ARTIFICIAL SATELLITES, Vol. 53, No. 1 – 2018
DOI: 10.2478/arsa-2018-0005

A NEW METHOD FOR MEASURING ANGULAR INCREMENTS BASED ON A TRI-AXIAL ACCELEROMETER AND A TRI-AXIAL MAGNETOMETER

X.N. Zhu[1], M. Ma[1], D.F. Cheng[1], Zh.J. Zhou[1]
College of Instrumentation & Electrical Engineering, Jilin University[1]
e-mails: zhuxn14@mails.jlu.edu.cn, 451079830@qq.com, chengdefu@jlu.edu.cn, zhouzhijian@jlu.edu.cn.

ABSTRACT. Tri-axial gyroscopes used to be the only instrument used to measure the angular increments of airplanes. However, because there were no reference devices, drift and accumulation errors affected the accuracy of the estimated attitude. In this paper, we propose a novel method for measuring angular increments based on a tri-axial accelerometer and a tri-axial magnetometer. Then, we mathematically proved the feasibility of the proposed method. The results of our simulation and experimental tests indicated that the proposed method accurately indicates the attitude of an airplane.

1. INTRODUCTION

The attitude and heading reference system (AHRS) is the most common device for airplane navigation. The system contains a tri-axial accelerometer, a tri-axial magnetometer, and a tri-axial gyroscope. The accelerometer and the magnetometer calculate approximate attitude and heading information including rough yaw, pitch and roll. The gyroscope calculates the angular increments that are combined with the approximate attitude and heading information to obtain the optimal estimation of attitude. The block diagram of AHRS was shown in Figure 1.

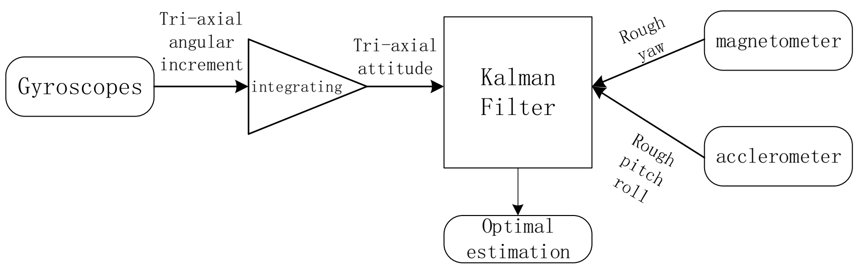

Fig. 1. Block diagram of AHRS

The accuracy of the estimate depends primarily on the fusion algorithm, but the approximations of yaw, pitch, roll, and tri-axial angular increment also affect the accuracy. In [1], the researchers used the following equations to calculate the approximate attitude and heading:

$$\theta = \sin^{-1}(\frac{A_x}{g}) \tag{1}$$

$$\varphi = \tan^{-1}(\frac{A_y}{A_z}) \tag{2}$$

$$\psi = \tan^{-1}(\frac{-B_y \cos(\gamma) + B_z \sin(\gamma)}{B_x \cos(\theta) + B_y \sin(\theta)\sin(\gamma) + B_z \sin(\theta)\cos(\gamma)}) \tag{3}$$

where $A_{x,y,z}$ are the outputs of the accelerometers, and $B_{x,y,z}$ are the outputs of the magnetometers. g is the gravitational field. During flight, the two sensors inevitably are interfused with motion acceleration and magnetic interference, so θ, φ, and ψ are the approximate pitch, roll, and yaw angle. In [2], the authors used a tri-axial gyroscope to provide the angular increments, and they used the following equation to determine the attitude:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -\sin\psi/\tan\theta & \cos\psi/\tan\theta & 1 \\ \cos\psi & \sin\psi & 0 \\ \sin\psi/\sin\theta & -\cos\psi/\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \tag{4}$$

where $\dot{\theta}$, $\dot{\phi}$, and $\dot{\psi}$ are the tri-axial angular increment in the navigation frame; ω_x, ω_y, and ω_z are the tri-axial angular increment in the body frame, which always are obtained by gyroscopes. Reference [3] illustrated a time-accumulating error in the gyroscope that corrupted the estimate of the attitude.

In this paper, a new method is proposed for calculating the tri-axial angular increments using a tri-axial accelerometer and a tri-axial magnetometer. The basic theory comes from the classic TRIAD algorithm with the exception of using the status of the previous moment as the initial moment. The proposed approach is illustrated in detail in the following sections. The simulations and experiments that we conducted proved the feasibility of this new, accurate resource for determining angular increments.

2. TRI-AXIAL ATTITUDE DETERMINATION (TRIAD)

TRIAD first was proposed in [3] to solve the attitude of satellites. TRIAD uses two non-parallel vectors to construct a substitute coordinate. In [4], the TRIAD algorithm was used in AHRS to calculate the approximate attitude (before fusing with the results from the gyroscope).

$$\begin{cases} \hat{r}_1 = \hat{V}_1 \\ \hat{r}_2 = (\hat{V}_1 \times \hat{V}_2)/|\hat{V}_1 \times \hat{V}_2| \\ \hat{r}_3 = (\hat{V}_1 \times \hat{r}_2)/|\hat{V}_1 \times \hat{r}_2| \end{cases} \qquad \begin{cases} \hat{s}_1 = \hat{W}_1^n \\ \hat{s}_2 = (\hat{W}_1^n \times \hat{W}_2^n)/|\hat{W}_1^n \times \hat{W}_2^n| \\ \hat{s}_3 = (\hat{W}_1^n \times \hat{s}_2^n)/|\hat{W}_1^n \times \hat{s}_2^n| \end{cases} \tag{5}$$

$$M_{ref} = [\hat{r}_1 \vdots \hat{r}_2 \vdots \hat{r}_3] \qquad M_{obs}^n = [\hat{s}_1 \vdots \hat{s}_2 \vdots \hat{s}_3] \tag{6}$$

where $\hat{V}_1$ and $\hat{V}_2$ are the initial status of the geomagnetic and gravitational vectors under the navigation frame. $\hat{W}_1$ and $\hat{W}_1$ are the observation vectors measured by magnetoemter and accelerometer. n represents the nth sampling point. M_{ref} is the reference matrix. M_{obs} is the observation matrix. Unlike equations (1-3), the attitude matrix calculated by TRIAD at the nth moment is:

$$A = M_{obs}^n / M_{ref} \tag{7}$$

where A is the attitude matrix. From equation (7), it can be seen that the attitude matrix is calculated only with the status of the initial moment and the current status, allowing the attitude and heading angle to be obtained simultaneously from the matrix.

3. TRI-AXIAL ATTITUDE DETERMINATION (TRIAD)

As illustrated in [4], changing the attitude is a dynamic process of summing countless small attitude changes. Figure 2 shows a one-dimensional attitude changing of pitch.

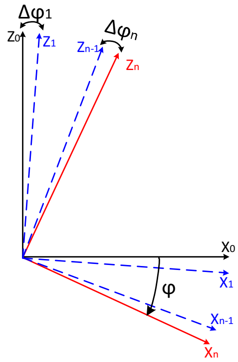

Fig. 2. Diagram of the attitude changing process

The TRIAD algorithm only uses the current attitude, and it ignores the dynamic changes that occur during the process. However, in this paper, the last moment is substituted by the initial moment, i.e., M_{ref} changes to M_{obs}^{n-1}.

$$\Delta A^n = M_{obs}^n / M_{obs}^{n-1} \tag{8}$$

where ΔA^n is the transform matrix calculated from the current moment and the previous moment. The next step is to prove that the angle obtained from ΔA^n can represent the angular increment.

In [4], the definition of angular increment was given as:

$$\begin{aligned}\omega &= \begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}\\ &= \begin{bmatrix}\dot{P}\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}1 & 0 & 0\\ 0 & \cos(P+\dot{P}) & \sin(P+\dot{P})\\ 0 & -\sin(P+\dot{P}) & \cos(P+\dot{P})\end{bmatrix}\begin{bmatrix}0\\ \dot{R}\\ 0\end{bmatrix}\\ &+ \begin{bmatrix}1 & 0 & 0\\ 0 & \cos(P+\dot{P}) & \sin(P+\dot{P})\\ 0 & -\sin(P+\dot{P}) & \cos(P+\dot{P})\end{bmatrix}\begin{bmatrix}\cos(R+\dot{R}) & 0 & -\sin(R+\dot{R})\\ 0 & 1 & 0\\ \sin(R+\dot{R}) & 0 & \cos(R+\dot{R})\end{bmatrix}\begin{bmatrix}0\\ 0\\ \dot{Q}\end{bmatrix}\end{aligned} \tag{9}$$

where $\dot{P}$, $\dot{R}$, and $\dot{Q}$ are the angular increment under the reference frame (the previous moment of the airplane), i.e., they are the variations in the angles between the current moment and the previous moment, and P and R are the initial pitch and roll angle, respectively. If the reference frame selects the previous status every time, $P = 0$, $R = 0$. Assume that $\dot{P}$, $\dot{R}$, and $\dot{Q}$ are all small quantities. Then, the rotation order is insignificant [7]. Furthermore, if $\dot{P}$ and $\dot{R}$ are infinitely small, the equation becomes:

$$\lim_{\dot{P}\to 0\dot{R}\to 0\dot{Q}\to 0}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}=\begin{bmatrix}\dot{P}\\ \dot{R}\\ \dot{Q}\end{bmatrix} \tag{10}$$

Equation (10) indicates that the angles calculated from ΔA^n are approximately equal to the angular increments. Note that equation (10) is the condition used by the TRIAID algorithm.

4. THE ERROR MODEL OF TRIAID

The accuracy of the angular increment depends on the accuracy of the field vectors. Assume that the scalar factor, non-orthogonal, and misalignment errors were already calibrated. The wideband noise is the only factor we are concerned with in this paper. The authors of [5} proposed the error model of TRIAD and, accordingly, it is derived here:

$$-\Omega\left(\delta\dot{\theta}\right)=\begin{bmatrix}0 & \delta\dot{\theta}_3 & -\delta\dot{\theta}_2\\ -\delta\dot{\theta}_3 & 0 & \delta\dot{\theta}_1\\ \delta\dot{\theta}_2 & -\delta\dot{\theta}_1 & 0\end{bmatrix}, \tag{11}$$

where $\delta\dot{\theta}_{1,2,3}$ is the angular increment error of the corresponding axis and Ω represents the skew symmetric matrix.

$$P=\left\langle \delta\dot{\theta}\delta\dot{\theta}^T\right\rangle \tag{12}$$

where P is the covariance matrix. According to equation (27) in [6], P was defined as:

$$P=\left\langle \delta M^t_{obs}\delta M^{t\,T}_{obs}\right\rangle+\Delta A\left\langle \delta M^{t-1}_{obs}\delta M^{t-1\,T}_{obs}\right\rangle\Delta A^T \tag{13}$$

However, the reference matrix, M_{ref}, changes to M^{n-1}_{obs}.

$$P=P^t_{obs}+\Delta A P^{t-1}_{obs}\Delta A \tag{14}$$

where P^t_{obs} is the covariance matrix of the observation at the tth moment. Therefore, the error model of TRIAID was given as:

$$P=\sigma_1^2 I+\frac{1}{\left|\hat{w}_1^t+\hat{w}_2^t\right|^2}\left[\sigma_1^2\left(\hat{w}_1^t\bullet\hat{w}_2^t\right)\left(\hat{w}_1^t\hat{w}_2^{t\,T}+\hat{w}_2^t\hat{w}_1^{t\,T}\right)+\left(\sigma_2^2-\sigma_1^2\right)\hat{w}_1^t\hat{w}_1^{t\,T}\right] \tag{15}$$

The equation above indicates that the accuracy of the determination of the angular increment is dominated by two components, i.e., the wideband noise and the including angle of vectors which caused by the dot product in equation.

5. SIMULATIONS

The condition of TRIAID is that the actual angular increment is a very small quantity. Therefore, this condition requires simulation to verify the feasibility of the algorithm. Assume that the angular increment was increasing 0.01° every sampling point within the range of 0°-5°.

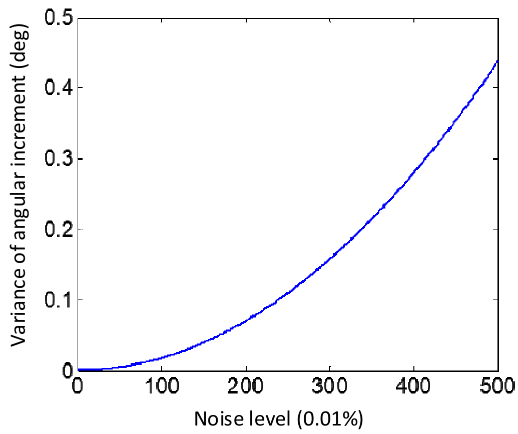

Fig. 3. Results of the TRIAID calculation with changing attitude

Fig. 3 shows that the accuracy of the angular increment using TRIAID is directly proportional to the actual variations in the angular increment. The algorithm because of the approximation in equation (10) creates this part of the error.

However, in practical work, it is inevitable that field sensors will have wideband noise, even if the sensors were already calibrated [6]. Therefore, it is necessary to simulate the effect of noise on the angular increment. Fig. 3 indicates that TRIAID has no error when the angular increment is zero. So, the simulation was performed without allowing the attitude to change. We assumed that that magnetometer and the accelerometer had the same noise level.

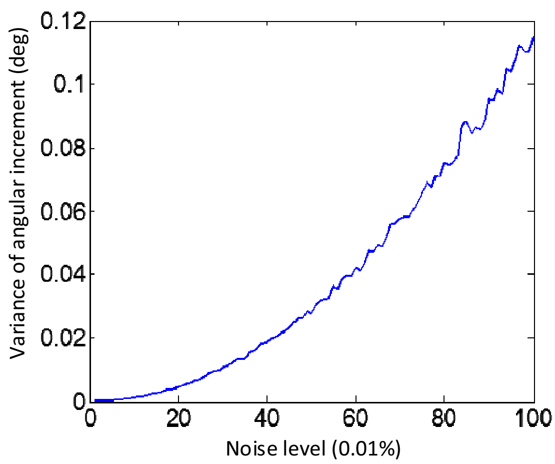

Fig. 4. TRIAID calculation with the noise level increasing

Fig. 4 shows that the accuracy of TRIAID is inversely proportional to the level of the wideband noise.

6. EXPERIMENTS

An AHRS (AH100B version) produced by RION Company was used in the experiment. It contained an HMC5883 magnetometer with a resolution of 500 nT, an ADXL345 accelerometer with a resolution of 10 mg, and an L3G4200 gyroscope with a resolution of 0.1

°/s. The output frequency was set at 100 Hz. The results of the experiment are provided below.

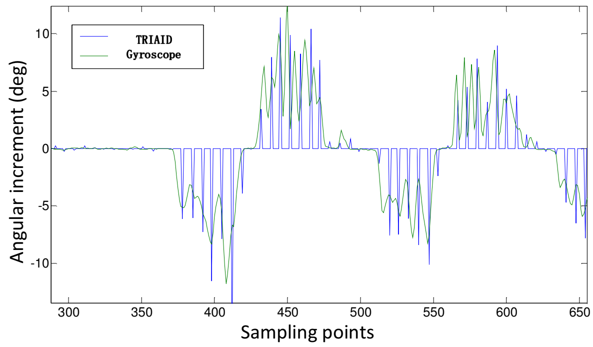

Fig. 5. Experimental results comparing the outputs of TRIAID and a Gyroscope

In Figure 5, the angular increment calculated by TRIAID is similaly with gyroscope. But the angular increment calculated by TRIAID always turn to zero, due to the response time of magnetometer and accelerometer is slower than gyroscope. Then, we updated the attitude of AHRS using equation (9) with the angular increment data obtained in the last experiment. The results are given in Figure 6.

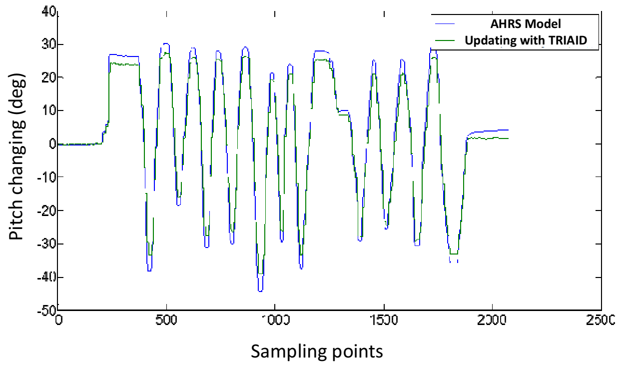

Fig. 6. Experimental results comparing attitude updating with angular increments obtained from gyroscope and TRIAID

7. CONCLUSIONS

In this paper, a new method for measuring angular increment named TRIAID was proposed. To overcome the drawbacks of the gyroscope, a combination of a magnetometer and an accelerometer was used to obtain angular increment information directly. Simulations were used to verify the feasible condition. The experiments showed that the superior accuracy of new angular increment resource.

Note that there were two parameters that determined the accuracy of the algorithm, i.e., a) then lower wideband noise of the sensors and b) the higher bandwidth of the sensors. These are two paradoxical elements, and the optimal set of parameter would be considered in future work.

REFERENCES

[1] Lee, D. C., Moon, G., & Lee, J. C. (2002). Mechanical dither design for ring laser gyroscope. KSME international journal, 16(4), 485-491.

[2] Titterton, D., & Weston, J. L. (2004). Strapdown inertial navigation technology (Vol. 17). IET.

[3] Georgy, J., Noureldin, A., Korenberg, M. J., & Bayoumi, M. M. (2010). Modeling the stochastic drift of a MEMS-based gyroscope in gyro/odometer/GPS integrated navigation. IEEE Transactions on Intelligent transportation systems, 11(4), 856-872.

[4] Gebre-Egziabher, D., Hayward, R. C., & Powell, J. D. (1998). A low-cost GPS/inertial attitude heading reference system (AHRS) for general aviation applications. Position Location and Navigation Symposium, IEEE 1998 (pp. 518-525).

[5] De Marina, H. G., Pereda, F. J., Giron-Sierra, J. M., & Espinosa, F. (2012). UAV attitude estimation using unscented Kalman filter and TRIAD. IEEE Transactions on Industrial Electronics, 59(11), 4465-4474.

[6] Shuster, M. D., & Oh, S. D. (2012). Three-axis attitude determination from vector observations. Journal of Guidance, Control, and Dynamics.

[7] Hasan, A. M., Samsudin, K., Ramli, A. R., Azmir, R. S., & Ismaeel, S. A. (2009). A review of navigation systems (integration and algorithms). Australian Journal of Basic and Applied Sciences, 3(2), 943-959.

Received: 2017-09-30,

Reviewed: 2018-01-17, and 2018-01-18, by He Junbo,

Accepted: 2018-02-02.