DOI: 10.2478/aucts-2018-0011

ACTA UNIVERSITATIS CIBINIENSIS – TECHNICAL SERIES
Vol. LXX 2018

NUMERICAL METHODS FOR BVP'S APPEARING IN NONDESTRUCTIVE ULTRASONIC TESTING

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Abstract: *In this article we studies BVP's that can not be solved as P.V.I or BVP with Matlab solvers ODExx or BVPxx since the solutions do not have limit in 0. We propose a numerical algorithm based on Cubic-spline written in Maple 16 [4].*

Key words: Fixed Points, Spline Approximations, Boundary value problems

1. Introduction

The non-destructive ultrasonic testing (US) is available since 2009 and is of-fered to the customers and projects that have high quality demands regarding the used materials.

The US testing detects internal (volumetric) defects of the materials and of the welding seams and is performed with state of the art equipment – Olympus EPOCH LT and EPOCH 600. The personnel of MIRAS own 2 stage accreditation according to SREN ISO 9712. Based on our acquired certifications can issue US conformity certificates according to SEP 1920, SEP 1921, EN 10160, EN 10308 and EN 10228-3.

The minimum dimensions for which the US testing can be performed are 6 mm for steel sheets and 20 mm for round steel bars.

Ultrasonic velocity $(5920\, m/s)$ are:

- axially on the side bumper,for steel sheet;
- radial ends of round steel bars.

Our idea is to determine a frequency band oscillator field limit of 62 decibels emitted ultrasound control for round steel bars. Bumper surface is coated with glycerin. If exist cracks inside the bumper see the third example, then spots appear on its surface, for otherwise not see second example.

First a check is made to see such control device see first example, which in our case consists in issuing a 62 decibels signal in bumper ends.

L.Greengard and L.Rohlin ([2]) studies frequency domain equation for vi-brating string using a parametric Bvp's problem with exact solution:

$$u(x) = \sin(kx)$$

In this article we use the following mathematical inequality:

$$\sin(kx) \le \sin\left(\frac{k}{x}\right), \forall |x| \le 1, x \ne 0, k \in R_+$$

2. Main results

2.1 First example

Let the differential equations:

$$y'(t) = x(t)$$

$$x'(t) = -y(t) + \sin\left(\frac{1}{t}\right) + \left(\frac{1}{t^4}\right) \cdot \left[2t\cos\left(\frac{1}{t}\right) - \sin\left(\frac{1}{t}\right)\right] \quad (1)$$

We define the Poincare map as:

$$f:(0,\infty) \to (-1,1) \quad (2)$$

$$f(x) = \sin\frac{1}{x}$$

$$f(x) = -f(-x) \quad (3)$$

We observe:

$$\nexists \lim_{x \to 0}\left(\sin\frac{1}{x}\right)$$

2.2 Second example

Let the differential equations:

$$y'(t) = x(t)$$

$$x'(t) = -y(t) + t\sin\frac{1}{t} - \frac{\sin\frac{1}{t}}{t^3} \quad (4)$$

We define Poincare map as:

$$f:(0,\infty) \to (-1,1)$$

$$f(x) = x\sin\frac{1}{x} \quad (5)$$

It is easy to observe that:

$$x_k = \frac{2}{(4k+1)\pi}, \quad k \in Z$$

are fixed points of *f* and $x_k \to 0$ when $k \to \infty$, and:

$$f(-x) = f(x)$$

2.3 Third example

Let the differential equations:

$$y'(t) = x(t)$$

$$x'(t) = -y(t) + t\sin\frac{1}{t} + \cos\frac{1}{t} - \frac{3t\sin\frac{1}{t} + \cos\frac{1}{t}}{t^4} \quad (6)$$

We define Poincare map as:

$$f:(0,\infty)\to(-1,2)$$
$$f(x)=x\sin\frac{1}{x}+\cos\frac{1}{x} \qquad (7)$$

It is easy to observe that:

$$x_k=\frac{2}{(4k+1)\pi},\quad k\in Z \quad (8)$$

are fixed points of *f* and $x_k\to 0$ when $k\to\infty$, and:

$$f(-x)=f(x)$$

Also:

$$\nexists \lim_{x\to 0}\left(xsin\frac{1}{x}+cos\frac{1}{x}\right)$$

2.4 Numerical results

Let the Bvp's:

$$y''(x)+y(x)=x\sin\frac{1}{x}-\frac{\sin\frac{1}{x}}{x^3},\quad x\in(0,\infty) \qquad (9)$$

$$y''(x)+y(x)=x\sin\frac{1}{x}+\cos\frac{1}{x}+\frac{-3x\sin\frac{1}{x}+\cos\frac{1}{x}}{x^4},\quad x\in(0,\infty) \quad (10)$$

$$y''(x)+y(x)=\sin\frac{1}{x}+\frac{1}{x^4}\left[2x\cos\frac{1}{x}-\sin\frac{1}{x}\right],\quad x\in(0,\infty) \qquad (11)$$

We observe that:

$$\lim_{x\to\infty}\left(cos\frac{1}{x}+xsin\frac{1}{x}\right)=2$$

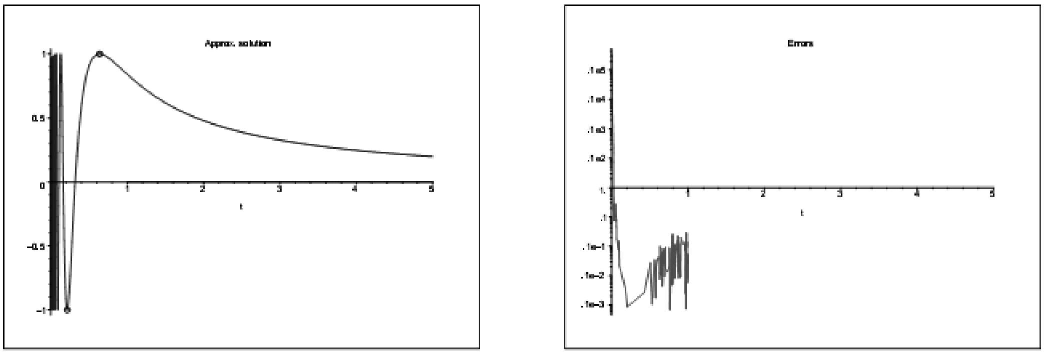
$$\lim_{x\to\infty}\left(xsin\frac{1}{x}\right)=1$$

$$\lim_{x\to\infty}\left(sin\frac{1}{x}\right)=0$$

The differential equations (9, 10, 11) has oscillatory solutions ([3, pp:29]). Choosing inside conditions is essential for getting a better accuracy so we take

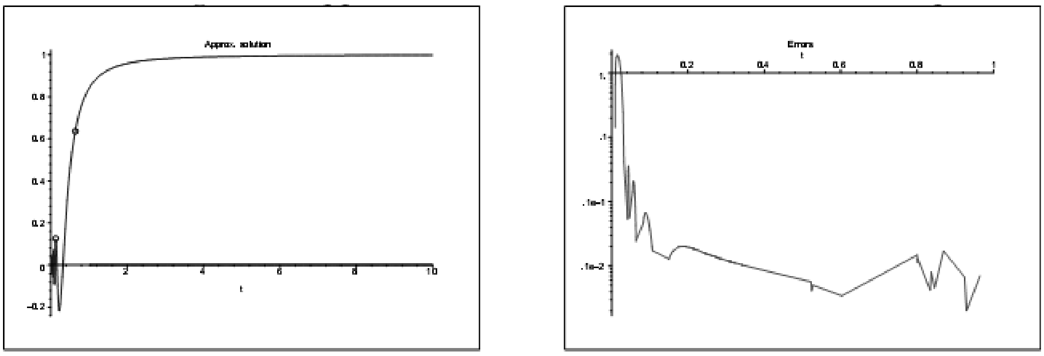
$$y(2/Pi)=2/Pi,\quad y(2/5Pi)=2/5Pi \quad (12)$$

To solve this kind of problems we used the numerical algorithm presented in the book [5, pp: 65-79], and for $n=150$ we obtained the following results depicted in figures (1), (2), (3):



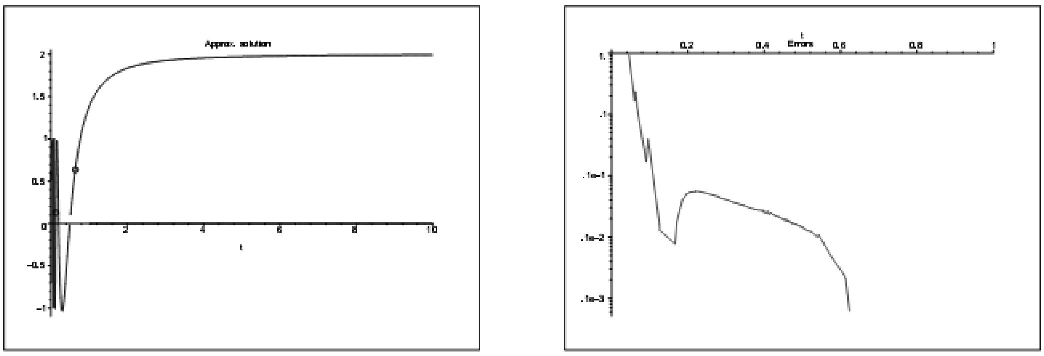
(a) Approximate solution (b) Errors

Figure 1: Approximate solution and Errors – First example



(a) Approximate solution (b) Errors

Figure 2: Approximate solution and Errors – Second example



(a) Approximate solution (b) Errors

Figure 3: Approximate solution and Errors – Third example

3. Conclusions

Using the functions profile and showprofile of Maple we obtained the following results:

Table 1:The results of the function profile and showprofile of Maple

function	depth	calls	time	time%	bytes	bytes%
genspline	1	1	80.903	100.00	3444968128	100.00
total	1	1	80.903	100.00	3444968128	100.00

Henceforth we must proved if choosing the inside conditions (12) we obtain the best approximation. We can generalized the function (7) like:

$$f(x,\varepsilon)=x\sin\frac{1}{x}+\varepsilon\cos\frac{1}{x}$$

$$|f(x,\varepsilon)|\le |x|+1$$

The choise of ε can be done so that can accept bars with cracks in the interval $(\varepsilon, 1)$.

Acknowledgements

The writing of this work benefited enormously from a lot of discussion with Assistant prof. dr. mat. Vasile Crăciunean from “Lucian Blaga” University of Sibiu and conf. dr. mat Radu T.Trimbiţaş from Babeş Bolyai, University of Cluj-Napoca.

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