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RELATIVISTIC ROTATION OF THE RIGID BODY IN THE RODRIGUES – HAMILTON PARAMETERS: LAGRANGE FUNCTION AND EQUATIONS OF MOTION

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ABSTRACT. The main purposes of this research are to obtain Lagrange function for the relativistic rotation of the rigid body, which is generated by metric properties of Riemann space of general relativity and to derive the differential equations, determining the rigid body rotation in the terms of the Rodrigues – Hamilton parameters. The Lagrange function for the relativistic rotation of the rigid body is derived from the Lagrange function of the non-rotation point of masses system in the relativistic approximation.

Keywords: the rigid body rotation, Rodrigues – Hamilton parameters, Lagrange function, the relativistic approximation.

1. BASIC EQUATIONS

The construction of the Lagrange function for the case when a certain set of point masses $m_{n_\alpha} = dm_n$ from the whole system of point masses m_i forms an 'absolutely rigid body' m_n (presented in Figures 1 – 2) in such a way that the condition $\Delta_{n_p n_q} \equiv const$ (Suslov, 1946) (Figure 1) holds for any point masses m_{n_p} and m_{n_q} from the set m_{n_α}. In this case, the body m_n can rotate around its own center of mass with angular velocity $|\overline{\omega}| \geq 0$, the remaining point bodies m_j from the set m_i do not rotate. Here m_j is the mass of the *j*-th point; O_n is the center of masses rigid body m_n; B is barycentre of a masses point system; $B\overline{I}_1\overline{I}_2\overline{I}_3$ is barycentric coordinate system; $O_n\overline{I}_1\overline{I}_2\overline{I}_3$ is a coordinate system of the rigid body, whose axes are parallel to the axes of the barycentric coordinate system; $O_n\overline{i_1}\overline{i_2}\overline{i_3}$ is a coordinate system of the rigid body m_n, whose axes are principal axes of inertia of this body; dm_n (Figure 2) is the element of a set of point of masses of the rigid body m_n; $\overline{\rho}$ is radius vector of the mass element dm_n; $\overline{R}_n^*$, $\overline{\Delta}_{nj}^*$ are barycentric and m_j body vectors of the mass elements dm_n respectively; $\overline{R}_n$, $\overline{\Delta}_{nj}$ are barycentric and m_j body vectors of the center of masses rigid body m_n, respectively; $\overline{R}_j$ is barycentric vector of the point bodies m_j. The coordinate system $O_n\overline{i_1}\overline{i_2}\overline{i_3}$ of the rigid body m_n rotates relatively to barycentric coordinate system with angular velocity $\overline{\omega}$.

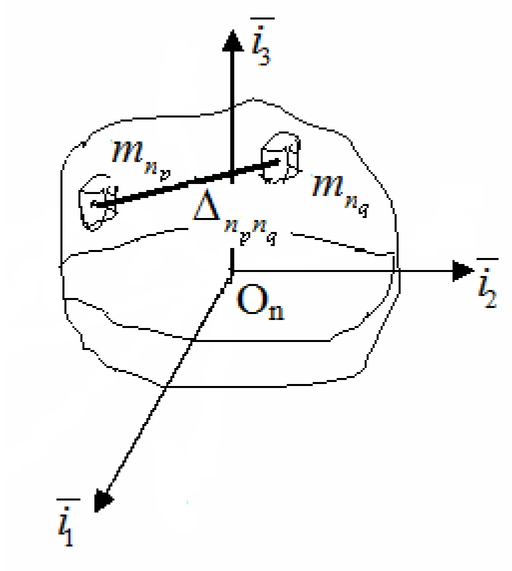

Fig. 1.

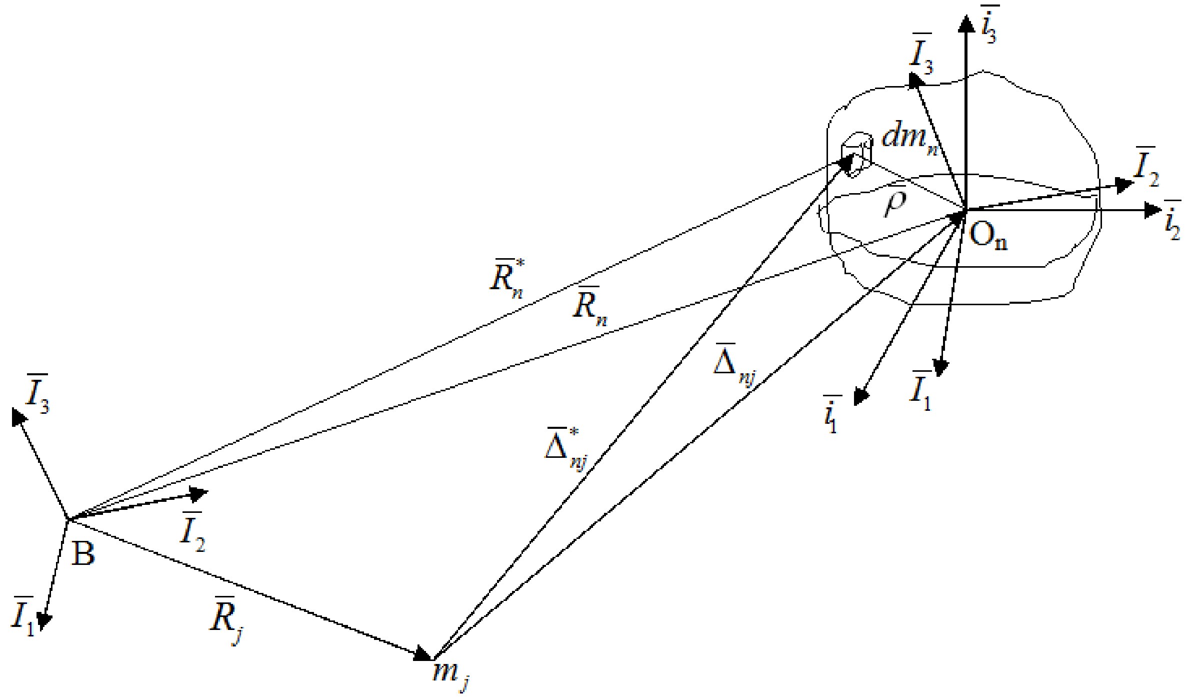

Fig. 2.

The radius vectors of the center of masses O_n of the rigid body m_n in coordinate system $B\overline{I}_1\overline{I}_2\overline{I}_3$ and in the projections on the axes system $O_n\overline{i}_1\overline{i}_2\overline{i}_3$ have forms: $\overline{R}_n = X_n\overline{I}_1 + Y_n\overline{I}_2 + Z_n\overline{I}_3$ and $\overline{R}_n = x_n\overline{i}_1 + y_n\overline{i}_2 + z_n\overline{i}_3$, respectively. The radius vectors of the mass element dm_n of the rigid body m_n in barycentric coordinate system $B\overline{I}_1\overline{I}_2\overline{I}_3$ and in the projections on the axes system $O_n\overline{i}_1\overline{i}_2\overline{i}_3$ have forms: $\overline{R}_n^* = X_n^*\overline{I}_1 + Y_n^*\overline{I}_2 + Z_n^*\overline{I}_3$ and $\overline{R}_n^* = x_n^*\overline{i}_1 + y_n^*\overline{i}_2 + z_n^*\overline{i}_3$, respectively. The radius vector of the mass element dm_n of the rigid body m_n in the coordinate system $O_n\overline{i}_1\overline{i}_2\overline{i}_3$ has a form: $\overline{\rho} = \xi\overline{i}_1 + \eta\overline{i}_2 + \zeta\overline{i}_3$, where ξ, η, ζ are constants. Relation of these three radius vectors has a form: $\overline{R}_n^* = \overline{R}_n + \overline{\rho}$.

After termwise differentiation with respect to time, the ratio of the velocity vectors was obtained: $\dot{\bar{R}}_n^* = \dot{\bar{R}}_n + \dot{\bar{\rho}}$. Velocity of vector of the mass element dm_n of the rigid body m_n in coordinate system $O_n\bar{I}_1\bar{I}_2\bar{I}_3$ has a form (Euler formula): $\dot{\bar{\rho}} = \bar{\omega}\times\bar{\rho} \Rightarrow \dot{\bar{R}}_n^* = \dot{\bar{R}}_n + \bar{\omega}\times\bar{\rho}$. Here and further sign $\times$ stands for the vector product; $\bar{\omega} = \omega_1\bar{i}_1 + \omega_2\bar{i}_2 + \omega_3\bar{i}_3$; ω_1, ω_2, ω_3 are the projections of angular velocity vector on the principal axes of the inertias of the rigid body m_n; $\bar{H}_n = A_n\omega_1\bar{i}_1 + B_n\omega_2\bar{i}_2 + C_n\omega_3\bar{i}_3$ is the angular momentum vector of the rotational motion of the rigid body m_n; A_n, B_n, C_n are the principal moments of inertia of the second order of the rigid body m_n.

$$A_n = \int_{m_n}\left(\eta^2+\zeta^2\right)dm_n;\ B_n = \int_{m_n}\left(\zeta^2+\xi^2\right)dm_n;\ C_n = \int_{m_n}\left(\xi^2+\eta^2\right)dm_n;\ dm_n = p(\xi,\eta,\zeta)\,d\xi d\eta d\zeta;$$

$p(\xi,\eta,\zeta)$ is mass distribution function of the body m_n.

In the particular case when the body m_n is a homogeneous triaxial ellipsoid with semiaxes a, b, c, its moments of inertia are determined by the following expressions (MacMillan,1936):

$$A_n = \frac{m_n}{5}\left(b^2+c^2\right),\quad B_n = \frac{m_n}{5}\left(c^2+a^2\right),\quad C_n = \frac{m_n}{5}\left(a^2+b^2\right).$$

It is easy to see that when $a, b, c \ \rightarrow\ 0$, then $A_n, B_n, C_n \ \rightarrow\ 0$.

The velocity vector of the mass element dm_n in the projections on the axes of the rotating coordinate system $O_n\overline{i_1 i_2 i_3}$ has a form:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} \dot{X}_n^* \\ \dot{Y}_n^* \\ \dot{Z}_n^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} \dot{X}_n \\ \dot{Y}_n \\ \dot{Z}_n \end{pmatrix} + \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$$

or

$$\begin{pmatrix} \dot{x}_n^* \\ \dot{y}_n^* \\ \dot{z}_n^* \end{pmatrix} = \begin{pmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{pmatrix} + \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$$

Here and further $\left(a_{ij}\right)$ is transformation matrix. Besides

$$\begin{pmatrix} \dot{X}_n^* \\ \dot{Y}_n^* \\ \dot{Z}_n^* \end{pmatrix} = \begin{pmatrix} \dot{X}_n \\ \dot{Y}_n \\ \dot{Z}_n \end{pmatrix} + \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}\begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$$

or equivalent formula

$$\begin{pmatrix} \dot{X}_n^* \\ \dot{Y}_n^* \\ \dot{Z}_n^* \end{pmatrix} = \begin{pmatrix} \dot{X}_n \\ \dot{Y}_n \\ \dot{Z}_n \end{pmatrix} + \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}\begin{pmatrix} 0 & -\zeta & \eta \\ \zeta & 0 & -\xi \\ -\eta & \xi & 0 \end{pmatrix}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

In celestial mechanics the rotation of a celestial body about its center of mass is described as the rotation of its principal axes of inertia with respect to the non-rotating body-centric coordinate system. As the variables of this problem, usually used four Rodrigues-Hamilton parameters:

$$\lambda_0 = \cos\frac{\theta}{2}\cos\frac{\psi+\varphi}{2},\ \lambda_1 = \sin\frac{\theta}{2}\cos\frac{\psi-\varphi}{2},\ \lambda_2 = \sin\frac{\theta}{2}\sin\frac{\psi-\varphi}{2},\ \lambda_3 = \cos\frac{\theta}{2}\sin\frac{\psi+\varphi}{2},$$

which are the functions of the Euler angles ψ, θ and φ. These parameters are bounded variables. It is very important for the numerical solution of the problem.

The transformation matrix and the angular velocity vector of the rigid body m_n can be expressed in terms of the Rodrigues-Hamilton parameters:

$$KK=\begin{pmatrix}-\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3\\ -\lambda_1 & \lambda_0 & \lambda_3 & -\lambda_2\\ -\lambda_2 & -\lambda_3 & \lambda_0 & \lambda_1\\ -\lambda_3 & \lambda_2 & -\lambda_1 & \lambda_0\end{pmatrix}\begin{pmatrix}-\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3\\ -\lambda_1 & \lambda_0 & \lambda_3 & -\lambda_2\\ -\lambda_2 & -\lambda_3 & \lambda_0 & \lambda_1\\ -\lambda_3 & \lambda_2 & -\lambda_1 & \lambda_0\end{pmatrix}=$$

$$=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \lambda_0^2+\lambda_1^2-\lambda_2^2-\lambda_3^2 & 2\left(\lambda_0\lambda_3+\lambda_1\lambda_2\right) & 2\left(\lambda_1\lambda_3-\lambda_0\lambda_2\right)\\ 0 & 2\left(\lambda_1\lambda_2-\lambda_0\lambda_3\right) & \lambda_0^2-\lambda_1^2+\lambda_2^2-\lambda_3^2 & 2\left(\lambda_0\lambda_1+\lambda_2\lambda_3\right)\\ 0 & 2\left(\lambda_0\lambda_2+\lambda_1\lambda_3\right) & 2\left(\lambda_2\lambda_3-\lambda_0\lambda_1\right) & \lambda_0^2-\lambda_1^2-\lambda_2^2+\lambda_3^2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & a_{11} & a_{12} & a_{13}\\ 0 & a_{21} & a_{22} & a_{23}\\ 0 & a_{31} & a_{32} & a_{33}\end{pmatrix}.$$

The transformation inverse matrix in terms of the Rodrigues-Hamilton parameters has a form:

$$K^{-1}K^{-1}=\begin{pmatrix}-\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3\\ -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2\\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1\\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0\end{pmatrix}\begin{pmatrix}-\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3\\ -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2\\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1\\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0\end{pmatrix}=$$

$$=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \lambda_0^2+\lambda_1^2-\lambda_2^2-\lambda_3^2 & 2\left(\lambda_1\lambda_2-\lambda_0\lambda_3\right) & 2\left(\lambda_0\lambda_2+\lambda_1\lambda_3\right)\\ 0 & 2\left(\lambda_0\lambda_3+\lambda_1\lambda_2\right) & \lambda_0^2-\lambda_1^2+\lambda_2^2-\lambda_3^2 & 2\left(\lambda_2\lambda_3-\lambda_0\lambda_1\right)\\ 0 & 2\left(\lambda_1\lambda_3-\lambda_0\lambda_2\right) & 2\left(\lambda_0\lambda_1+\lambda_2\lambda_3\right) & \lambda_0^2-\lambda_1^2-\lambda_2^2+\lambda_3^2\end{pmatrix}=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & a_{11} & a_{21} & a_{31}\\ 0 & a_{12} & a_{22} & a_{32}\\ 0 & a_{13} & a_{23} & a_{33}\end{pmatrix}.$$

The angular velocity vector of the rigid body m_n in terms of the Rodrigues-Hamilton parameters has a form:

$$\begin{pmatrix}0\\ \omega_1\\ \omega_2\\ \omega_3\end{pmatrix}=2\begin{pmatrix}-\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3\\ -\lambda_1 & \lambda_0 & \lambda_3 & -\lambda_2\\ -\lambda_2 & -\lambda_3 & \lambda_0 & \lambda_1\\ -\lambda_3 & \lambda_2 & -\lambda_1 & \lambda_0\end{pmatrix}\begin{pmatrix}\dot\lambda_0\\ \dot\lambda_1\\ \dot\lambda_2\\ \dot\lambda_3\end{pmatrix}\quad \begin{pmatrix}0\\ \omega_X\\ \omega_Y\\ \omega_Z\end{pmatrix}=2\begin{pmatrix}-\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3\\ -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2\\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1\\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0\end{pmatrix}\begin{pmatrix}\dot\lambda_0\\ \dot\lambda_1\\ \dot\lambda_2\\ \dot\lambda_3\end{pmatrix}. \tag{1.1}$$

Here $\omega_1, \omega_2, \omega_3$ are the components of the vector $\overline{\omega}$ in the coordinate system $O_n\overline{i_1}\overline{i_2}\overline{i_3}$; ω_X, ω_Y, ω_Z are the projections of this vector on the axes of the non-rotating $O_n\overline{I}_1\overline{I}_2\overline{I}_3$ reference system.

Relation between these projections in the different coordinate systems is:

$$\begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \lambda_0^2+\lambda_1^2-\lambda_2^2-\lambda_3^2 & 2(\lambda_0\lambda_3+\lambda_1\lambda_2) & 2(\lambda_1\lambda_3-\lambda_0\lambda_2) \\ 0 & 2(\lambda_1\lambda_2-\lambda_0\lambda_3) & \lambda_0^2-\lambda_1^2+\lambda_2^2-\lambda_3^2 & 2(\lambda_0\lambda_1+\lambda_2\lambda_3) \\ 0 & 2(\lambda_0\lambda_2+\lambda_1\lambda_3) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) & \lambda_0^2-\lambda_1^2-\lambda_2^2+\lambda_3^2 \end{pmatrix} \begin{pmatrix} 0 \\ \omega_X \\ \omega_Y \\ \omega_Z \end{pmatrix} =$$

$$= KK \begin{pmatrix} 0 \\ \omega_X \\ \omega_Y \\ \omega_Z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a_{11} & a_{12} & a_{13} \\ 0 & a_{21} & a_{22} & a_{23} \\ 0 & a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 0 \\ \omega_X \\ \omega_Y \\ \omega_Z \end{pmatrix}$$

The velocity vector in terms of the Rodrigues-Hamilton parameters is:

$$\begin{pmatrix} 0 \\ \dot{X}_n^* \\ \dot{Y}_n^* \\ \dot{Z}_n^* \end{pmatrix} = \begin{pmatrix} 0 \\ \dot{X}_n \\ \dot{Y}_n \\ \dot{Z}_n \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a_{11} & a_{21} & a_{31} \\ 0 & a_{12} & a_{22} & a_{32} \\ 0 & a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\omega_3 & \omega_2 \\ 0 & \omega_3 & 0 & -\omega_1 \\ 0 & -\omega_2 & \omega_1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \xi \\ \eta \\ \zeta \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ \dot{X}_n \\ \dot{Y}_n \\ \dot{Z}_n \end{pmatrix} + K^{-1}K^{-1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\omega_3 & \omega_2 \\ 0 & \omega_3 & 0 & -\omega_1 \\ 0 & -\omega_2 & \omega_1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \xi \\ \eta \\ \zeta \end{pmatrix}$$

The coordinate system $O_n\overline{i_1}\overline{i_2}\overline{i_3}$ rotates relative to the coordinate system $O_n\overline{I_1}\overline{I_2}\overline{I_3}$ with an angular velocity $\bar{\omega}$, which can be equal to zero. The coordinate system $O_n\overline{I_1}\overline{I_2}\overline{I_3}$ rotates relative to the coordinate system $O_n\overline{i_1}\overline{i_2}\overline{i_3}$ with angular velocity $-\bar{\omega}$.

2. CALCULATION OF LAGRANGE FUNCTION

Lagrange function for the relativistic rotation of the rigid body is derived from Lagrange function of the non-rotating point of masses system in the relativistic approximation, which has a form (Landau, Lifshitz, 1967):

$$\begin{aligned} L = {} & \frac{1}{2}\sum_i m_i \dot{\bar{R}}_i^2 + \frac{1}{2}\sum_i\sum_{k\neq i}\frac{Gm_im_k}{\Delta_{ik}} + \frac{1}{c^2}\left\{\frac{3}{2}\sum_i\sum_{k\neq i}\frac{Gm_im_k}{\Delta_{ik}}\dot{\bar{R}}_i^2 + \frac{1}{8}\sum_i m_i\dot{\bar{R}}_i^4 - \right. \\ & \left. -\frac{1}{4}\sum_i\sum_{k\neq i}\frac{Gm_im_k}{\Delta_{ik}}\left[7\dot{\bar{R}}_i\cdot\dot{\bar{R}}_k + \dot{\bar{R}}_i\cdot\frac{\bar{R}_i-\bar{R}_k}{\Delta_{ik}}\dot{\bar{R}}_k\cdot\frac{\bar{R}_i-\bar{R}_k}{\Delta_{ik}} + 2\sum_{s\neq i}\frac{Gm_s}{\Delta_{is}}\right]\right\}. \end{aligned} \tag{2.1}$$

Here m_i, m_k, m_s are the mass of the i-th, k-th and s-th points, respectively; $\Delta_{ij} = \sqrt{(X_i-X_j)^2+(Y_i-Y_j)^2+(Z_i-Z_j)^2} = |\bar{R}_i-\bar{R}_j|$, where $j=k$ or s; $\bar{R}_i, \dot{\bar{R}}_i, \bar{R}_k, \dot{\bar{R}}_k$ are barycentric positions and velocities of these points of mass; c is velocity of light in vacuum; G is the gravitational constant.

In the following decompositions of the sums, only in body terms for the single sum and perturbing terms between masses of the rigid body and others point of masses in the second sums are retained:

$$\sum_i f_i = \sum_n f_n + \sum_j f_j = \qquad\qquad \sum_i \sum_{l \neq i} f_{il} = \sum_n \sum_j f_{nj} + \sum_j \sum_{k \neq j} f_{jk} + \sum_j \sum_n f_{jn} =$$

$$= \underbrace{\sum_n f_n}_{\text{in body terms}} + \underbrace{\cancel{\sum_j f_j}}_{\text{out body terms}} \qquad\qquad = \underbrace{\sum_n \sum_j f_{nj}}_{\text{pertubations mixed terms}} + \underbrace{\cancel{\sum_j \sum_{k \neq j} f_{jk}}}_{\text{out body terms}} + \underbrace{\sum_j \sum_n f_{jn}}_{\text{pertubations mixed terms}}$$

$$\Rightarrow \sum_i \sum_{l \neq i} f_{il} \sum_{s \neq i} g_{is} = \sum_n \sum_j f_{nj} \sum_j g_{nj} + \sum_j \sum_n f_{jn} \sum_n g_{jn}$$

Here f_i, f_{il}, g_{is} are some functions of point bodies under sums of the expression (2.1).

$$\begin{aligned}
L = {} & \frac{1}{2}\sum_n dm_n \dot{\bar R}_n^{*2} + \frac{1}{2}\sum_j m_j \dot{\bar R}_j^{2} + \frac{1}{2}\sum_n \sum_j \frac{Gdm_n m_j}{\Delta^*_{nj}} + \frac{1}{2}\sum_j \sum_{k \neq j} \frac{Gm_j m_k}{\Delta_{jk}} + \frac{1}{2}\sum_j \sum_n \frac{Gm_j dm_n}{\Delta^*_{nj}} + \\
& + \frac{1}{c^2}\left\{ \frac{3}{2}\sum_n \sum_j \frac{Gdm_n m_j}{\Delta^*_{nj}} \dot{\bar R}_n^{*2} + \frac{3}{2}\sum_j \sum_{k \neq j} \frac{Gm_j m_k}{\Delta_{jk}} \dot{\bar R}_j^{2} + \frac{3}{2}\sum_j \sum_n \frac{Gm_j dm_n}{\Delta^*_{nj}} \dot{\bar R}_j^{2} + \right. \\
& + \frac{1}{8}\sum_n dm_n \dot{\bar R}_n^{*4} + \frac{1}{8}\sum_j m_j \dot{\bar R}_j^{4} - \\
& - \frac{1}{4}\sum_n \sum_j \frac{Gdm_n m_j}{\Delta^*_{nj}} \left[7\dot{\bar R}_n^{*} \cdot \dot{\bar R}_j + \dot{\bar R}_n^{*} \cdot \frac{\bar R_n^* - \bar R_j}{\Delta^*_{nj}} \dot{\bar R}_j \cdot \frac{\bar R_n^* - \bar R_j}{\Delta^*_{nj}} + 2\sum_j \frac{Gm_j}{\Delta^*_{nj}} \right] - \\
& \left. - \frac{1}{4}\sum_j \sum_n \frac{Gm_j dm_n}{\Delta^*_{nj}} \left[7\dot{\bar R}_j \cdot \dot{\bar R}_n^{*} + \dot{\bar R}_j \cdot \frac{\bar R_j - \bar R_n^*}{\Delta^*_{nj}} \dot{\bar R}_n^{*} \cdot \frac{\bar R_j - \bar R_n^*}{\Delta^*_{nj}} + 2\sum_n \frac{Gdm_n}{\Delta^*_{nj}} \right] \right\} =
\end{aligned}$$

$$\begin{aligned}
= {} & \frac{1}{2}\sum_n dm_n \dot{\bar R}_n^{*2} + \cancel{\frac{1}{2}\sum_j m_j \dot{\bar R}_j^{2}} + \frac{1}{2}\sum_n \sum_j \frac{Gdm_n m_j}{\Delta^*_{nj}} + \cancel{\frac{1}{2}\sum_j \sum_{k \neq j} \frac{Gm_j m_k}{\Delta_{jk}}} + \frac{1}{2}\sum_j \sum_n \frac{Gm_j dm_n}{\Delta^*_{nj}} + \\
& + \frac{1}{c^2}\left\{ \frac{3}{2}\sum_n \sum_j \frac{Gdm_n m_j}{\Delta^*_{nj}} \dot{\bar R}_n^{*2} + \cancel{\frac{3}{2}\sum_j \sum_{k \neq j} \frac{Gm_j m_k}{\Delta_{jk}} \dot{\bar R}_j^{2}} + \frac{3}{2}\sum_j \sum_n \frac{Gm_j dm_n}{\Delta^*_{nj}} \dot{\bar R}_j^{2} + \right. \\
& + \frac{1}{8}\sum_n dm_n \dot{\bar R}_n^{*4} + \cancel{\frac{1}{8}\sum_j m_j \dot{\bar R}_j^{4}} - \\
& - \frac{1}{4}\sum_n \sum_j \frac{Gdm_n m_j}{\Delta^*_{nj}} \left[7\dot{\bar R}_n^{*} \cdot \dot{\bar R}_j + \dot{\bar R}_n^{*} \cdot \frac{\bar R_n^* - \bar R_j}{\Delta^*_{nj}} \dot{\bar R}_j \cdot \frac{\bar R_n^* - \bar R_j}{\Delta^*_{nj}} + 2\sum_j \frac{Gm_j}{\Delta^*_{nj}} \right] - \\
& \left. - \frac{1}{4}\sum_j \sum_n \frac{Gm_j dm_n}{\Delta^*_{nj}} \left[7\dot{\bar R}_j \cdot \dot{\bar R}_n^{*} + \dot{\bar R}_j \cdot \frac{\bar R_j - \bar R_n^*}{\Delta^*_{nj}} \dot{\bar R}_n^{*} \cdot \frac{\bar R_j - \bar R_n^*}{\Delta^*_{nj}} + 2\sum_n \frac{Gdm_n}{\Delta^*_{nj}} \right] \right\}
\end{aligned}$$

After the reduction of similar terms Lagrange function for the relativistic rotation of the rigid body m_n has the form:

$$L_n = \frac{1}{2}\sum_n dm_n \dot{\overline{R}}_n^{*2} + \sum_n\sum_j \frac{Gdm_n m_j}{\Delta_{nj}^*} +$$
$$+\frac{1}{c^2}\left\{\frac{3}{2}\sum_n\sum_j \frac{Gdm_n m_j}{\Delta_{nj}^*}\dot{\overline{R}}_n^{*2} + \frac{3}{2}\sum_n\sum_j \frac{Gdm_n m_j}{\Delta_{nj}^*}\dot{\overline{R}}_j^{\,2} + \frac{1}{8}\sum_n dm_n \dot{\overline{R}}_n^{*4} -\right.$$
$$-\frac{1}{2}\sum_n\sum_j \frac{Gdm_n m_j}{\Delta_{nj}^*}\left[7\dot{\overline{R}}_n^* \cdot \dot{\overline{R}}_j + \dot{\overline{R}}_n^* \cdot \frac{\overline{R}_n^* - \overline{R}_j}{\Delta_{nj}^*}\dot{\overline{R}}_j \cdot \frac{\overline{R}_n^* - \overline{R}_j}{\Delta_{nj}^*}\right] - \tag{2.2}$$
$$\left.-\frac{1}{2}\sum_n dm_n\left(\sum_j \frac{Gm_j}{\Delta_{nj}^*}\right)^2 - \frac{1}{2}\sum_j m_j\left(\sum_n \frac{Gdm_n}{\Delta_{nj}^*}\right)^2\right\}$$

After the easy replacement of the summation signs with respect to the index n by the integration signs over the range m_n these terms take the form:

$$L_n = \frac{1}{2}\int_{m_n} \dot{\overline{R}}_n^{*2} dm_n + \sum_{j\neq n} Gm_j \int_{m_n} \frac{dm_n}{\Delta_{nj}^*} +$$
$$+\frac{1}{8c^2}\int_{m_n} \dot{\overline{R}}_n^{*4} dm_n + \sum_{j\neq n}\frac{Gm_j}{c^2}\left\{\frac{3}{2}\int_{m_n}\dot{\overline{R}}_n^{*2}\frac{dm_n}{\Delta_{nj}^*} + \frac{3}{2}\dot{\overline{R}}_j^{\,2}\int_{m_n}\frac{dm_n}{\Delta_{nj}^*} -\right.$$
$$\left.-\frac{7}{2}\int_{m_n}\dot{\overline{R}}_n^* \cdot \dot{\overline{R}}_j \frac{dm_n}{\Delta_{nj}^*} - \frac{1}{2}\int_{m_n}\dot{\overline{R}}_n^* \cdot \frac{\left(\overline{R}_n^* - \overline{R}_j\right)}{\Delta_{nj}^*}\dot{\overline{R}}_j \cdot \frac{\left(\overline{R}_n^* - \overline{R}_j\right)}{\Delta_{nj}^*}\frac{dm_n}{\Delta_{nj}^*}\right\} - \tag{2.3}$$
$$-\frac{1}{2c^2}\int_{m_n}\left(\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^*}\right)^2 dm_n - \frac{1}{2c^2}\sum_{j\neq n} m_j\left(\int_{m_n}\frac{Gdm_n}{\Delta_{nj}^*}\right)^2.$$

Here and further $\Delta_{nj}^* = \left|\overline{R}_n - \overline{R}_j + \overline{\rho}\right|$. Usually in celestial mechanics $\overline{\rho} << \overline{\Delta}_{nj}$. The integrands are expanded in Taylor series in powers of the parameter $\frac{|\overline{\rho}|}{\Delta_{nj}}$, which is a small quantity, for example, in view of the fact that the dimensions of the large bodies of the solar system are small in comparison with the distances between them. By the definition of the coordinate system $O_n\overline{i_1}\overline{i_2}\overline{i_3}$ all integrals of the form

$\int_{m_n}\xi dm_n, \int_{m_n}\eta dm_n, \int_{m_n}\zeta dm_n, \int_{m_n}\xi\eta dm_n, \int_{m_n}\eta\zeta dm_n, \int_{m_n}\zeta\xi dm_n$ are identically equal to zero (2.4) (MacMillan, 1936).

In order to calculate certain integrals in Lagrange function (2.3), it is convenient to have some formulas and expressions, while using the identical transformations of the mixed product of vectors $\overline{a}\cdot\left(\overline{b}\times\overline{c}\right) = \overline{b}\cdot\left(\overline{c}\times\overline{a}\right) = \overline{c}\cdot\left(\overline{a}\times\overline{b}\right)$:

$$\dot{\overline{R}}_n^{*2} = \dot{\overline{R}}_n^2 + 2\dot{\overline{R}}_n\cdot\left(\overline{\omega}\times\overline{\rho}\right) + \left(\overline{\omega}\times\overline{\rho}\right)^2 = \dot{\overline{R}}_n^2 + 2\overline{\rho}\cdot\left(\dot{\overline{R}}_n\times\overline{\omega}\right) + \left(\overline{\omega}\times\overline{\rho}\right)^2;$$

$$\dot{\overline{R}}_n^*\cdot\dot{\overline{R}}_j = \dot{\overline{R}}_n\cdot\dot{\overline{R}}_j + \dot{\overline{R}}_j\cdot\left(\overline{\omega}\times\overline{\rho}\right) = \dot{\overline{R}}_n\cdot\dot{\overline{R}}_j + \overline{\rho}\cdot\left(\dot{\overline{R}}_j\times\overline{\omega}\right);$$

$$\Delta_{nj}^* = \sqrt{\Delta_{nj}^2 + 2\overline{\rho}\cdot\left(\overline{R}_n - \overline{R}_j\right) + \overline{\rho}^2} \Rightarrow$$

$$\frac{1}{\Delta_{nj}^{*}}=\frac{1}{\Delta_{nj}}\left[1+2\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^{2}}+\frac{\bar{\rho}^{2}}{\Delta_{nj}^{2}}\right]^{-\frac{1}{2}}=\frac{1}{\Delta_{nj}}-\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^{3}}-\frac{\bar{\rho}^{2}}{2\Delta_{nj}^{3}}+\frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^{2}}{\Delta_{nj}^{5}}+\ldots;$$

$$\frac{1}{\Delta_{nj}^{*2}}=\frac{1}{\Delta_{nj}^{2}}\left[1+2\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^{2}}+\frac{\bar{\rho}^{2}}{\Delta_{nj}^{2}}\right]^{-2}=\frac{1}{\Delta_{nj}^{2}}\left\{1-4\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^{2}}-2\frac{\bar{\rho}^{2}}{\Delta_{nj}^{2}}+12\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^{2}}{\Delta_{nj}^{4}}+\ldots\right\};$$

$$\frac{1}{\Delta_{nj}^{*3}}=\frac{1}{\Delta_{nj}^{3}}\left[1+2\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^{2}}+\frac{\bar{\rho}^{2}}{\Delta_{nj}^{2}}\right]^{-\frac{3}{2}}=\frac{1}{\Delta_{nj}^{3}}-3\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^{5}}-\frac{3}{2}\frac{\bar{\rho}^{2}}{\Delta_{nj}^{5}}+\frac{15}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^{2}}{\Delta_{nj}^{7}}+\ldots;$$

Analytical calculations of auxiliary formulas, expressions and integrals necessary for calculating the Lagrange function (2.3) for the relativistic rotation of the rigid body m_n are given in Appendix A.

2. 1 Calculation of the Newtonian part of the Lagrange function

The first line of the expression (2.3) turned out to be the Newtonian part of Lagrange function for the relativistic rotation of the rigid body:

$$L_n^{Newton}=\frac{1}{2}\int_{m_n}\dot{\bar{R}}_n^{*2}dm_n+\sum_{j\neq n}Gm_j\int_{m_n}\frac{dm_n}{\Delta_{nj}^{*}}. \tag{2.5}$$

The first certain integral, which is dependent from $\dot{\bar{R}}_n^{*}$, is computed by using integral (A.1):

$\frac{1}{2}\int_{m_n}\dot{\bar{R}}_n^{*2}dm_n=\frac{1}{2}\dot{\bar{R}}_n^{2}m_n+\frac{1}{2}\int_{m_n}\left(\bar{\omega}\times\bar{\rho}\right)^{2}dm_n=\frac{1}{2}\dot{\bar{R}}_n^{2}m_n+\frac{1}{2}\bar{H}_n\cdot\bar{\omega}$ is the kinetic energy of the body m_n (the sum of the kinetic energy of the translational motion and of the rotational motion).

The second certain integral, which is independent from $\dot{\bar{R}}_n^{*}$, is computed by using integrals (A.2) and (A.3):

$$\sum_{j\neq n}Gm_j\int_{m_n}\frac{dm_n}{\Delta_{nj}^{*}}=\sum_{j\neq n}Gm_j\left\langle\frac{m_n}{\Delta_{nj}}-\frac{1}{2\Delta_{nj}^{3}}\int_{m_n}\bar{\rho}^{2}dm_n+\frac{3}{2\Delta_{nj}^{5}}\int_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^{2}dm_n\right\rangle=$$

$$=\sum_{j\neq n}Gm_j\left\langle\frac{m_n}{\Delta_{nj}}+\right. \tag{2.6}$$

$$\left.+\frac{1}{2\Delta_{nj}^{3}}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^{2}}\left[A_n\left(x_n-x_j\right)^{2}+B_n\left(y_n-y_j\right)^{2}+C_n\left(z_n-z_j\right)^{2}\right]\right\}\right\rangle;$$

It is the force function of gravitational interaction of the body m_n with other bodies.

2. 2 Calculation of the relativistic part of the Lagrange function

The relativistic part of the Lagrange function for the relativistic rotation of the rigid body, taking into account the interaction of the body m_n with other bodies, can be represented as follows:

$$\Delta L_n = \frac{1}{8c^2}\int_{m_n} \dot{\bar{R}}_n^{*4} dm_n + \sum_{j\neq n}\frac{Gm_j}{c^2}\left\{\frac{3}{2}\int_{m_n}\dot{\bar{R}}_n^{*2}\frac{dm_n}{\Delta_{nj}^*} + \frac{3}{2}\dot{\bar{R}}_j^{\,2}\int_{m_n}\frac{dm_n}{\Delta_{nj}^*} - \frac{7}{2}\int_{m_n}\dot{\bar{R}}_n^*\cdot\dot{\bar{R}}_j\frac{dm_n}{\Delta_{nj}^*} - \right.$$
$$\left. -\frac{1}{2}\int_{m_n}\dot{\bar{R}}_n^*\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\dot{\bar{R}}_j\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\frac{dm_n}{\Delta_{nj}^*}\right\} - \qquad (2.7)$$
$$-\frac{1}{2c^2}\int_{m_n}\left(\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^*}\right)^2 dm_n - \frac{1}{2c^2}\sum_{j\neq n} m_j\left(\int_{m_n}\frac{Gdm_n}{\Delta_{nj}^*}\right)^2.$$

2. 2.1 Calculation of definite integrals dependent on $\dot{\bar{R}}_n^*$

The geodesic rotation of the set of point masses forming the body m_n is generated by the relativistic terms of the Lagrange function ΔL_n, which contain the angular velocity of rotation of the body m_n around its own center of mass $\bar{\omega}$, that is depending on the barycentric velocity vector $\dot{\bar{R}}_n^*$ of the element dm_n :

$$\Delta L_n^G = \frac{1}{8c^2}\int_{m_n}\dot{\bar{R}}_n^{*4}dm_n + \sum_{j\neq n}\frac{Gm_j}{c^2}\left\{\frac{3}{2}\int_{m_n}\dot{\bar{R}}_n^{*2}\frac{dm_n}{\Delta_{nj}^*} - \frac{7}{2}\int_{m_n}\dot{\bar{R}}_n^*\cdot\dot{\bar{R}}_j\frac{dm_n}{\Delta_{nj}^*} - \right.$$
$$\left. -\frac{1}{2}\int_{m_n}\dot{\bar{R}}_n^*\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\dot{\bar{R}}_j\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\frac{dm_n}{\Delta_{nj}^*}\right\}. \qquad (2.8)$$

As a result of combining integrals (A.4) and (A.1), the integral $\frac{1}{8}\int_{m_n}\dot{\bar{R}}_n^{*4}dm_n$ is computed:

$$\frac{1}{8}\int_{m_n}\dot{\bar{R}}_n^{*4}dm_n = \frac{1}{8}\dot{\bar{R}}_n^4 m_n + \frac{1}{2}\int_{m_n}\left[\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)\right]^2 dm_n + \frac{1}{4}\dot{\bar{R}}_n^2\int_{m_n}\left(\bar{\omega}\times\bar{\rho}\right)^2 dm_n =$$
$$= \frac{1}{8}\dot{\bar{R}}_n^4 m_n + \frac{1}{4}\dot{\bar{R}}_n^2\left(A_n\omega_1^2 + B_n\omega_2^2 + C_n\omega_3^2\right) + \left(\dot{y}_n\omega_3 - \dot{z}_n\omega_2\right)^2\frac{B_n + C_n - A_n}{4} +$$
$$+\left(\dot{z}_n\omega_1 - \dot{x}_n\omega_3\right)^2\frac{C_n + A_n - B_n}{4} + \left(\dot{x}_n\omega_2 - \dot{y}_n\omega_1\right)^2\frac{A_n + B_n - C_n}{4};$$

As a result of combining integrals (A.1), (A.5), (A.2) and (A.3), the integral $\frac{3}{2}\int_{m_n}\frac{\dot{\bar{R}}_n^{*2}}{\Delta_{nj}^*}dm_n$ is computed:

$$\frac{3}{2}\int_{m_n}\frac{\dot{\bar{R}}_n^{*2}}{\Delta_{nj}^*}dm_n = \frac{3}{2}\frac{\dot{\bar{R}}_n^{\,2}}{\Delta_{nj}}m_n + \frac{3}{2\Delta_{nj}}\int_{m_n}\left(\bar{\omega}\times\bar{\rho}\right)^2 dm_n - \frac{3}{\Delta_{nj}^3}\int_{m_n}\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)dm_n -$$
$$-\frac{3\dot{\bar{R}}_n^2}{4\Delta_{nj}^3}\int_{m_n}\bar{\rho}^2 dm_n + \frac{9}{4}\frac{\dot{\bar{R}}_n^2}{\Delta_{nj}^5}\int_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)\right]^2 dm_n = \frac{3}{2}\frac{\dot{\bar{R}}_n^{\,2}}{\Delta_{nj}}m_n + \frac{3\dot{\bar{R}}_n^2}{4\Delta_{nj}^3}\left\{A_n + B_n + C_n - \right.$$
$$\left.-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n - x_j\right)^2 + B_n\left(y_n - y_j\right)^2 + C_n\left(z_n - z_j\right)^2\right]\right\} - \frac{3}{2\Delta_{nj}^3}\left\{\bar{H}_n\cdot\left(\bar{R}_n - \bar{R}_j\right)\times\dot{\bar{R}}_n + \right.$$
$$+C_n\left(z_n - z_j\right)\left(\omega_1\dot{y}_n - \omega_2\dot{x}_n\right) + B_n\left(y_n - y_j\right)\left(\omega_3\dot{x}_n - \omega_1\dot{z}_n\right) + A_n\left(x_n - x_j\right)\left(\omega_2\dot{z}_n - \omega_3\dot{y}_n\right) +$$
$$+C_n\dot{z}_n\left[\omega_1\left(y_n - y_j\right) - \omega_2\left(x_n - x_j\right)\right] + B_n\dot{y}_n\left[\omega_3\left(x_n - x_j\right) - \omega_1\left(z_n - z_j\right)\right] +$$

$$+A_n\dot{x}_n\left[\omega_2\left(z_n-z_j\right)-\omega_3\left(y_n-y_j\right)\right]\Big\}+\frac{3}{2\Delta_{nj}}\left(A_n\omega_1^2+B_n\omega_2^2+C_n\omega_3^2\right);$$

As a result of combining integrals (A.6), (A.2) and (A.3), the integral $-\frac{7}{2}\int\limits_{m_n}\frac{\dot{\bar{R}}_n^*\cdot\dot{\bar{R}}_j}{\Delta_{nj}^*}dm_n$ is computed:

$$-\frac{7}{2}\int\limits_{m_n}\frac{\dot{\bar{R}}_n^*\cdot\dot{\bar{R}}_j}{\Delta_{nj}^*}dm_n=-\frac{7}{2}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}}m_n+\frac{7}{2\Delta_{nj}^3}\int\limits_{m_n}\bar{\rho}\cdot\left(\dot{\bar{R}}_j\times\bar{\omega}\right)\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)dm_n+\frac{7\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{4\Delta_{nj}^3}\int\limits_{m_n}\bar{\rho}^2dm_n-$$

$$-\frac{21}{4}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}^5}\int\limits_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2dm_n=-\frac{7}{2}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}}m_n-\frac{7}{4}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}^3}\Big\{A_n+B_n+C_n-$$

$$-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\Big\}+\frac{7}{4\Delta_{nj}^3}\Big\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_j+$$

$$+C_n\left(z_n-z_j\right)\left(\omega_1\dot{y}_n-\omega_2\dot{x}_n\right)+B_n\left(y_n-y_j\right)\left(\omega_3\dot{x}_n-\omega_1\dot{z}_n\right)+A_n\left(x_n-x_j\right)\left(\omega_2\dot{z}_n-\omega_3\dot{y}_n\right)+$$

$$+C_n\dot{z}_n\left[\omega_1\left(y_n-y_j\right)-\omega_2\left(x_n-x_j\right)\right]+B_n\dot{y}_n\left[\omega_3\left(x_n-x_j\right)-\omega_1\left(z_n-z_j\right)\right]+$$

$$+A_n\dot{x}_n\left[\omega_2\left(z_n-z_j\right)-\omega_3\left(y_n-y_j\right)\right]\Big\};$$

As a result of combining integrals (A.2), (A.3), (A.7), (A.8), (A.9), (A.10) and (A.12), next integral is computed:

$$-\frac{1}{2}\int\limits_{m_n}\dot{\bar{R}}_n^*\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\dot{\bar{R}}_j\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\frac{1}{\Delta_{nj}^*}dm_n=-\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{2\Delta_{nj}^3}\int\limits_{m_n}dm_n+$$

$$+\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{3}{4\Delta_{nj}^5}\int\limits_{m_n}\bar{\rho}^2dm_n-\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{15}{4\Delta_{nj}^7}\int\limits_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2dm_n+$$

$$+\frac{3\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\int\limits_{m_n}\bar{\rho}\cdot\dot{\bar{R}}_j\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)dm_n+\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\int\limits_{m_n}\bar{\rho}\cdot\dot{\bar{R}}_n\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)dm_n-$$

$$-\frac{1}{2\Delta_{nj}^3}\int\limits_{m_n}\bar{\rho}\cdot\dot{\bar{R}}_n\bar{\rho}\cdot\dot{\bar{R}}_jdm_n+\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\int\limits_{m_n}\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)dm_n-$$

$$-\frac{1}{2\Delta_{nj}^3}\int\limits_{m_n}\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\bar{\rho}\cdot\dot{\bar{R}}_jdm_n=-\frac{1}{2}\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{\Delta_{nj}^3}m_n+$$

$$+\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{15}{4\Delta_{nj}^7}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]-$$

$$-\frac{3\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\left[\dot{x}_j\left(x_n-x_j\right)A_n+\dot{y}_j\left(y_n-y_j\right)B_n+\dot{z}_j\left(z_n-z_j\right)C_n\right]-$$

$$-\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\left[\dot{x}_n\left(x_n-x_j\right)A_n+\dot{y}_n\left(y_n-y_j\right)B_n+\dot{z}_n\left(z_n-z_j\right)C_n\right]-$$

$$-\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{4\Delta_{nj}^3}\left(A_n+B_n+C_n\right)+\frac{1}{2\Delta_{nj}^3}\left[\dot{x}_n\dot{x}_jA_n+\dot{y}_n\dot{y}_jB_n+\dot{z}_n\dot{z}_jC_n\right]+$$

$$+\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\Big[\left(x_n-x_j\right)\left(y_n-y_j\right)\omega_3\left(B_n-A_n\right)+\left(z_n-z_j\right)\left(x_n-x_j\right)\omega_2\left(A_n-C_n\right)+$$
$$+\left(y_n-y_j\right)\left(z_n-z_j\right)\omega_1\left(C_n-B_n\right)\Big]+$$
$$+\frac{1}{4\Delta_{nj}^3}\Big\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_j-C_n\left(z_n-z_j\right)\left(\omega_1\dot{y}_n-\omega_2\dot{x}_n\right)-B_n\left(y_n-y_j\right)\left(\omega_3\dot{x}_n-\omega_1\dot{z}_n\right)-$$
$$-A_n\left(x_n-x_j\right)\left(\omega_2\dot{z}_n-\omega_3\dot{y}_n\right)-C_n\dot{z}_n\left[\omega_1\left(y_n-y_j\right)-\omega_2\left(x_n-x_j\right)\right]-$$
$$-B_n\dot{y}_n\left[\omega_3\left(x_n-x_j\right)-\omega_1\left(z_n-z_j\right)\right]-A_n\dot{x}_n\left[\omega_2\left(z_n-z_j\right)-\omega_3\left(y_n-y_j\right)\right]\Big\};$$

2. 2.2 Calculation of definite integrals independent of $\dot{\bar{R}}_n^*$

$$L_n^{NG}=\sum_{j\neq n}\frac{Gm_j}{c^2}\frac{3}{2}\dot{\bar{R}}_j^2\int_{m_n}\frac{dm_n}{\Delta_{nj}^*}-\frac{1}{2c^2}\int_{m_n}\left(\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^*}\right)^2dm_n-\frac{1}{2c^2}\sum_{j\neq n}m_j\left(\int_{m_n}\frac{Gdm_n}{\Delta_{nj}^*}\right)^2. \tag{2.9}$$

As a result of combining integrals (A.2) and (A.3), the integral $\frac{3}{2}\dot{\bar{R}}_j^2\int_{m_n}\frac{dm_n}{\Delta_{nj}^*}$ is computed:

$$\frac{3}{2}\dot{\bar{R}}_j^2\int_{m_n}\frac{dm_n}{\Delta_{nj}^*}=\frac{3}{2}\dot{\bar{R}}_j^2\frac{m_n}{\Delta_{nj}}-\frac{3\dot{\bar{R}}_j^2}{4\Delta_{nj}^3}\int_{m_n}\bar{\rho}^2dm_n+\frac{9\dot{\bar{R}}_j^2}{4\Delta_{nj}^5}\int_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2dm_n=$$
$$=\frac{3}{2}\dot{\bar{R}}_j^2\frac{m_n}{\Delta_{nj}}-\frac{3\dot{\bar{R}}_j^2}{8\Delta_{nj}^3}\left(A_n+B_n+C_n\right)+$$
$$+\frac{9\dot{\bar{R}}_j^2}{8\Delta_{nj}^3}\left\{A_n+B_n+C_n-\frac{2}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}=$$
$$=\frac{3}{2}\dot{\bar{R}}_j^2\frac{m_n}{\Delta_{nj}}+\frac{3\dot{\bar{R}}_j^2}{4\Delta_{nj}^3}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\};$$

As a result of combining integrals (A.11), (A.2) and (A.3), computing the integral $-\frac{1}{2c^2}\int_{m_n}\left(\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^*}\right)^2dm_n$:

$$-\frac{1}{2c^2}\int_{m_n}\left(\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^*}\right)^2dm_n=-\frac{1}{2c^2}\int_{m_n}\left(\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^*}\right)\left(\sum_{k\neq n}\frac{Gm_k}{\Delta_{nk}^*}\right)dm_n=$$
$$=-\frac{1}{2c^2}\sum_{j\neq n}\sum_{k\neq n}Gm_jGm_k\int_{m_n}\frac{1}{\Delta_{nj}^*}\frac{1}{\Delta_{nk}^*}dm_n=$$
$$=-\frac{1}{2c^2}\sum_{j\neq n}\sum_{k\neq n}\frac{Gm_j}{\Delta_{nj}}\frac{Gm_k}{\Delta_{nk}}\Big\langle m_n+\frac{1}{\Delta_{nj}^2}\frac{1}{\Delta_{nk}^2}\int_{m_n}\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_k\right)dm_n-$$
$$-\frac{1}{2\Delta_{nj}^2}\int_{m_n}\bar{\rho}^2dm_n+\frac{3}{2\Delta_{nj}^4}\int_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2dm_n-\frac{1}{2\Delta_{nk}^2}\int_{m_n}\bar{\rho}^2dm_n+\frac{3}{2\Delta_{nk}^4}\int_{m_n}\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_k\right)\right]^2dm_n\Big\rangle=$$

$$= -\frac{1}{2c^2}\sum_{j\neq n}\sum_{k\neq n}\frac{Gm_j}{\Delta_{nj}}\frac{Gm_k}{\Delta_{nk}}\left\{ m_n + \frac{1}{2}\left[\frac{\left(\bar{R}_n - \bar{R}_j\right)\cdot\left(\bar{R}_n - \bar{R}_k\right)}{\Delta_{nj}^2\Delta_{nk}^2} + \frac{1}{2}\left(\frac{1}{\Delta_{nj}^2} + \frac{1}{\Delta_{nk}^2}\right)\right]\left(A_n + B_n + C_n\right) - \right.$$

$$- \frac{1}{\Delta_{nj}^2\Delta_{nk}^2}\left[A_n\left(x_n - x_j\right)\left(x_n - x_k\right) + B_n\left(y_n - y_j\right)\left(y_n - y_k\right) + C_n\left(z_n - z_j\right)\left(z_n - z_k\right)\right] -$$

$$- \frac{3}{2\Delta_{nj}^4}\left[A_n\left(x_n - x_j\right)^2 + B_n\left(y_n - y_j\right)^2 + C_n\left(z_n - z_j\right)^2\right] -$$

$$\left. - \frac{3}{2\Delta_{nk}^4}\left[A_n\left(x_n - x_k\right)^2 + B_n\left(y_n - y_k\right)^2 + C_n\left(z_n - z_k\right)^2\right]\right\};$$

As it was obtained earlier

$$\int_{m_n}\frac{dm_n}{\Delta_{nj}^*} = \frac{m_n}{\Delta_{nj}} + \frac{1}{2\Delta_{nj}^3}\left\{A_n + B_n + C_n - \frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n - x_j\right)^2 + B_n\left(y_n - y_j\right)^2 + C_n\left(z_n - z_j\right)^2\right]\right\}.$$

Accordingly the integral $-\frac{1}{2c^2}\sum_{j\neq n} m_j\left(\int_{m_n}\frac{Gdm_n}{\Delta_{nj}^*}\right)^2$ is computed:

$$-\frac{1}{2c^2}\sum_{j\neq n} m_j\left(\int_{m_n}\frac{Gdm_n}{\Delta_{nj}^*}\right)^2 =$$

$$= -\frac{1}{2c^2}\sum_{j\neq n} m_j G^2\left\langle\frac{m_n}{\Delta_{nj}} + \frac{1}{2\Delta_{nj}^3}\left\{A_n + B_n + C_n - \frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n - x_j\right)^2 + B_n\left(y_n - y_j\right)^2 + C_n\left(z_n - z_j\right)^2\right]\right\}\right\rangle^2 =$$

$$= -\frac{1}{2c^2}\sum_{j\neq n} m_j G^2\left\langle\frac{m_n^2}{\Delta_{nj}^2} + \frac{m_n}{\Delta_{nj}^4}\left\{A_n + B_n + C_n - \frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n - x_j\right)^2 + B_n\left(y_n - y_j\right)^2 + C_n\left(z_n - z_j\right)^2\right]\right\} + \ldots\right\rangle;$$

after omitted small terms $\leq o\left(\Delta_{nj}^{-6}\right)$:

$$-\frac{1}{2c^2}\sum_{j\neq n} m_j\left(\int_{m_n}\frac{Gdm_n}{\Delta_{nj}^*}\right)^2 =$$

$$= -\frac{Gm_n}{2c^2}\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^2}\left\langle m_n + \frac{1}{\Delta_{nj}^2}\left\{A_n + B_n + C_n - \frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n - x_j\right)^2 + B_n\left(y_n - y_j\right)^2 + C_n\left(z_n - z_j\right)^2\right]\right\}\right\rangle.$$

2. 2.3 Results

Thus, the relativistic part of the Lagrange function associated with the body m_n can be represented in the form:

$$\Delta L_n = \Delta L_n^{(0)}\left(\omega_k^0\right) + \Delta L_n^{(1)}\left(\omega_k^1\right) + \Delta L_n^{(2)}\left(\omega_k^2\right),$$

where $\Delta L_n^{(0)}\left(\omega_k^0\right)$ independent from the components of the angular velocity ω_k, has the form:

$$\Delta L_n^{(0)}\left(\omega_k^0\right) = \frac{1}{8c^2}\dot{\bar{R}}_n^4 m_n +$$

$$+\sum_{j\neq n}\frac{Gm_j}{c^2}\left\langle\frac{3}{2}\frac{\dot{\bar{R}}_n^{\,2}}{\Delta_{nj}}m_n+\frac{3\dot{\bar{R}}_n^2}{4\Delta_{nj}^3}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}+\right.$$

$$+\frac{3}{2}\dot{\bar{R}}_j^{\,2}\frac{m_n}{\Delta_{nj}}+\frac{3\dot{\bar{R}}_j^{\,2}}{4\Delta_{nj}^3}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}-$$

$$-\frac{7}{2}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}}m_n-\frac{7}{4}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}^3}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}-$$

$$-\frac{1}{2}\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{\Delta_{nj}^3}m_n+$$

$$+\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{15}{4\Delta_{nj}^7}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]-$$

$$-\frac{3\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\left[\dot{x}_j\left(x_n-x_j\right)A_n+\dot{y}_j\left(y_n-y_j\right)B_n+\dot{z}_j\left(z_n-z_j\right)C_n\right]-$$

$$-\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\left[\dot{x}_n\left(x_n-x_j\right)A_n+\dot{y}_n\left(y_n-y_j\right)B_n+\dot{z}_n\left(z_n-z_j\right)C_n\right]-$$

$$\left.-\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{4\Delta_{nj}^3}\left(A_n+B_n+C_n\right)+\frac{1}{2\Delta_{nj}^3}\left[\dot{x}_n\dot{x}_jA_n+\dot{y}_n\dot{y}_jB_n+\dot{z}_n\dot{z}_jC_n\right]\right\rangle-$$

$$-\frac{1}{2c^2}\sum_{j\neq n}\sum_{k\neq n}\frac{Gm_j}{\Delta_{nj}}\frac{Gm_k}{\Delta_{nk}}\left\{m_n+\frac{1}{2}\left[\frac{\left(\bar{R}_n-\bar{R}_j\right)\cdot\left(\bar{R}_n-\bar{R}_k\right)}{\Delta_{nj}^2\Delta_{nk}^2}+\frac{1}{2}\left(\frac{1}{\Delta_{nj}^2}+\frac{1}{\Delta_{nk}^2}\right)\right]\left(A_n+B_n+C_n\right)-\right.$$

$$-\frac{1}{\Delta_{nj}^2\Delta_{nk}^2}\left[A_n\left(x_n-x_j\right)\left(x_n-x_k\right)+B_n\left(y_n-y_j\right)\left(y_n-y_k\right)+C_n\left(z_n-z_j\right)\left(z_n-z_k\right)\right]-$$

$$-\frac{3}{2\Delta_{nj}^4}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]-$$

$$\left.-\frac{3}{2\Delta_{nk}^4}\left[A_n\left(x_n-x_k\right)^2+B_n\left(y_n-y_k\right)^2+C_n\left(z_n-z_k\right)^2\right]\right\}-$$

$$-\sum_{j\neq n}\frac{Gm_j}{c^2}\frac{Gm_n}{2\Delta_{nj}^2}\left\langle m_n+\frac{1}{\Delta_{nj}^2}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}\right\rangle.$$

After the reduction of similar terms the relativistic part of the Lagrange function, which relate to the force function of gravitational interaction the body m_n with other bodies has form:

$$\Delta L_n^{(0)}\left(\omega_k^0\right)=\frac{1}{8c^2}\dot{\bar{R}}_n^4m_n+\frac{1}{c^2}\sum_{j\neq n}Gm_j\left\langle\left(\frac{3}{2}\frac{\dot{\bar{R}}_n^{\,2}}{\Delta_{nj}}+\frac{3}{2}\frac{\dot{\bar{R}}_j^{\,2}}{\Delta_{nj}}-\frac{7}{2}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}}\right)\right.$$

$$\left\langle m_n+\frac{1}{2\Delta_{nj}^2}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}\right\rangle-$$

$$-\frac{\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^3}\left\{m_n-\frac{15}{2\Delta_{nj}^4}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}-$$

$$-\frac{3\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\left[\dot{x}_j\left(x_n-x_j\right)A_n+\dot{y}_j\left(y_n-y_j\right)B_n+\dot{z}_j\left(z_n-z_j\right)C_n\right]-$$

$$-\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\left[\dot{x}_n\left(x_n-x_j\right)A_n+\dot{y}_n\left(y_n-y_j\right)B_n+\dot{z}_n\left(z_n-z_j\right)C_n\right]-$$

$$-\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{4\Delta_{nj}^3}\left(A_n+B_n+C_n\right)+\frac{1}{2\Delta_{nj}^3}\left[\dot{x}_n\dot{x}_jA_n+\dot{y}_n\dot{y}_jB_n+\dot{z}_n\dot{z}_jC_n\right]\Bigg\rangle-$$

$$-\frac{1}{2c^2}\sum_{j\neq n}\sum_{k\neq n}\frac{Gm_j}{\Delta_{nj}}\frac{Gm_k}{\Delta_{nk}}\left\{m_n+\frac{1}{2}\left[\frac{\left(\bar{R}_n-\bar{R}_j\right)\cdot\left(\bar{R}_n-\bar{R}_k\right)}{\Delta_{nj}^2\Delta_{nk}^2}+\frac{1}{2}\left(\frac{1}{\Delta_{nj}^2}+\frac{1}{\Delta_{nk}^2}\right)\right]\left(A_n+B_n+C_n\right)-\right.$$

$$-\frac{1}{\Delta_{nj}^2\Delta_{nk}^2}\left[A_n\left(x_n-x_j\right)\left(x_n-x_k\right)+B_n\left(y_n-y_j\right)\left(y_n-y_k\right)+C_n\left(z_n-z_j\right)\left(z_n-z_k\right)\right]-$$

$$-\frac{3}{2\Delta_{nj}^4}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]-$$

$$\left.-\frac{3}{2\Delta_{nk}^4}\left[A_n\left(x_n-x_k\right)^2+B_n\left(y_n-y_k\right)^2+C_n\left(z_n-z_k\right)^2\right]\right\}-\frac{Gm_n}{2c^2}\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}}\Bigg\langle\frac{m_n}{\Delta_{nj}}+$$

$$+\frac{1}{\Delta_{nj}^3}\left\{A_n+B_n+C_n-\frac{3}{\Delta_{nj}^2}\left[A_n\left(x_n-x_j\right)^2+B_n\left(y_n-y_j\right)^2+C_n\left(z_n-z_j\right)^2\right]\right\}\Bigg\rangle.$$

If the body m_n is spherically symmetric, that is $A_n=B_n=C_n=I_n$, then $\Delta L_n^{(0)}\left(\omega_k^0\right)$ becomes:

$$\Delta L_n^{(0)}\left(\omega_k^0\right)=\frac{1}{8c^2}\dot{\bar{R}}_n^4m_n+\sum_{j\neq n}\frac{Gm_j}{c^2}\left[\frac{3}{2}\frac{\dot{\bar{R}}_n^{\,2}}{\Delta_{nj}}m_n-\frac{7}{2}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}}m_n-\right.$$

$$\left.-\frac{1}{2}\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{\Delta_{nj}^3}m_n+\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{15}{4\Delta_{nj}^5}I_n-\frac{3\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{4\Delta_{nj}^3}I_n\right].$$

An additional part $\Delta L_n^{(1)}\left(\omega_k^1\right)$ that depends linearly on the components ω_k has the form:

$$\Delta L_n^{(1)}\left(\omega_k^1\right)=\sum_{j\neq n}\frac{Gm_j}{c^2}\Bigg\langle-\frac{3}{2\Delta_{nj}^3}\left\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_n+\right.$$

$$+\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\dot{z}_n+\left(z_n-z_j\right)\dot{y}_n\right]+\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\dot{x}_n+\left(x_n-x_j\right)\dot{z}_n\right]+$$

$$\left.+\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\dot{y}_n+\left(y_n-y_j\right)\dot{x}_n\right]\right\}+$$

$$+\frac{7}{4\Delta_{nj}^3}\left\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_j+\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\dot{z}_j+\left(z_n-z_j\right)\dot{y}_j\right]+\right.$$

$$+\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\dot{x}_j+\left(x_n-x_j\right)\dot{z}_j\right]+\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\dot{y}_j+\left(y_n-y_j\right)\dot{x}_j\right]\Big\}+$$

$$+\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^5}\Big[\left(x_n-x_j\right)\left(y_n-y_j\right)\omega_3\left(B_n-A_n\right)+\left(z_n-z_j\right)\left(x_n-x_j\right)\omega_2\left(A_n-C_n\right)+$$

$$+\left(y_n-y_j\right)\left(z_n-z_j\right)\omega_1\left(C_n-B_n\right)\Big]+$$

$$+\frac{1}{4\Delta_{nj}^3}\Big\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_j-$$

$$-\left(C_n-B_n\right)\omega_1\left[\dot{y}_j\left(z_n-z_j\right)+\dot{z}_j\left(y_n-y_j\right)\right]-\left(A_n-C_n\right)\omega_2\left[\dot{z}_j\left(x_n-x_j\right)+\dot{x}_j\left(z_n-z_j\right)\right]-$$

$$-\left(B_n-A_n\right)\omega_3\left[\dot{x}_j\left(y_n-y_j\right)+\dot{y}_j\left(x_n-x_j\right)\right]\Big]\Big\}\Big\rangle.$$

After the reduction of similar terms an additional part $\Delta L_n^{(1)}\left(\omega_k^1\right)$ has the form:

$$\Delta L_n^{(1)}\left(\omega_k^1\right)=-\sum_{j\neq n}\frac{Gm_j}{c^2}\frac{1}{\Delta_{nj}^3}\Big\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\left(\frac{3}{2}\dot{\bar{R}}_n-2\dot{\bar{R}}_j\right)+$$

$$+\frac{3}{2}\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\left(\dot{z}_n-\dot{z}_j\right)+\left(z_n-z_j\right)\left(\dot{y}_n-\dot{y}_j\right)\right]+$$

$$+\frac{3}{2}\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\left(\dot{x}_n-\dot{x}_j\right)+\left(x_n-x_j\right)\left(\dot{z}_n-\dot{z}_j\right)\right]+$$

$$+\frac{3}{2}\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\left(\dot{y}_n-\dot{y}_j\right)+\left(y_n-y_j\right)\left(\dot{x}_n-\dot{x}_j\right)\right]- \tag{2.10}$$

$$-\frac{3\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2\Delta_{nj}^2}\Big[\left(x_n-x_j\right)\left(y_n-y_j\right)\omega_3\left(B_n-A_n\right)+\left(z_n-z_j\right)\left(x_n-x_j\right)\omega_2\left(A_n-C_n\right)+$$

$$+\left(y_n-y_j\right)\left(z_n-z_j\right)\omega_1\left(C_n-B_n\right)\Big]\Big\}.$$

It is Lagrange function for the geodetic rotation of the rigid body, which was received in the previous investigations (Eroshkin, Pashkevich, 1997), (Pashkevich, 2000).
The first line turned out to be the relativistic components of the Lagrange function, which relate to the body m_n as a point mass or as spherically symmetric:

$$\Delta L_n^{(1)}\left(\omega_k^1\right)=-\sum_{j\neq n}\frac{Gm_j}{c^2}\frac{1}{\Delta_{nj}^3}\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\left(\frac{3}{2}\dot{\bar{R}}_n-2\dot{\bar{R}}_j\right).$$

An additional part $\Delta L_n^{(2)}\left(\omega_k^2\right)$ that depends square on the components ω_k^2 has the form:

$$\Delta L_n^{(2)}\left(\omega_k^2\right)=\frac{1}{4c^2}\dot{\bar{R}}_n^2\left(A_n\omega_1^2+B_n\omega_2^2+C_n\omega_3^2\right)+\sum_{j\neq n}\frac{Gm_j}{c^2}\Big[\frac{3}{2\Delta_{nj}}\left(A_n\omega_1^2+B_n\omega_2^2+C_n\omega_3^2\right)+$$

$$+\left(\dot{x}_n\omega_2-\dot{y}_n\omega_1\right)^2\frac{A_n+B_n-C_n}{4}+\left(\dot{x}_n\omega_3-\dot{z}_n\omega_1\right)^2\frac{C_n+A_n-B_n}{4}+$$

$$+\left(\dot{y}_n\omega_3-\dot{z}_n\omega_2\right)^2\frac{B_n+C_n-A_n}{4}\Big]=\frac{1}{2}\bar{H}_n\cdot\bar{\omega}\frac{1}{c^2}\left(\frac{\dot{\bar{R}}_n^2}{2}+3\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}}\right)+$$

$$+\sum_{j\neq n}\frac{Gm_j}{c^2}\Big[\left(\dot{x}_n\omega_2-\dot{y}_n\omega_1\right)^2\frac{A_n+B_n-C_n}{4}+\left(\dot{x}_n\omega_3-\dot{z}_n\omega_1\right)^2\frac{C_n+A_n-B_n}{4}+$$

$$+\left(\dot{y}_n\omega_3-\dot{z}_n\omega_2\right)^2\frac{B_n+C_n-A_n}{4}\Bigg].$$

If the body m_n is spherically symmetric, then $\Delta L_n^{(2)}\left(\omega_k^2\right)$ becomes:

$$\Delta L_n^{(2)}\left(\omega_k^2\right)=\frac{1}{c^2}\left[\frac{1}{2}\overline{H}_n\cdot\overline{\omega}\left(\frac{\dot{\overline{R}}_n^2}{2}+3\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}}\right)+\frac{I_n}{4}\left(\dot{\overline{R}}_n\times\overline{\omega}\right)^2\sum_{j\neq n}Gm_j\right].$$

3. DERIVATION OF THE DIFFERENTIAL EQUATIONS

The differential equations of the rigid body rotation are derived in the terms of the Rodrigues – Hamilton parameters.

Lagrange differential equations of the second kind have the form:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\lambda}_i}-\frac{\partial L}{\partial\lambda_i}=0,\quad i=0,1,2,3; \tag{3.1}$$

where $L=T+U+\Delta L$, $T=\frac{1}{2}\overline{H}_n\cdot\overline{\omega}=\frac{1}{2}(A_n\omega_1^2+B_n\omega_2^2+C_n\omega_3^2)$, T is the kinetic energy of the rotational motion of the rigid body; U is the force function of the gravitational interaction of the rigid body with the disturbing bodies; ΔL is the additional part of the Lagrange function, which generating geodetic perturbations (2.10). This expression in other form is:

$$\Delta L=\overline{H}_n\left(\overline{\sigma}_n+\overline{\sigma}_R\right)=\overline{H}_n\overline{\sigma}_g,$$

where $\overline{\sigma}_n=-\sum_{j\neq n}\frac{Gm_j}{c^2\Delta_{nj}^3}\left(\overline{R}_n-\overline{R}_j\right)\times\left(\frac{3}{2}\dot{\overline{R}}_n-2\dot{\overline{R}}_j\right)$;

$$\begin{aligned}\overline{H}_n\overline{\sigma}_R=-\frac{3}{2}\sum_{j\neq n}\frac{Gm_j}{c^2\Delta_{nj}^3}\Big\{&\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\left(\dot{z}_n-\dot{z}_j\right)+\left(z_n-z_j\right)\left(\dot{y}_n-\dot{y}_j\right)\right]+\\&+\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\left(\dot{x}_n-\dot{x}_j\right)+\left(x_n-x_j\right)\left(\dot{z}_n-\dot{z}_j\right)\right]+\\&+\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\left(\dot{y}_n-\dot{y}_j\right)+\left(y_n-y_j\right)\left(\dot{x}_n-\dot{x}_j\right)\right]-\\&-\frac{\dot{\overline{R}}_j\cdot\left(\overline{R}_n-\overline{R}_j\right)}{\Delta_{nj}^2}\Big[\left(C_n-B_n\right)\omega_1\left(y_n-y_j\right)\left(z_n-z_j\right)+\\&+\left(A_n-C_n\right)\omega_2\left(z_n-z_j\right)\left(x_n-x_j\right)+\left(B_n-A_n\right)\omega_3\left(x_n-x_j\right)\left(y_n-y_j\right)\Big]\Big\}.\end{aligned}$$

Thus relations for relativistic angular velocity vectors have forms:

$$\begin{aligned}\overline{\sigma}_n=-\frac{1}{c^2}\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^3}\Big\{&\left[y_{nj}\left(\frac{3}{2}\dot{z}_n-2\dot{z}_j\right)-z_{nj}\left(\frac{3}{2}\dot{y}_n-2\dot{y}_j\right)\right]\overline{i}_1+\\&+\left[z_{nj}\left(\frac{3}{2}\dot{x}_n-2\dot{x}_j\right)-x_{nj}\left(\frac{3}{2}\dot{z}_n-2\dot{z}_j\right)\right]\overline{i}_2+\\&+\left[x_{nj}\left(\frac{3}{2}\dot{y}_n-2\dot{y}_j\right)-y_{nj}\left(\frac{3}{2}\dot{x}_n-2\dot{x}_j\right)\right]\overline{i}_3\Big\};\end{aligned}$$

$$\bar{\sigma}_R = -\frac{3}{2}\sum_{j\neq n}\frac{Gm_j}{c^2\Delta_{nj}^3}\left\{\alpha_n\left[y_{nj}\dot{z}_{nj}+z_{nj}\dot{y}_{nj}-\frac{\dot{\bar{R}}_j\cdot\bar{\Delta}_{nj}}{\Delta_{nj}^2}y_{nj}z_{nj}\right]\bar{i}_1-\right.$$

$$\left.-\beta_n\left[z_{nj}\dot{x}_{nj}+x_{nj}\dot{z}_{nj}-\frac{\dot{\bar{R}}_j\cdot\bar{\Delta}_{nj}}{\Delta_{nj}^2}z_{nj}x_{nj}\right]\bar{i}_2+\gamma_n\left[x_{nj}\dot{y}_{nj}+y_{nj}\dot{x}_{nj}-\frac{\dot{\bar{R}}_j\cdot\bar{\Delta}_{nj}}{\Delta_{nj}^2}x_{nj}y_{nj}\right]\bar{i}_3\right\}.$$

Here $\alpha_n=(C_n-B_n)/A_n,\ \beta_n=(C_n-A_n)/B_n,\ \gamma_n=(B_n-A_n)/C_n; x_{nj}=x_n-x_j, y_{nj}=y_n-y_j,$ $z_{nj}=z_n-z_j, \dot{x}_{nj}=\dot{x}_n-\dot{x}_j, \dot{y}_{nj}=\dot{y}_n-\dot{y}_j, \dot{z}_{nj}=\dot{z}_n-\dot{z}_j; \bar{\Delta}_{nj}=\bar{R}_n-\bar{R}_j; \Delta_{nj}=\left|\bar{R}_n-\bar{R}_j\right|.$

The first derivatives of the relativistic angular velocity vectors are given by the following expressions:

$$\dot{\bar{\sigma}}_n=-\frac{1}{c^2}\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^3}\left\{\left[-3\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}\bar{\Delta}_{nj}\times\left(\frac{3}{2}\dot{\bar{R}}_n-2\dot{\bar{R}}_j\right)+\dot{\bar{\Delta}}_{nj}\times\left(\frac{3}{2}\dot{\bar{R}}_n-2\dot{\bar{R}}_j\right)+\right.\right.$$

$$\left.\left.+\bar{\Delta}_{nj}\times\left(\frac{3}{2}\ddot{\bar{R}}_n-2\ddot{\bar{R}}_j\right)\right]\right\};$$

$$\dot{\bar{\sigma}}_R=-\frac{3}{2c^2}\sum_{j\neq n}\frac{Gm_j}{\Delta_{nj}^3}\left\{\alpha_n\left[-3\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}\left(y_{nj}\dot{z}_{nj}+z_{nj}\dot{y}_{nj}\right)+2\dot{y}_{nj}\dot{z}_{nj}+y_{nj}\ddot{z}_{nj}+z_{nj}\ddot{y}_{nj}-\right.\right.$$

$$\left.-\frac{1}{\Delta_{nj}^2}\left(\dot{\bar{R}}_j\cdot\bar{\Delta}_{nj}\left(-5\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}y_{nj}z_{nj}+\dot{y}_{nj}z_{nj}+y_{nj}\dot{z}_{nj}\right)+\left(\dot{\bar{R}}_j\cdot\dot{\bar{\Delta}}_{nj}+\ddot{\bar{R}}_j\cdot\bar{\Delta}_{nj}\right)y_{nj}z_{nj}\right)\right]\bar{i}_1-$$

$$-\beta_n\left[-3\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}\left(z_{nj}\dot{x}_{nj}+x_{nj}\dot{z}_{nj}\right)+2\dot{z}_{nj}\dot{x}_{nj}+z_{nj}\ddot{x}_{nj}+x_{nj}\ddot{z}_{nj}-\right.$$

$$\left.-\frac{1}{\Delta_{nj}^2}\left(\dot{\bar{R}}_j\cdot\bar{\Delta}_{nj}\left(-5\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}z_{nj}x_{nj}+\dot{z}_{nj}x_{nj}+z_{nj}\dot{x}_{nj}\right)+\left(\dot{\bar{R}}_j\cdot\dot{\bar{\Delta}}_{nj}+\ddot{\bar{R}}_j\cdot\bar{\Delta}_{nj}\right)z_{nj}x_{nj}\right)\right]\bar{i}_2+$$

$$+\gamma_n\left[-3\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}\left(x_{nj}\dot{y}_{nj}+y_{nj}\dot{x}_{nj}\right)+2\dot{x}_{nj}\dot{y}_{nj}+x_{nj}\ddot{y}_{nj}+y_{nj}\ddot{x}_{nj}-\right.$$

$$\left.\left.-\frac{1}{\Delta_{nj}^2}\left(\dot{\bar{R}}_j\cdot\bar{\Delta}_{nj}\left(-5\frac{\dot{\Delta}_{nj}}{\Delta_{nj}}x_{nj}y_{nj}+\dot{x}_{nj}y_{nj}+x_{nj}\dot{y}_{nj}\right)+\left(\dot{\bar{R}}_j\cdot\dot{\bar{\Delta}}_{nj}+\ddot{\bar{R}}_j\cdot\bar{\Delta}_{nj}\right)x_{nj}y_{nj}\right)\right]\bar{i}_3\right\}.$$

To define new operators:

$$\begin{aligned}
\langle\ \rangle_0 &= \lambda_0\{\ \}_0+\lambda_1\{\ \}_1+\lambda_2\{\ \}_2+\lambda_3\{\ \}_3,\\
\langle\ \rangle_1 &= \lambda_1\{\ \}_0-\lambda_0\{\ \}_1-\lambda_3\{\ \}_2+\lambda_2\{\ \}_3,\\
\langle\ \rangle_2 &= \lambda_2\{\ \}_0+\lambda_3\{\ \}_1-\lambda_0\{\ \}_2-\lambda_1\{\ \}_3,\\
\langle\ \rangle_3 &= \lambda_3\{\ \}_0-\lambda_2\{\ \}_1+\lambda_1\{\ \}_2-\lambda_0\{\ \}_3;
\end{aligned} \tag{3.2}$$

where $\{\ \}_i=\dfrac{d}{dt}\dfrac{\partial}{\partial\dot{\lambda}_i}-\dfrac{\partial}{\partial\lambda_i};$

$$
\begin{aligned}
(\ )_0 &= \lambda_0 \frac{\partial}{\partial \dot{\lambda}_0} + \lambda_1 \frac{\partial}{\partial \dot{\lambda}_1} + \lambda_2 \frac{\partial}{\partial \dot{\lambda}_2} + \lambda_3 \frac{\partial}{\partial \dot{\lambda}_3}, \\
(\ )_1 &= \lambda_1 \frac{\partial}{\partial \dot{\lambda}_0} - \lambda_0 \frac{\partial}{\partial \dot{\lambda}_1} - \lambda_3 \frac{\partial}{\partial \dot{\lambda}_2} + \lambda_2 \frac{\partial}{\partial \dot{\lambda}_3}, \\
(\ )_2 &= \lambda_2 \frac{\partial}{\partial \dot{\lambda}_0} + \lambda_3 \frac{\partial}{\partial \dot{\lambda}_1} - \lambda_0 \frac{\partial}{\partial \dot{\lambda}_2} - \lambda_1 \frac{\partial}{\partial \dot{\lambda}_3}, \\
(\ )_3 &= \lambda_3 \frac{\partial}{\partial \dot{\lambda}_0} - \lambda_2 \frac{\partial}{\partial \dot{\lambda}_1} + \lambda_1 \frac{\partial}{\partial \dot{\lambda}_2} - \lambda_0 \frac{\partial}{\partial \dot{\lambda}_3};
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
[\ ]_0 &= \lambda_0 \frac{\partial}{\partial \lambda_0} + \lambda_1 \frac{\partial}{\partial \lambda_1} + \lambda_2 \frac{\partial}{\partial \lambda_2} + \lambda_3 \frac{\partial}{\partial \lambda_3}, \\
[\ ]_1 &= \lambda_1 \frac{\partial}{\partial \lambda_0} - \lambda_0 \frac{\partial}{\partial \lambda_1} - \lambda_3 \frac{\partial}{\partial \lambda_2} + \lambda_2 \frac{\partial}{\partial \lambda_3}, \\
[\ ]_2 &= \lambda_2 \frac{\partial}{\partial \lambda_0} + \lambda_3 \frac{\partial}{\partial \lambda_1} - \lambda_0 \frac{\partial}{\partial \lambda_2} - \lambda_1 \frac{\partial}{\partial \lambda_3}, \\
[\ ]_3 &= \lambda_3 \frac{\partial}{\partial \lambda_0} - \lambda_2 \frac{\partial}{\partial \lambda_1} + \lambda_1 \frac{\partial}{\partial \lambda_2} - \lambda_0 \frac{\partial}{\partial \lambda_3}.
\end{aligned}
\tag{3.4}
$$

The force function (2.6) is independent from $\dot{\bar{R}}_n^*$, accordingly one is independent from $\dot{\lambda}_i$:

$$\frac{\partial U}{\partial \dot{\lambda}_i} = 0.$$

From (3.1) is received: $\{T\}_i = \frac{d}{dt}\frac{\partial T}{\partial \dot{\lambda}_i} - \frac{\partial T}{\partial \lambda_i} = -\frac{d}{dt}\frac{\partial \Delta L}{\partial \dot{\lambda}_i} + \frac{\partial (U+\Delta L)}{\partial \lambda_i}$, then

$$\langle T \rangle_i = -\langle \Delta L \rangle_i + [U]_i . \tag{3.5}$$

After apply (see Appendix B: (16-19)) operator (3.2) to the kinetic energy has a form:

$$
\begin{aligned}
\langle T \rangle_0 &= -4(A_n\omega_1^2 + B_n\omega_2^2 + C_n\omega_3^2) = -8T, \\
\langle T \rangle_1 &= 2(-A_n\dot{\omega}_1 + B_n\omega_2\omega_3 - C_n\omega_2\omega_3), \\
\langle T \rangle_2 &= 2(-B_n\dot{\omega}_2 + C_n\omega_1\omega_3 - A_n\omega_1\omega_3), \\
\langle T \rangle_3 &= 2(-C_n\dot{\omega}_3 + A_n\omega_1\omega_2 - B_n\omega_1\omega_2).
\end{aligned}
\tag{3.6}
$$

From (3.5) and (3.6) received the first derivatives of the angular velocity components are given by the following equations:

$$
\begin{aligned}
\dot{\omega}_1 &= -\alpha_n\omega_2\omega_3 + \frac{1}{2A_n}(\langle \Delta L \rangle_1 - [\mathrm{U}]_1), \\
\dot{\omega}_2 &= \beta_n\omega_1\omega_3 + \frac{1}{2B_n}(\langle \Delta L \rangle_2 - [\mathrm{U}]_2), \\
\dot{\omega}_3 &= -\gamma_n\omega_1\omega_2 + \frac{1}{2C_n}(\langle \Delta L \rangle_3 - [\mathrm{U}]_3),
\end{aligned}
$$

where $\langle \Delta L \rangle_i = \langle \bar{H} \rangle_i \bar{\sigma}_g + (\bar{H})_i \dot{\bar{\sigma}}_g - \bar{H}[\bar{\sigma}_g]_i$;

After apply (see Appendix B: (B.5)) operator (3.2) to the angular momentum vector of the rotational motion of the rigid body has a form:

$$\left\langle \bar{H} \right\rangle_0 = -2\bar{H},\ \left\langle \bar{H} \right\rangle_1 = 2\left(B_n \omega_3 \vec{i}_2 - C_n \omega_2 \vec{i}_3 \right),$$
$$\left\langle \bar{H} \right\rangle_2 = 2\left(-A_n \omega_3 \vec{i}_1 + C_n \omega_1 \vec{i}_3 \right),\ \left\langle \bar{H} \right\rangle_3 = 2\left(A_n \omega_2 \vec{i}_1 + B_n \omega_1 \vec{i}_2 \right); \tag{3.7}$$

After apply (see Appendix B) operator (3.3) to the angular momentum vector of the rotational motion of the rigid body has a form:

$$\left(\bar{H} \right)_0 = 0,\ \left(\bar{H} \right)_1 = -2A_n \vec{i}_1,\ \left(\bar{H} \right)_2 = -2B_n \vec{i}_2,\ \left(\bar{H} \right)_3 = -2C_n \vec{i}_3. \tag{3.8}$$

After apply operators (3.2), (3.3) and (3.4) to ΔL the first derivatives of the angular velocity components are given by the following equations:

$$\dot{\omega}_1 + \dot{\sigma}_{1g} = -\alpha_n \left\{ \omega_2 \omega_3 + \omega_2 \sigma_{3n} + \sigma_{2n} \omega_3 - \omega_2 \frac{\sigma_{3R}}{\gamma_n} - \frac{\sigma_{2R}}{\beta_n} \omega_3 + \omega_1 F_1 \right\} - \frac{1}{2A_n} [\mathrm{U}]_1,$$

$$\dot{\omega}_2 + \dot{\sigma}_{2g} = \beta_n \left\{ \omega_1 \omega_3 + \omega_1 \sigma_{3n} + \sigma_{1n} \omega_3 + \omega_1 \frac{\sigma_{3R}}{\gamma_n} - \frac{\sigma_{1R}}{\alpha_n} \omega_3 + \omega_2 F_2 \right\} - \frac{1}{2B_n} [\mathrm{U}]_2,$$

$$\dot{\omega}_3 + \dot{\sigma}_{3g} = -\gamma_n \left\{ \omega_1 \omega_2 + \omega_1 \sigma_{2n} + \sigma_{1n} \omega_2 + \omega_1 \frac{\sigma_{2R}}{\beta_n} + \frac{\sigma_{1R}}{\alpha_n} \omega_2 + \omega_3 F_3 \right\} - \frac{1}{2C_n} [\mathrm{U}]_3;$$

where $\bar{\sigma}_g = \bar{\sigma}_n + \bar{\sigma}_R, \dot{\bar{\sigma}}_g = \dot{\bar{\sigma}}_n + \dot{\bar{\sigma}}_R,$

$$F_1 = \frac{3}{2} \sum_{j \neq n} \frac{Gm_j}{c^2 \Delta_{nj}^3} \left[\frac{\dot{\bar{R}}_j \cdot \bar{\Delta}_{nj}}{\Delta_{nj}^2} (y_{nj}^2 - z_{nj}^2) - 2(y_{nj} \dot{y}_{nj} - z_{nj} \dot{z}_{nj}) \right],$$

$$F_2 = \frac{3}{2} \sum_{j \neq n} \frac{Gm_j}{c^2 \Delta_{nj}^3} \left[\frac{\dot{\bar{R}}_j \cdot \bar{\Delta}_{nj}}{\Delta_{nj}^2} (z_{nj}^2 - x_{nj}^2) - 2(z_{nj} \dot{z}_{nj} - x_{nj} \dot{x}_{nj}) \right],$$

$$F_3 = \frac{3}{2} \sum_{j \neq n} \frac{Gm_j}{c^2 \Delta_{nj}^3} \left[\frac{\dot{\bar{R}}_j \cdot \bar{\Delta}_{nj}}{\Delta_{nj}^2} (x_{nj}^2 - y_{nj}^2) - 2(x_{nj} \dot{x}_{nj} - y_{nj} \dot{y}_{nj}) \right].$$

The differential equations, determining the rigid body rotation in the terms of the Rodrigues – Hamilton parameters are obtained from expressions of the angular velocity vector of the rigid body:

$$\begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = 2K \begin{pmatrix} \dot{\lambda}_0 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{pmatrix}, K = \begin{pmatrix} -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 \\ -\lambda_1 & \lambda_0 & \lambda_3 & -\lambda_2 \\ -\lambda_2 & -\lambda_3 & \lambda_0 & \lambda_1 \\ -\lambda_3 & \lambda_2 & -\lambda_1 & \lambda_0 \end{pmatrix},$$

by differentiating with respect to time. The differential equations, determining the rigid body rotation in the terms of the Rodrigues – Hamilton parameters are:

$$\begin{pmatrix} \ddot{\lambda}_0 \\ \ddot{\lambda}_1 \\ \ddot{\lambda}_2 \\ \ddot{\lambda}_3 \end{pmatrix} = \frac{1}{2} K^{-1} \begin{pmatrix} \frac{1}{2}\omega^2 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix}, K^{-1} = \begin{pmatrix} -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 \\ -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2 \\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1 \\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0 \end{pmatrix}.$$

Here $\omega^2 = |\bar{\omega}|^2$; $\lambda_0^2 + \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 1$; $\lambda_0 \dot{\lambda}_0 + \lambda_1 \dot{\lambda}_1 + \lambda_2 \dot{\lambda}_2 + \lambda_3 \dot{\lambda}_3 = 0.$

4. CONCLUSIONS

Thus Lagrange function for the relativistic rotation of the rigid body, which is generated by metric properties of Riemann space of general relativity, was received:

$$L_n = L_n^{Newton} + \Delta L_n.$$

The differential equations, determining the rigid body rotation in the terms of the Rodrigues – Hamilton parameters are derived.

Also are derived the differential equations, which are determining the rigid body rotation in the terms of the Rodrigues – Hamilton parameters:

$$\begin{pmatrix} \ddot{\lambda}_0 \\ \ddot{\lambda}_1 \\ \ddot{\lambda}_2 \\ \ddot{\lambda}_3 \end{pmatrix} = \frac{1}{2}K^{-1}\begin{pmatrix} \frac{1}{2}\omega^2 \\ \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{pmatrix}, K^{-1} = \begin{pmatrix} -\lambda_0 & -\lambda_1 & -\lambda_2 & -\lambda_3 \\ -\lambda_1 & \lambda_0 & -\lambda_3 & \lambda_2 \\ -\lambda_2 & \lambda_3 & \lambda_0 & -\lambda_1 \\ -\lambda_3 & -\lambda_2 & \lambda_1 & \lambda_0 \end{pmatrix}.$$

APPENDIX A

In the following decompositions of the integrands of Lagrange function (2.3), only the principal terms of the expansion and terms that depend on the angular velocity $\bar{\omega}$ or contain $\bar{\rho}$ not higher and $\bar{\rho}$ not less (2.4) than the second order are retained:

1) $$\dot{\bar{R}}_n^{*2} = \dot{\bar{R}}_n^2 + 2\dot{\bar{R}}_n\cdot(\bar{\omega}\times\bar{\rho}) + (\bar{\omega}\times\bar{\rho})^2 = \dot{\bar{R}}_n^2 + \cancel{2\bar{\rho}\cdot(\dot{\bar{R}}_n\times\bar{\omega})} + (\bar{\omega}\times\bar{\rho})^2;$$

2) $$\dot{\bar{R}}_n^{*4} = \left[\dot{\bar{R}}_n^2 + 2\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right) + (\bar{\omega}\times\bar{\rho})^2\right]\left[\dot{\bar{R}}_n^2 + 2\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right) + (\bar{\omega}\times\bar{\rho})^2\right] =$$
$$= \dot{\bar{R}}_n^4 + \cancel{4\dot{\bar{R}}_n^2\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)} + 4\left[\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)\right]^2 + 2\dot{\bar{R}}_n^2(\bar{\omega}\times\bar{\rho})^2 + \ldots;$$

3) $$\frac{1}{\Delta_{nj}^*}\frac{1}{\Delta_{nk}^*} = \frac{1}{\Delta_{nj}}\left\{1 - \frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)}{\Delta_{nj}^2} - \frac{\bar{\rho}^2}{2\Delta_{nj}^2} + \frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)\right]^2}{\Delta_{nj}^4} + \ldots\right\}$$
$$\frac{1}{\Delta_{nk}}\left\{1 - \frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_k\right)}{\Delta_{nk}^2} - \frac{\bar{\rho}^2}{2\Delta_{nk}^2} + \frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_k\right)\right]^2}{\Delta_{nk}^4} + \ldots\right\} =$$
$$= \frac{1}{\Delta_{nj}}\frac{1}{\Delta_{nk}}\left\{1 - \cancel{\frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)}{\Delta_{nj}^2}} - \cancel{\frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_k\right)}{\Delta_{nk}^2}} + \frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)}{\Delta_{nj}^2}\frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_k\right)}{\Delta_{nk}^2} - \right.$$
$$\left. - \frac{\bar{\rho}^2}{2\Delta_{nj}^2} + \frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)\right]^2}{\Delta_{nj}^4} - \frac{\bar{\rho}^2}{2\Delta_{nk}^2} + \frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_k\right)\right]^2}{\Delta_{nk}^4} + \ldots\right\};$$

4) $$\frac{\dot{\bar{R}}_n^{*2}}{\Delta_{nj}^*} = \left[\dot{\bar{R}}_n^2 + 2\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right) + (\bar{\omega}\times\bar{\rho})^2\right]\left[\frac{1}{\Delta_{nj}} - \frac{\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)}{\Delta_{nj}^3} - \frac{\bar{\rho}^2}{2\Delta_{nj}^3} + \frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n - \bar{R}_j\right)\right]^2}{\Delta_{nj}^5}\right] =$$

$$=\frac{\dot{\bar{R}}_n^2}{\Delta_{nj}}+\cancel{2\frac{\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)}{\Delta_{nj}}}+\frac{\left(\bar{\omega}\times\bar{\rho}\right)^2}{\Delta_{nj}}-\cancel{\dot{\bar{R}}_n^2\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^3}}-2\frac{\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^3}-$$

$$-\dot{\bar{R}}_n^2\frac{\bar{\rho}^2}{2\Delta_{nj}^3}+\dot{\bar{R}}_n^2\frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^5}+\ldots;$$

5) $$\frac{\dot{\bar{R}}_n^*\cdot\dot{\bar{R}}_j}{\Delta_{nj}^*}=\left[\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j+\bar{\rho}\cdot\left(\dot{\bar{R}}_j\times\bar{\omega}\right)\right]\left[\frac{1}{\Delta_{nj}}-\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^3}-\frac{\bar{\rho}^2}{2\Delta_{nj}^3}+\frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^5}\right]=$$

$$=\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j}{\Delta_{nj}}+\cancel{\frac{\bar{\rho}\cdot\left(\dot{\bar{R}}_j\times\bar{\omega}\right)}{\Delta_{nj}}}+\cancel{\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^3}}-\frac{\bar{\rho}\cdot\left(\dot{\bar{R}}_j\times\bar{\omega}\right)\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^3}-$$

$$-\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j\bar{\rho}^2}{2\Delta_{nj}^3}+\frac{3}{2}\frac{\dot{\bar{R}}_n\cdot\dot{\bar{R}}_j\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^5};$$

6) $$\frac{1}{\Delta_{nj}^*}=\frac{1}{\Delta_{nj}}\left[1+2\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^2}+\frac{\bar{\rho}^2}{\Delta_{nj}^2}\right]^{-\frac{1}{2}}=\frac{1}{\Delta_{nj}}-\cancel{\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^3}}-\frac{\bar{\rho}^2}{2\Delta_{nj}^3}+\frac{3}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^5}+\ldots;$$

7) $$\dot{\bar{R}}_n^*\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\dot{\bar{R}}_j\cdot\frac{\left(\bar{R}_n^*-\bar{R}_j\right)}{\Delta_{nj}^*}\frac{1}{\Delta_{nj}^*}=\left[\dot{\bar{R}}_n\cdot\left(\bar{R}_n^*-\bar{R}_j\right)+\left(\bar{\omega}\times\bar{\rho}\right)\cdot\left(\bar{R}_n^*-\bar{R}_j\right)\right]\dot{\bar{R}}_j\cdot\left(\bar{R}_n^*-\bar{R}_j\right)\frac{1}{\Delta_{nj}^{*3}}=$$

$$=\left\{\dot{\bar{R}}_n\cdot\left(\bar{R}_n^*-\bar{R}_j\right)+\bar{\rho}\cdot\left[\left(\bar{R}_n^*-\bar{R}_j\right)\times\bar{\omega}\right]\right\}\dot{\bar{R}}_j\cdot\left(\bar{R}_n^*-\bar{R}_j\right)\frac{1}{\Delta_{nj}^{*3}}=$$

$$=\left\{\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)+\bar{\rho}\cdot\dot{\bar{R}}_n+\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]+\cancel{\bar{\rho}\cdot\left(\bar{\rho}\times\bar{\omega}\right)}\equiv 0\right\}\left[\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)+\bar{\rho}\cdot\dot{\bar{R}}_j\right]$$

$$\left\{\frac{1}{\Delta_{nj}^3}-3\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^5}-\frac{3}{2}\frac{\bar{\rho}^2}{\Delta_{nj}^5}+\frac{15}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^7}\right\}=$$

$$=\left[\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)+\bar{\rho}\cdot\dot{\bar{R}}_j\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)+\bar{\rho}\cdot\dot{\bar{R}}_n\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)+\bar{\rho}\cdot\dot{\bar{R}}_n\bar{\rho}\cdot\dot{\bar{R}}_j+\right.$$

$$\left.+\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)+\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\bar{\rho}\cdot\dot{\bar{R}}_j\right]$$

$$\left\{\frac{1}{\Delta_{nj}^3}-3\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^5}-\frac{3}{2}\frac{\bar{\rho}^2}{\Delta_{nj}^5}+\frac{15}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^7}\right\}=$$

$$=\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{\Delta_{nj}^3}-\cancel{\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)3\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^5}}-$$

$$-\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{3}{2}\frac{\bar{\rho}^2}{\Delta_{nj}^5}+\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{15}{2}\frac{\left[\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)\right]^2}{\Delta_{nj}^7}+$$

$$+\cancel{\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\bar{\rho}\cdot\dot{\bar{R}}_j\frac{1}{\Delta_{nj}^3}}-3\dot{\bar{R}}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\bar{\rho}\cdot\dot{\bar{R}}_j\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^5}+$$

$$+\cancel{\bar{\rho}\cdot\dot{\bar{R}}_n\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{\Delta_{nj}^3}}-\bar{\rho}\cdot\dot{\bar{R}}_n\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)3\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^5}+\bar{\rho}\cdot\dot{\bar{R}}_n\bar{\rho}\cdot\dot{\bar{R}}_j\frac{1}{\Delta_{nj}^3}+$$

$$+\cancel{\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)\frac{1}{\Delta_{nj}^3}}-\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)3\frac{\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)}{\Delta_{nj}^5}+$$

$$+\bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\bar{\rho}\cdot\dot{\bar{R}}_j\frac{1}{\Delta_{nj}^3}+\dots .$$

To compute some 12 integrals:

$$\int_{m_n}\left(\bar{\omega}\times\bar{\rho}\right)^2 dm_n=\int_{m_n}\left[\left(\omega_2\zeta-\omega_3\eta\right)^2+\left(\omega_3\xi-\omega_1\zeta\right)^2+\left(\omega_1\eta-\omega_2\xi\right)^2\right]dm_n=$$

$$=\left[\left(\omega_1^2+\omega_2^2\right)\frac{A_n+B_n-C_n}{2}+\left(\omega_3^2+\omega_1^2\right)\frac{C_n+A_n-B_n}{2}+\left(\omega_3^2+\omega_2^2\right)\frac{B_n+C_n-A_n}{2}\right]=$$

$$=\left(A_n\omega_1^2+B_n\omega_2^2+C_n\omega_3^2\right)=\bar{H}_n\cdot\bar{\omega}; \tag{A.1}$$

$$\int_{m_n}\bar{\rho}^2 dm_n=\int_{m_n}(\xi^2+\eta^2+\zeta^2)dm_n=\frac{1}{2}\left(A_n+B_n+C_n\right); \tag{A.2}$$

$$\int_{m_n}\left[\left(\bar{R}_n-\bar{R}_j\right)\cdot\bar{\rho}\right]^2 dm_n=\int_{m_n}\left[\left(x_n-x_j\right)^2\xi^2+\left(y_n-y_j\right)^2\eta^2+\left(z_n-z_j\right)^2\zeta^2\right]dm_n=$$

$$=\frac{1}{2}\left[\left(x_n-x_j\right)^2\left(B_n+C_n-A_n\right)+\left(y_n-y_j\right)^2\left(C_n+A_n-B_n\right)+\left(z_n-z_j\right)^2\left(A_n+B_n-C_n\right)\right]=$$

$$=\frac{1}{2}\left\{\Delta_{nj}^2(A_n+B_n+C_n)-2\left[\left(x_n-x_j\right)^2A_n+\left(y_n-y_j\right)^2B_n+\left(z_n-z_j\right)^2C_n\right]\right\}; \tag{A.3}$$

$$\int_{m_n}\left[\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)\right]^2 dm_n =$$

$$= \int_{m_n}\xi^2\left(\dot{y}_n\omega_3-\dot{z}_n\omega_2\right)^2 dm_n + \int_{m_n}\eta^2\left(\dot{z}_n\omega_1-\dot{x}_n\omega_3\right)^2 dm_n + \int_{m_n}\zeta^2\left(\dot{x}_n\omega_2-\dot{y}_n\omega_1\right)^2 dm_n =$$

$$=\left(\dot{y}_n\omega_3-\dot{z}_n\omega_2\right)^2\frac{B_n+C_n-A_n}{2}+\left(\dot{z}_n\omega_1-\dot{x}_n\omega_3\right)^2\frac{C_n+A_n-B_n}{2}+\left(\dot{x}_n\omega_2-\dot{y}_n\omega_1\right)^2\frac{A_n+B_n-C_n}{2};\quad (A.4)$$

$$\int_{m_n}\bar{\rho}\cdot\left(\dot{\bar{R}}_n\times\bar{\omega}\right)\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)dm_n=\left[\int_{m_n}\xi^2\left(\dot{y}_n\omega_3-\dot{z}_n\omega_2\right)\left(x_n-x_j\right)dm_n+\right.$$

$$\left.+\int_{m_n}\eta^2\left(\dot{z}_n\omega_1-\dot{x}_n\omega_3\right)\left(y_n-y_j\right)dm_n+\int_{m_n}\zeta^2\left(\dot{x}_n\omega_2-\dot{y}_n\omega_1\right)\left(z_n-z_j\right)dm_n\right]=$$

$$=\frac{1}{2}\Big[\left(\dot{y}_n\omega_3-\dot{z}_n\omega_2\right)\left(x_n-x_j\right)\left(B_n+C_n-A_n\right)+\left(\dot{z}_n\omega_1-\dot{x}_n\omega_3\right)\left(y_n-y_j\right)\left(C_n+A_n-B_n\right)+$$

$$+\left(\dot{x}_n\omega_2-\dot{y}_n\omega_1\right)\left(z_n-z_j\right)\left(A_n+B_n-C_n\right)\Big]=$$

$$=\frac{1}{2}\Big\{A_n\omega_1\left[\left(y_n-y_j\right)\dot{z}_n-\left(z_n-z_j\right)\dot{y}_n\right]+B_n\omega_2\left[\left(z_n-z_j\right)\dot{x}_n-\left(x_n-x_j\right)\dot{z}_n\right]+$$

$$+C_n\omega_3\left[\left(x_n-x_j\right)\dot{y}_n-\left(y_n-y_j\right)\dot{x}_n\right]+\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\dot{z}_n+\left(z_n-z_j\right)\dot{y}_n\right]+$$

$$+\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\dot{x}_n+\left(x_n-x_j\right)\dot{z}_n\right]+\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\dot{y}_n+\left(y_n-y_j\right)\dot{x}_n\right]\Big\}=$$

$$=\frac{1}{2}\Big\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_n+$$

$$+\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\dot{z}_n+\left(z_n-z_j\right)\dot{y}_n\right]+\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\dot{x}_n+\left(x_n-x_j\right)\dot{z}_n\right]+$$

$$+\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\dot{y}_n+\left(y_n-y_j\right)\dot{x}_n\right]\Big\};\quad (A.5)$$

$$\int_{m_n}\bar{\rho}\cdot\left(\dot{\bar{R}}_j\times\bar{\omega}\right)\bar{\rho}\cdot\left(\bar{R}_n-\bar{R}_j\right)dm_n=\frac{1}{2}\Big\{\bar{H}_n\cdot\left(\bar{R}_n-\bar{R}_j\right)\times\dot{\bar{R}}_j+$$

$$+\left(C_n-B_n\right)\omega_1\left[\left(y_n-y_j\right)\dot{z}_j+\left(z_n-z_j\right)\dot{y}_j\right]+\left(A_n-C_n\right)\omega_2\left[\left(z_n-z_j\right)\dot{x}_j+\left(x_n-x_j\right)\dot{z}_j\right]+$$

$$+\left(B_n-A_n\right)\omega_3\left[\left(x_n-x_j\right)\dot{y}_j+\left(y_n-y_j\right)\dot{x}_j\right]\Big\};\quad (A.6)$$

$$\int_{m_n}\dot{\bar{R}}_j\cdot\bar{\rho}\left(\bar{R}_n-\bar{R}_j\right)\cdot\bar{\rho}dm_n=\frac{1}{2}\Big[\dot{x}_j\left(x_n-x_j\right)\left(B_n+C_n-A_n\right)+$$

$$+\dot{y}_j\left(y_n-y_j\right)\left(C_n+A_n-B_n\right)+\dot{z}_j\left(z_n-z_j\right)\left(A_n+B_n-C_n\right)\Big]=$$

$$=\frac{\dot{\bar{R}}_j\cdot\left(\bar{R}_n-\bar{R}_j\right)}{2}\left(A_n+B_n+C_n\right)-\left[\dot{x}_j\left(x_n-x_j\right)A_n+\dot{y}_j\left(y_n-y_j\right)B_n+\dot{z}_j\left(z_n-z_j\right)C_n\right];\quad (A.7)$$

$$\int_{m_n} \dot{\bar{R}}_n \cdot \bar{\rho}\left(\bar{R}_n - \bar{R}_j\right) \cdot \bar{\rho} dm_n = \frac{1}{2}\left[\dot{x}_n\left(x_n - x_j\right)\left(B_n + C_n - A_n\right) + \right.$$

$$\left. + \dot{y}_n\left(y_n - y_j\right)\left(C_n + A_n - B_n\right) + \dot{z}_n\left(z_n - z_j\right)\left(A_n + B_n - C_n\right)\right] =$$

$$= \frac{\dot{\bar{R}}_n \cdot \left(\bar{R}_n - \bar{R}_j\right)}{2}\left(A_n + B_n + C_n\right) - \left[\dot{x}_n\left(x_n - x_j\right)A_n + \dot{y}_n\left(y_n - y_j\right)B_n + \dot{z}_n\left(z_n - z_j\right)C_n\right]; \qquad (A.8)$$

$$\int_{m_n} \dot{\bar{R}}_n \cdot \bar{\rho}\dot{\bar{R}}_j \cdot \bar{\rho} dm_n = \frac{1}{2}\left[\dot{x}_n\dot{x}_j\left(B_n + C_n - A_n\right) + \dot{y}_n\dot{y}_j\left(C_n + A_n - B_n\right) + \right.$$

$$\left. + \dot{z}_n\dot{z}_j\left(A_n + B_n - C_n\right)\right] = \frac{\dot{\bar{R}}_n \cdot \dot{\bar{R}}_j}{2}\left(A_n + B_n + C_n\right) - \left[\dot{x}_n\dot{x}_jA_n + \dot{y}_n\dot{y}_jB_n + \dot{z}_n\dot{z}_jC_n\right]; \qquad (A.9)$$

$$\int_{m_n} \bar{\rho} \cdot \left[\left(\bar{R}_n - \bar{R}_j\right) \times \bar{\omega}\right]\bar{\rho} \cdot \left(\bar{R}_n - \bar{R}_j\right) dm_n = \int_{m_n} \xi^2\left(x_n - x_j\right)\left[\left(y_n - y_j\right)\omega_3 - \left(z_n - z_j\right)\omega_2\right] dm_n +$$

$$+ \int_{m_n} \eta^2\left(y_n - y_j\right)\left[\left(z_n - z_j\right)\omega_1 - \left(x_n - x_j\right)\omega_3\right] dm_n + \int_{m_n} \zeta^2\left(z_n - z_j\right)\left[\left(x_n - x_j\right)\omega_2 - \left(y_n - y_j\right)\omega_1\right] dm_n =$$

$$= \frac{1}{2}\left\{\left(x_n - x_j\right)\left[\omega_3\left(y_n - y_j\right) - \omega_2\left(z_n - z_j\right)\right]\left(B_n + C_n - A_n\right) + \right.$$

$$+ \left(y_n - y_j\right)\left[\omega_1\left(z_n - z_j\right) - \omega_3\left(x_n - x_j\right)\right]\left(C_n + A_n - B_n\right) +$$

$$\left. + \left(z_n - z_j\right)\left[\omega_2\left(x_n - x_j\right) - \omega_1\left(y_n - y_j\right)\right]\left(A_n + B_n - C_n\right)\right\} =$$

$$= \left(x_n - x_j\right)\left(y_n - y_j\right)\omega_3\left(B_n - A_n\right) + \left(z_n - z_j\right)\left(x_n - x_j\right)\omega_2\left(A_n - C_n\right) +$$

$$+ \left(y_n - y_j\right)\left(z_n - z_j\right)\omega_1\left(C_n - B_n\right); \qquad (A.10)$$

$$\int_{m_n} \bar{\rho} \cdot \left(\bar{R}_n - \bar{R}_j\right)\bar{\rho} \cdot \left(\bar{R}_n - \bar{R}_k\right) dm_n = \frac{1}{2}\left[\left(x_n - x_j\right)\left(x_n - x_k\right)\left(B_n + C_n - A_n\right) + \right.$$

$$\left. + \left(y_n - y_j\right)\left(y_n - y_k\right)\left(C_n + A_n - B_n\right) + \left(z_n - z_j\right)\left(z_n - z_k\right)\left(A_n + B_n - C_n\right)\right] =$$

$$= \frac{\left(\bar{R}_n - \bar{R}_j\right) \cdot \left(\bar{R}_n - \bar{R}_k\right)}{2}\left(A_n + B_n + C_n\right) -$$

$$- \left[A_n\left(x_n - x_j\right)\left(x_n - x_k\right) + B_n\left(y_n - y_j\right)\left(y_n - y_k\right) + C_n\left(z_n - z_j\right)\left(z_n - z_k\right)\right]; \qquad (A.11)$$

$$\int_{m_n} \bar{\rho}\cdot\left[\left(\bar{R}_n-\bar{R}_j\right)\times\bar{\omega}\right]\bar{\rho}\cdot\dot{\bar{R}}_j dm_n = \int_{m_n}\xi^2\dot{x}_j\left[\left(y_n-y_j\right)\omega_3-\left(z_n-z_j\right)\omega_2\right]dm_n +$$

$$+\int_{m_n}\eta^2\dot{y}_j\left[\left(z_n-z_j\right)\omega_1-\left(x_n-x_j\right)\omega_3\right]dm_n+\int_{m_n}\zeta^2\dot{z}_j\left[\left(x_n-x_j\right)\omega_2-\left(y_n-y_j\right)\omega_1\right]dm_n=$$

$$=\frac{1}{2}\Big\{\dot{x}_j\left[\omega_3\left(y_n-y_j\right)-\omega_2\left(z_n-z_j\right)\right]\left(B_n+C_n-A_n\right)+$$

$$+\dot{y}_j\left[\omega_1\left(z_n-z_j\right)-\omega_3\left(x_n-x_j\right)\right]\left(C_n+A_n-B_n\right)+\dot{z}_j\left[\omega_2\left(x_n-x_j\right)-\omega_1\left(y_n-y_j\right)\right]\left(A_n+B_n-C_n\right)\Big\}=$$

$$=\frac{1}{2}\Big\{A_n\omega_1\left[\dot{y}_j\left(z_n-z_j\right)-\dot{z}_j\left(y_n-y_j\right)\right]+B_n\omega_2\left[\dot{z}_j\left(x_n-x_j\right)-\dot{x}_j\left(z_n-z_j\right)\right]+$$

$$+C_n\omega_3\left[\dot{x}_j\left(y_n-y_j\right)-\dot{y}_j\left(x_n-x_j\right)\right]+\left(C_n-B_n\right)\omega_1\left[\dot{y}_j\left(z_n-z_j\right)+\dot{z}_j\left(y_n-y_j\right)\right]+$$

$$+\left(A_n-C_n\right)\omega_2\left[\dot{z}_j\left(x_n-x_j\right)+\dot{x}_j\left(z_n-z_j\right)\right]+\left(B_n-A_n\right)\omega_3\left[\dot{x}_j\left(y_n-y_j\right)+\dot{y}_j\left(x_n-x_j\right)\right]\Big\}=$$

$$=\frac{1}{2}\Big\{\bar{H}_n\cdot\dot{\bar{R}}_j\times\left(\bar{R}_n-\bar{R}_j\right)+\left(C_n-B_n\right)\omega_1\left[\dot{y}_j\left(z_n-z_j\right)+\dot{z}_j\left(y_n-y_j\right)\right]+$$

$$+\left(A_n-C_n\right)\omega_2\left[\dot{z}_j\left(x_n-x_j\right)+\dot{x}_j\left(z_n-z_j\right)\right]+\left(B_n-A_n\right)\omega_3\left[\dot{x}_j\left(y_n-y_j\right)+\dot{y}_j\left(x_n-x_j\right)\right]\Big\}. \qquad (A.12)$$

APPENDIX B

The partial derivatives of the kinetic energy are computed by using (1.1):

$$\begin{aligned}
&\frac{\partial T}{\partial\lambda_i}=A_n\omega_1\frac{\partial\omega_1}{\partial\lambda_i}+B_n\omega_2\frac{\partial\omega_2}{\partial\lambda_i}+C_n\omega_3\frac{\partial\omega_3}{\partial\lambda_i},\\
&\frac{\partial\omega_1}{\partial\lambda_0}=2\dot{\lambda}_1;\frac{\partial\omega_1}{\partial\lambda_1}=-2\dot{\lambda}_0;\frac{\partial\omega_1}{\partial\lambda_2}=-2\dot{\lambda}_3;\frac{\partial\omega_1}{\partial\lambda_3}=2\dot{\lambda}_2;\\
&\frac{\partial\omega_2}{\partial\lambda_0}=2\dot{\lambda}_2;\frac{\partial\omega_2}{\partial\lambda_1}=2\dot{\lambda}_3;\frac{\partial\omega_2}{\partial\lambda_2}=-2\dot{\lambda}_0;\frac{\partial\omega_2}{\partial\lambda_3}=-2\dot{\lambda}_1;\\
&\frac{\partial\omega_3}{\partial\lambda_0}=2\dot{\lambda}_3;\frac{\partial\omega_3}{\partial\lambda_1}=-2\dot{\lambda}_2;\frac{\partial\omega_3}{\partial\lambda_2}=2\dot{\lambda}_1;\frac{\partial\omega_3}{\partial\lambda_3}=-2\dot{\lambda}_0;
\end{aligned} \qquad (B.1)$$

$$\begin{aligned}
&\frac{\partial T}{\partial\dot{\lambda}_i}=A_n\omega_1\frac{\partial\omega_1}{\partial\dot{\lambda}_i}+B_n\omega_2\frac{\partial\omega_2}{\partial\dot{\lambda}_i}+C_n\omega_3\frac{\partial\omega_3}{\partial\dot{\lambda}_i},\\
&\frac{\partial\omega_1}{\partial\dot{\lambda}_0}=-2\lambda_1;\frac{\partial\omega_1}{\partial\dot{\lambda}_1}=2\lambda_0;\frac{\partial\omega_1}{\partial\dot{\lambda}_2}=2\lambda_3;\frac{\partial\omega_1}{\partial\dot{\lambda}_3}=-2\lambda_2;\\
&\frac{\partial\omega_2}{\partial\dot{\lambda}_0}=-2\lambda_2;\frac{\partial\omega_2}{\partial\dot{\lambda}_1}=-2\lambda_3;\frac{\partial\omega_2}{\partial\dot{\lambda}_2}=2\lambda_0;\frac{\partial\omega_2}{\partial\dot{\lambda}_3}=2\lambda_1;\\
&\frac{\partial\omega_3}{\partial\dot{\lambda}_0}=-2\lambda_3;\frac{\partial\omega_3}{\partial\dot{\lambda}_1}=2\lambda_2;\frac{\partial\omega_3}{\partial\dot{\lambda}_2}=-2\lambda_1;\frac{\partial\omega_3}{\partial\dot{\lambda}_3}=2\lambda_0;
\end{aligned} \qquad (B.2)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\lambda}_i} = A_n\dot{\omega}_1\frac{\partial \omega_1}{\partial \dot{\lambda}_i} + B_n\dot{\omega}_2\frac{\partial \omega_2}{\partial \dot{\lambda}_i} + C_n\dot{\omega}_3\frac{\partial \omega_3}{\partial \dot{\lambda}_i} + A_n\omega_1\frac{d}{dt}\frac{\partial \omega_1}{\partial \dot{\lambda}_i} + B_n\omega_2\frac{d}{dt}\frac{\partial \omega_2}{\partial \dot{\lambda}_i} + C_n\omega_3\frac{d}{dt}\frac{\partial \omega_3}{\partial \dot{\lambda}_i},$$

$$\frac{d}{dt}\frac{\partial \omega_1}{\partial \dot{\lambda}_0} = -2\dot{\lambda}_1; \frac{d}{dt}\frac{\partial \omega_1}{\partial \dot{\lambda}_1} = 2\dot{\lambda}_0; \frac{d}{dt}\frac{\partial \omega_1}{\partial \dot{\lambda}_2} = 2\dot{\lambda}_3; \frac{d}{dt}\frac{\partial \omega_1}{\partial \dot{\lambda}_3} = -2\dot{\lambda}_2;$$

$$\frac{d}{dt}\frac{\partial \omega_2}{\partial \dot{\lambda}_0} = -2\dot{\lambda}_2; \frac{d}{dt}\frac{\partial \omega_2}{\partial \dot{\lambda}_1} = -2\dot{\lambda}_3; \frac{d}{dt}\frac{\partial \omega_2}{\partial \dot{\lambda}_2} = 2\dot{\lambda}_0; \frac{d}{dt}\frac{\partial \omega_2}{\partial \dot{\lambda}_3} = 2\dot{\lambda}_1;$$

$$\frac{d}{dt}\frac{\partial \omega_3}{\partial \dot{\lambda}_0} = -2\dot{\lambda}_3; \frac{d}{dt}\frac{\partial \omega_3}{\partial \dot{\lambda}_1} = 2\dot{\lambda}_2; \frac{d}{dt}\frac{\partial \omega_3}{\partial \dot{\lambda}_2} = -2\dot{\lambda}_1; \frac{d}{dt}\frac{\partial \omega_3}{\partial \dot{\lambda}_3} = 2\dot{\lambda}_0;$$

Thus $\frac{d}{dt}\frac{\partial \omega_j}{\partial \dot{\lambda}_i} = -\frac{\partial \omega_j}{\partial \lambda_i}, i = 1,2,3,4; j = 1,2,3.$ (B.3)

$$\begin{aligned}
\{T\}_0 &= 2\left(-\lambda_1 A_n\dot{\omega}_1 - \lambda_2 B_n\dot{\omega}_2 - \lambda_3 C_n\dot{\omega}_3\right) + 4\left(-\dot{\lambda}_1 A_n\omega_1 - \dot{\lambda}_2 B_n\omega_2 - \dot{\lambda}_3 C_n\omega_3\right);\\
\{T\}_1 &= 2\left(\lambda_0 A_n\dot{\omega}_1 - \lambda_3 B_n\dot{\omega}_2 + \lambda_2 C_n\dot{\omega}_3\right) + 4\left(\dot{\lambda}_0 A_n\omega_1 - \dot{\lambda}_3 B_n\omega_2 + \dot{\lambda}_2 C_n\omega_3\right);\\
\{T\}_2 &= 2\left(\lambda_3 A_n\dot{\omega}_1 + \lambda_0 B_n\dot{\omega}_2 - \lambda_1 C_n\dot{\omega}_3\right) + 4\left(\dot{\lambda}_3 A_n\omega_1 + \dot{\lambda}_0 B_n\omega_2 - \dot{\lambda}_1 C_n\omega_3\right);\\
\{T\}_3 &= 2\left(-\lambda_2 A_n\dot{\omega}_1 + \lambda_1 B_n\dot{\omega}_2 + \lambda_0 C_n\dot{\omega}_3\right) + 4\left(-\dot{\lambda}_2 A_n\omega_1 + \dot{\lambda}_1 B_n\omega_2 + \dot{\lambda}_0 C_n\omega_3\right).
\end{aligned} \qquad \text{(B.4)}$$

The partial derivatives of the angular momentum vector of the rotational motion of the rigid body are computed:

$$\begin{aligned}
&\frac{\partial \bar{H}}{\partial \lambda_i} = A_n\frac{\partial \omega_1}{\partial \lambda_i}\bar{i}_1 + B_n\frac{\partial \omega_2}{\partial \lambda_i}\bar{i}_2 + C_n\frac{\partial \omega_3}{\partial \lambda_i}\bar{i}_3,\\
&\frac{\partial \bar{H}}{\partial \dot{\lambda}_i} = A_n\frac{\partial \omega_1}{\partial \dot{\lambda}_i}\bar{i}_1 + B_n\frac{\partial \omega_2}{\partial \dot{\lambda}_i}\bar{i}_2 + C_n\frac{\partial \omega_3}{\partial \dot{\lambda}_i}\bar{i}_3,\\
&\frac{d}{dt}\frac{\partial \bar{H}}{\partial \dot{\lambda}_i} = A_n\frac{d}{dt}\frac{\partial \omega_1}{\partial \dot{\lambda}_i}\bar{i}_1 + B_n\frac{d}{dt}\frac{\partial \omega_2}{\partial \dot{\lambda}_i}\bar{i}_2 + C_n\frac{d}{dt}\frac{\partial \omega_3}{\partial \dot{\lambda}_i}\bar{i}_3,\\
&\frac{d}{dt}\frac{\partial \bar{H}}{\partial \dot{\lambda}_i} = -\frac{\partial \bar{H}}{\partial \lambda_i};\\
&\{\bar{H}\}_i = -2\left(A_n\frac{\partial \omega_1}{\partial \lambda_i}\bar{i}_1 + B_n\frac{\partial \omega_2}{\partial \lambda_i}\bar{i}_2 + C_n\frac{\partial \omega_3}{\partial \lambda_i}\bar{i}_3\right).
\end{aligned} \qquad \text{(B.5)}$$

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