cases} (-1)^j \binom{n}{n-j}, & \textit{if } 0 \le j \le n, \\ 0, & \textit{otherwise} \end{cases}$$

for $i = 0, \ldots, (\lambda - 1)n$ *and* $j = -i, \ldots, \lambda n - i$. *Let* a_i *denote the* i^{th} *row in* A *and let* $b = (\beta^{(0)}, \ldots, \beta^{(\lambda n)})$, *where*

$$\beta^{(j)} = \begin{cases} (-1)^{\frac{j}{\lambda}} \binom{n}{n-\frac{j}{\lambda}}, & \text{if } \lambda \mid j, \\ 0, & \text{if } \lambda \nmid j, \end{cases} \qquad j = 0, \ldots, \lambda n.$$

Then there exist positive integers $K_0, \ldots, K_{(\lambda-1)n}$ *such that*

$$K_0 a_0 + \ldots + K_{(\lambda-1)n} a_{(\lambda-1)n} = b$$

and

$$K_0 + \ldots + K_{(\lambda-1)n} = \lambda^n.$$

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Attila Gilányi

5. Problem.

The following problem was found in the paper:

R. Daniel Mauldin, *Infinite iterated function systems: theory and applications*, Progr. Probab. **37** (1995), 91–110.

It is known that the following equation

$$f(x) - f(x+1) = \frac{1}{(x+1)^2} f\Big(\frac{1}{x+1}\Big)$$

has a unique continuous solution on $(0, \infty)$: $f(x) = \frac{1}{c+x}$, where $c = \frac{1}{\log 2}$.

Consider another functional equation (eq. (8.9) in cited article)

$$f(x) - f(x+2) = \frac{1}{(x+2)^{2h}} f\Big(\frac{1}{x+2}\Big). \tag{1}$$

The author says that:

1. there is exactly one constant $h \in (\frac{1}{2}, 1)$ such that (1) has a nontrivial continuous solution, but
2. the exact value of the constant h is unknown.

Is the first sentence true? Find desired value of $\boldsymbol{h}$**.**

Notice that the meaning of h is strictly connected with Hausdorff dimension of some fractal set.

Grzegorz Guzik

6. Remark.

We show how functional equations can be used to calculate the derivative of the power functions.

Our aim is to derive the formula for the deriative of the power function x^a.

Fix $x > 0$ and let

$$f(a) = (x^a)'.$$

We have

$$f(a+b) = (x^{a+b})' = (x^a)'x^b + x^a(x^b)' = x^b f(a) + x^a f(b).$$

Thus

$$\frac{f(a+b)}{x^{a+b}} = \frac{f(a)}{x^a} + \frac{f(b)}{x^b},$$

which yields

$$\frac{f(a)}{x^a} = ca.$$

Since $f(1) = 1$ we get $c = 1/x$, so

$$f(a) = ax^{a-1}.$$

Similar approach can be used for calculating the derivative of the exponential function.

Alfred Witkowski

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