APPLICATION OF GREY MODEL GM(1, 1) TO

ULTRA SHORT-TERM PREDICTIONS OF UNIVERSAL TIME

Yu Lei[1, 2, 3], Min Guo[4], Danning Zhao[1, 3], Hongbing Cai[1, 2] and Dandan Hu[4]

[1](National Time Service Center, Chinese Academy of Sciences, China)
[2](Key Laboratory of Time and Frequency Primary Standards, Chinese Academy of Sciences, China)
[3](University of Chinese Academy of Sciences, China)
[4](Xi'an Institute of Optics and Precision Mechanics, Chinese Academy of Sciences, China)
(E-mail: leiyu@ntsc.ac.cn)

ABSTRACT. A mathematical model known as one-order one-variable grey differential equation model GM(1, 1) has been herein employed successfully for the ultra short-term (<10days) predictions of universal time (UT1-UTC). The results of predictions are analyzed and compared with those obtained by other methods. It is shown that the accuracy of the predictions is comparable with that obtained by other prediction methods. The proposed method is able to yield an exact prediction even though only a few observations are provided. Hence it is very valuable in the case of a small size dataset since traditional methods, e.g., least-squares (LS) extrapolation, require longer data span to make a good forecast. In addition, these results can be obtained without making any assumption about an original dataset, and thus is of high reliability. Another advantage is that the developed method is easy to use. All these reveal a great potential of the GM(1, 1) model for UT1-UTC predictions.

KEYWORDS: 1. Universal Time (UT1-UTC). 2. Prediction. 3. Grey Model.

1. INTRODUCTION

The near real-time estimates and short-term forecasts of Earth Orientation Parameters (EOPs): universal time (UT1-UTC), x_p, y_p pole coordinates and nutation-precession corrections $d\psi$, $d\varepsilon$, are needed in various fields linked to reference systems such as precise orbit determinations of the Earth artificial satellites, space navigation, time-keeping and positional astronomy (Gambis and Luzum, 2011). EOP

predictions enable the real-time transformation between the International Celestial and Terrestrial Reference Frames (ICRF and ITRF). Regularly generated EOP predictions are provided by several international and national services, e.g., the International Earth Rotation and Reference Systems Service (IERS), or the EOP Service of the Institute for Applied Astronomy (IAA) in Saint Petersburg, Russia (Malkin and Skurikhina, 1996).

The forecast accuracy of the nutation-precession corrections $d\psi$, $d\varepsilon$ for 1-year in the future is of the order of their observational error, which is of the order of 100μas, because the current IAU2000/IAU2006 nutation-precession model fits very well to the observations (Kalarus et al., 2010). Nevertheless, x_p, y_p pole coordinates and in particular UT1-UTC vary rapidly and are unpredictable in time. Therefore, determinations and forecasts of the three parameters are an ongoing challenge. Usually, the prediction accuracy of the three parameters even for a few days in the future are much worse than their observational accuracy, which now corresponds to 50–100μas in the case of x_p, y_p pole coordinates and 5–10μs in the case of UT1-UTC (Kalarus et al., 2010). Among these five EOPs, the UT1-UTC or its first derivative, length of day (LOD), which represents the variations of the Earth's rotation rate, is the most difficult to forecast. The greatest difficulties in UT1-UTC or LOD forecasts are owing to the occurrence of extremes in the LOD signal caused by the collapse of the tropical monsoon during an El Niño event (Gross et al., 1996).

Various prediction strategies and techniques were employed in the past for improvement of the UT1-UTC or LOD predictions, e.g., autocovariance (AC) techniques (Kosek et al., 1998), artificial neural networks (ANN) (Schuh et al., 2002; Liao et al., 2012; Zhang et al., 2012; Lei et al., 2015(a)), fuzzy inference systems (FIS) (Akyilmaz and Kutterer, 2004), Gaussian process regression (GPR) (Lei et al., 2015(b)), autoregressive (AR) and multivariate autoregressive (MAR) techniques (Niedzielski and Kosek, 2008). In October 2005 the Earth Orientation Parameters Prediction Comparison Campaign (EOP PCC) was started in an effort to evaluate different methods available for forecasting EOP data under the same rules and conditions. A main conclusion of the EOP PCC is that there is not one particular prediction method superior to the others for all EOP components and all prediction intervals (Kalarus et al., 2010). The campaign has prompted researchers for more activity in this domain to improve the existing prediction approaches (Guo et al., 2013; Xu and Zhou, 2015).

The grey system theory was initially presented by Deng (1986). A grey model adopts the essential part of the theory, and it has some advantages, including: (a) it can be used to model a system with insufficient information; (b) relativity a little data are required for establishing a grey model, i.e., the minimum data down to four observations; (c) it does not rely on statistical methods to approximate a time-series, and therefore avoids any assumption about an original dataset. A grey model has

already been successfully applied in many domains due to its merits, e.g., atom clock predictions.

This paper focuses on predicting UT1–UTC. The first-order one-variable grey differential equation model, abbreviate as GM(1, 1), is used as a method to predict UT1-UTC data. In order to demonstrate the performance of the presented approach, we compared the results with those of the combination of least-squares (LS) extrapolation and AR prediction, denoted as LS+AR, the combination of LS extrapolation and ANN prediction, referred to as LS+ANN, as well as the EOP PCC.

This paper is divided into four sections. Following the introduction, Section 2 describes the methodology for forecasting UT1-UTC based on the GM(1, 1) model, including the detailed review of the operation of grey forecasting, building of the prediction model and criterion for prediction error estimates. The results of the predictions are analysed and compared with those obtained by other methods in Section 3, followed by a discussion in Section 4.

2. METHODOLOGY

2.1. GM(1, 1) MODEL

A grey model has the three basic operations: (1) accumulated generating operation (AGO), (2) grey modeling, (3) inverse accumulated generating operation (IAGO). Generally speaking, it utilizes the operations of accumulated generation to construct a differential equation. The GM(1, 1) model, which has already been used most widely among various grey models, is summarized as follows (Deng, 1986).

Suppose there is an original time-series $X^{(0)}$ with n samples, which can be expressed as

$$X^{(0)} = \left\{ x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n) \right\}, \qquad (1)$$

where $x^{(0)}(t)$ ($t = 1, 2, \cdots, n$) is the data point at time t, and n should be equal to or greater than 4.

In order to reveal the objective law of systems, the grey system theory usually adopts a unique data pre-processing strategy before a grey model is to be built. It utilizes AGO to accumulate the original time-series $X^{(0)}$ and then obtain a new time-series $X^{(r)}$, that is

$$X^{(r)} = \left\{ x^{(r)}(1), x^{(r)}(2), \cdots, x^{(r)}(n) \right\}, \qquad (2)$$

where $x^{(r)}(k) = \sum_{t=1}^{k} x^{(r-1)}(t)$, and r represents r-order AGO (r-AGO).

Now the time-series $X^{(r)}$ satisfies the following one-order linear differential equation.

$$\frac{dx^{(r)}}{dt} + ax^{(r)} = b, \tag{3}$$

where a and b are called the development coefficient and grey input, respectively.

Using discrete one-order linear difference Equation 3, one can get a matrix as

$$\begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}[x^{(r)}(1) + x^{(r)}(2)] & 1 \\ -\frac{1}{2}[x^{(r)}(2) + x^{(r)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(r)}(n-1) + x^{(r)}(n)] & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}. \tag{4}$$

Let

$$Y = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, \quad A = \begin{bmatrix} -\frac{1}{2}[x^{(r)}(1) + x^{(r)}(2)] & 1 \\ -\frac{1}{2}[x^{(r)}(2) + x^{(r)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(r)}(n-1) + x^{(r)}(n)] & 1 \end{bmatrix}. \quad U = \begin{bmatrix} a \\ b \end{bmatrix},$$

Furthermore, this matrix can be expressed as

$$Y = AU. \tag{5}$$

In the Equation 5, Y and A can be derived from the original data, but U has to be estimated. One usually employs the LS solution to obtain LS approximation, and therefore the Equation 5 can be expressed as

$$Y = A\hat{U} + e, \tag{6}$$

where e is an error item. U can be estimated using matrix deviation formula.

$$\hat{U} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (A^{\mathrm{T}}A)^{-1}(A^{\mathrm{T}}Y). \tag{7}$$

Consequently, $\hat{a}$ and $\hat{b}$ are got. Furthermore, one can find the solution to Equation 3.

$$x^{(r)}(t) = \left[x^{(r)}(1) - \frac{\hat{b}}{\hat{a}} \right] e^{-\hat{a}t} + \frac{\hat{b}}{\hat{a}}. \tag{8}$$

Since $x^{(r)}(1) = x^{(0)}(1)$, the time response function of the GM(1, 1) model is also obtained.

$$\hat{x}^{(r)}(k) = \left[x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right] e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a}}, \quad k = 1, 2, \cdots. \tag{9}$$

The predicted value $\hat{x}^{(0)}(k)$ then can be yielded from the following r-order IAGO (r-IAGO).

$$\begin{cases} \hat{x}^{(0)}(1) = \hat{x}^{(1)}(1) = \cdots = \hat{x}^{(r)}(1) = x^{(0)}(1) \\ \hat{x}^{(r-1)}(k+1) = \hat{x}^{(r)}(k+1) - \hat{x}^{(r)}(k) \end{cases}, \quad k = 1, 2, \cdots. \tag{10}$$

2.2. BUILDING OF THE UT1-UTC PREDICTION MODEL

In this contribution, UT1-UTC time-series with daily sampling interval from the IERS EOP 05 C04 series are served as data base. Leap seconds are first removed from original UT1-UTC data to get UT1-TAI. The effects of the solid Earth tides with periods from five days up to 18.6 years as well as the diurnal and semi-diurnal ocean tides are then also removed using the tidal models recommended in the IERS Conventions 2010 (Petit and Luzum, 2010) to obtain UT1R-TAI. Next, UT1R-TAI (see Figure 1) data are forecasted by the grey prediction technique, which adopts the one-order AGO (1-AGO) strategy in this work, i.e., $r = 1$. At last, in order to obtain UT1-UTC forecasts, the UT1-UTC predictions must be corrected for leap seconds together with tide model. However, leap second adjustments and tidal models do not contribute to UT1-UTC prediction error. Figure 1 shows the algorithm for UT1-UTC data processing and forecasting using the GM(1, 1) model.
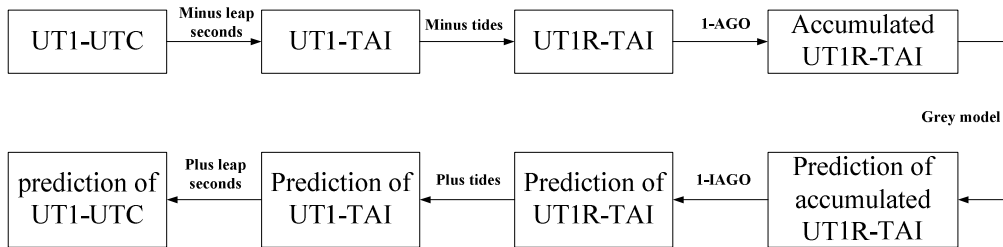

Fig. 1. Algorithm of UT1-UTC predictions by the GM(1, 1) model.

Application of the GM(1, 1) model requires lots of complex operations which are difficult to analytically calculate. Nevertheless, the GM(1, 1) model is very easy to use in the MATLAB platform. Since the prediction performance of the GM(1, 1) model is related to the sample number n, how to select the optimal sample number in the GM(1, 1) model needs to be carefully taken into account. In this work, the optimal sample number is determined by the fitting error criterion that corresponds to the minimum mean relative error (MRE), which is defined by

$$MRE_n = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{x^{(0)}(t) - \hat{x}^{(0)}(t)}{x^{(0)}(t)} \right|. \tag{11}$$

2.3. PREDICTION ERROR ESTIMATES

Out of many error statistic criteria, we selected the root mean square error (RMSE) and mean absolute error (MAE) as the statistics to evaluate the prediction accuracy of the proposed method. The two statistics are counted by

$$\text{RMS}_p = \sqrt{\frac{1}{l}\sum_{i=1}^{l}(O_d^i - F_d^i)^2} \,, \tag{12}$$

and

$$\text{MAE}_p = \frac{1}{l}\sum_{i=1}^{l}\left|O_d^i - F_d^i\right| , \tag{13}$$

respectively. Herein O denotes the observed UT1-UTC from the IERS EOP 05 C04 series, F represents the forecasted UT1-UTC obtained by the prediction model, and l is the number of predictions made for the particular prediction day.

3. PREDICTION RESULTS AND COMPARISON WITH OTHER METHODS

Daily UT1-UTC values of the IERS EOP 05 C04 series from 1 January 1994 to 10 December 1999 are used for building and validation of the GM(1, 1) model. The whole dataset are split up into two parts in such a way that the time-series from 1 January 1994 to 31 December 1997 are used to build the GM(1, 1) model with optimal sample number and the remaining part between 1 January 1998 and 10 December 1999 for evaluation of the established model. A comparison with the LS+AR and LS+ANN approaches which had been realized by Xu and Zhou (2015) and Schuh et al. (2002) respectively is given in Figure 2 and Table 1, where the RMS error for the prediction intervals of $1\sim10$ days is summarized, and 700 predictions starting at different days have been made for each prediction day to compute the RMS error, i.e. $l = 700$. In addition, the estimated value of sample number n of the GM(1, 1) model is shown in Figure 3. As can be seen in Figure 2 and Table 1, the GM(1, 1) model is able to provide predictions which are equal to or even better than those obtained by the other two methods as far as the prediction intervals of $1\sim4$ days is concerned. After the 5th day, the prediction accuracy of the GM(1, 1) model is slighter worse than that of the LS+ANN approach, but the comparison reveals good agreement of the predictions of the GM(1, 1) model with those of the LS+AR technology. Roughly speaking, therefore, the presented method is comparable with the other two techniques. For the optimum sample number of the GM(1, 1) model, from Figure 3 it can be seen that 4 observations are sufficient for establishing the optimized GM(1, 1) model in most cases, although the estimated n is not constant, indicating that it is associated with starting prediction epochs more or less. In the given case of a small size dataset, consequently, the GM(1, 1)-based UT1-UTC prediction method has remarkable advantages over the LS+AR and LS+ANN techniques, that require much longer time span data to make a good forecast. On the other hand, the GM(1, 1) model is rather simple to use in comparison with the LS+AR model and especially the LS+ANN technique.
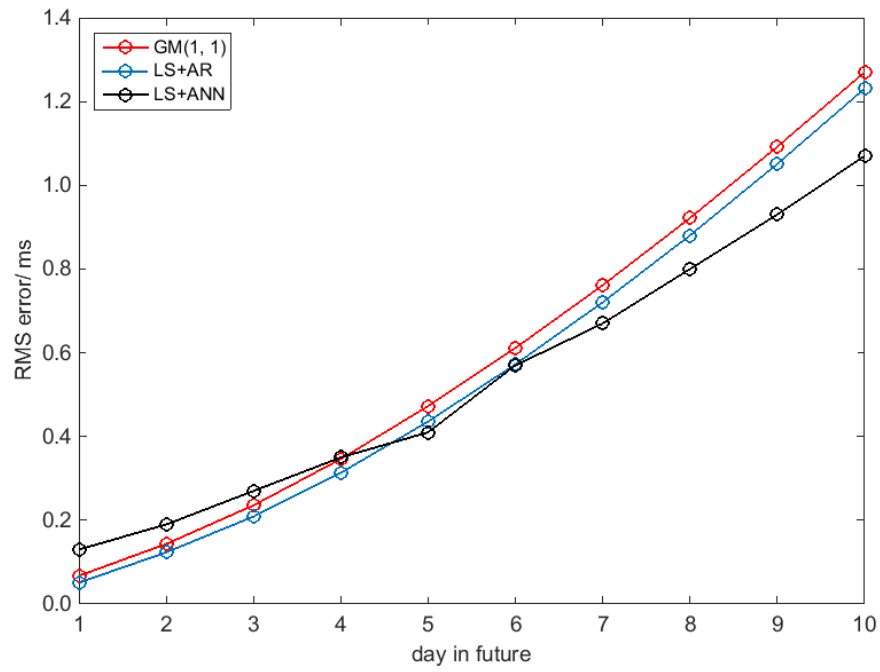
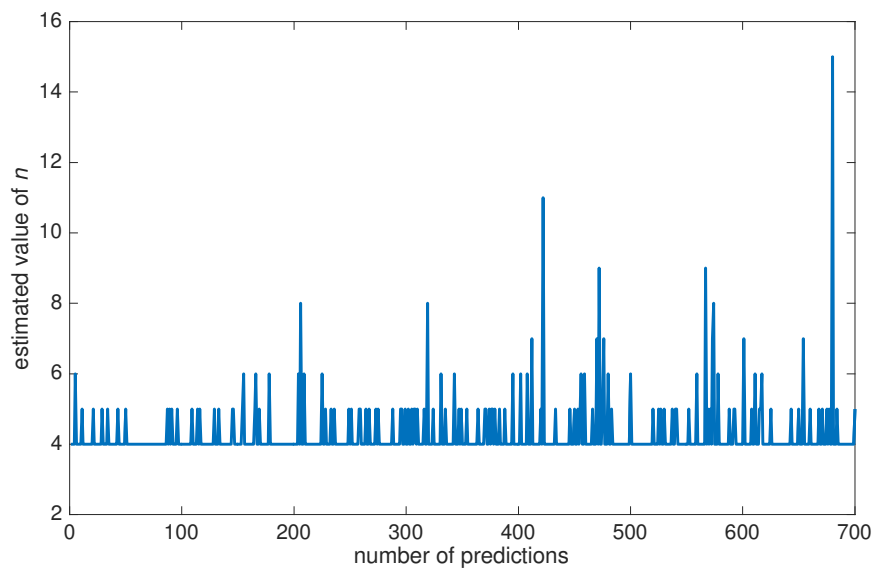**Fig. 2.** Comparison of RMS prediction error of the LS+AR, LS+ANN and GM(1, 1) models

Fig. 3. Estimated value of n using the MRE criterion

Table 1. Comparison of RMS prediction error of the LS+AR, LS+ANN and GM(1, 1) models (in units of ms).

Prediction day	GM(1, 1)	LS+AR	LS+ANN
1	0.07	0.05	0.13
2	0.14	0.12	0.19
3	0.24	0.21	0.27
4	0.35	0.31	0.35
5	0.47	0.44	0.41
6	0.61	0.57	0.57
7	0.76	0.72	0.67
8	0.92	0.88	0.80
9	1.09	1.05	0.93
10	1.27	1.23	1.07

In order to furthermore assess the performance of the GM(1, 1)-based prediction model, the results of the developed model is also compared with those of the EOP PCC lasting from 1 October 2005 to 28 February 2008, where prediction period and validation scheme had been clearly specified in advance. Using the well-defined rules a comparison with other prediction strategies and techniques participating in the EOP PCC is shown in Figure 4, where the prediction period is the same, the MAE is computed using the Equation 13, and the statistics are referred to the IERS EOP 05 C04 series.

What can be said with the information available from the comparison is that the accuracy of the predictions for 1 ∼ 5 days in the future by the GM(1, 1) model is equal to the prediction accuracy of the most accurate prediction techniques for UT1-UTC forecasts, namely Kalman filter (Gross et al., 1998) and adaptive transform from atmospheric angular momentum (AAM) to LODR (Gambis et al., 2011), respectively. Since the 6th day, the prediction error of the GM(1, 1) model gradually increases and is bigger than that of the two best techniques, but is noticeably lower than that of the other strategies yet.
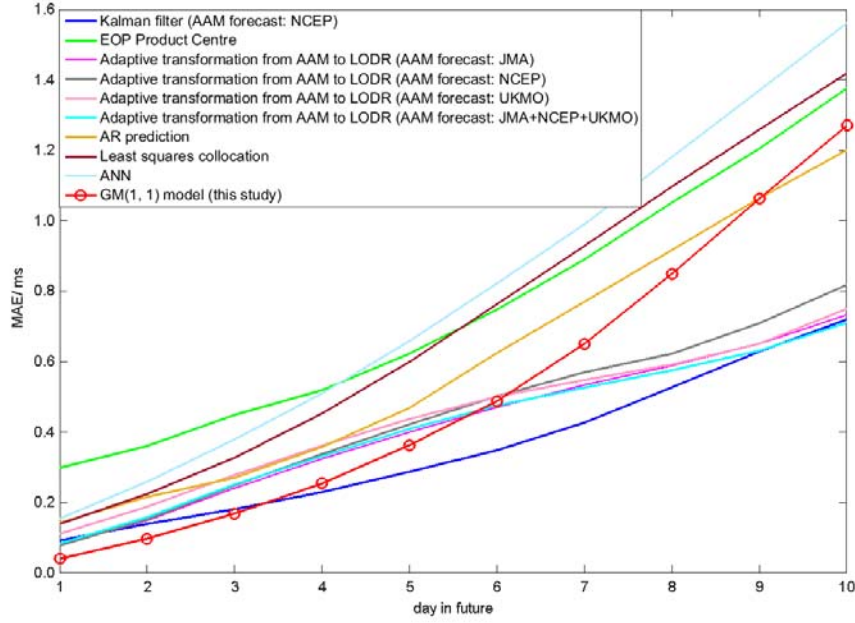
Fig. 4. Comparison of MAE prediction errors of different prediction strategies and techniques

4. CONCLUSIONS

The comparison with the existing prediction strategies and techniques has demonstrated the high effectiveness of the GM(1, 1)-based prediction model. Unlike traditional methods, e.g., LS+AR and LS+ANN, which require a lot of observations to make a good prediction, the developed method is able to yield an exact prediction with only a little data, i.e., the minimum data down to four observations. This can then be very valuable in the case of a small size dataset. Moreover, the forecasted UT1-UTC can be obtained without making any assumption about an original time-series, depicting high reliability of the method. Another advantage is that the presented method is quite simple to use when compared with other strategies and techniques, e.g., ANN.

From what has been discussed above, it is concluded that the proposed method is of high reliability, effectiveness and practicality. Since the variations in the Earth's rotation rate is partly caused by the exchange of angular momentum between the atmosphere and the Earth's crust, a further idea is to incorporate the AMM forecast data into a grey model so as to enhance the prediction quality. Generally speaking, UT1-UTC could be predicted by means of a first-order multivariate grey model (GM(1, 2) model) in combination of the AAM forecast data and UT1-UTC data.

28

REFERENCES

Akyilmaz, O. and Kutterer, H. (2004). Prediction of Earth Rotation Parameters by Fuzzy Inference Systems. *Journal of Geodesy*, **78**(1-2), 82-93.

Deng, J.L. (1986). Grey Forecasting and Decision. *Huazhong University of Science and Technology Press,* Wuhan, 97-134.

Gambis, D. and Luzum, B. (2011). Earth Rotation Monitoring, UT1 Determination and Prediction. *Metrologia*, **48**(4), 65-170.

Gambis, D., Salstein, D.A. and Lambert, L. (2011). Use of Atmospheric Angular Momentum Forecasts for UT1 Predictions: Analyses over CONT08. *Journal of Geodesy*, **85**(7), 435-441.

Gross, R.S., Marcus, S.L., Eubanks, T.M., Dickey, J.O. and Keppenne, C.L. (1996). Detection of an ENSO Signal in Seasonal Length-of-day Variations. *Geophysical Research Letters*, **23**(23), 3373-3376.

Gross, R.S., Eubanks, T.M., Steppe, J.A., Freedman, A.P., Dickey, J.O. and Runge,T.F. (1998). A Kalman-filter-based Approach to Combining Independent Earth-orientation Series. *Journal of Geodesy*, **72**(4), 215-235.

Guo, J.Y., Li, Y.B., Dai, C.L. and Shum, C.K. (2013). A Technique to Improve the Accuracy of Earth Orientation Prediction Algorithms Based on Least Squares Extrapolation. *Journal of Geodynamics*, **70**(10), 36-48.

Kalarus, M., Schuh, H., Kosek, W., Akyilmaz, O. and Bizouard, Ch. (2010). Achievements of the Earth Orientation Parameters Prediction Comparison Campaign. *Journal of Geodynamics*, **84**(10), 587-596.

Kosek, W., McCarthy, D.D. and Luzum, B.J. (1998). Possible Improvement of Earth Orientation Forecast Using Autocovariance Prediction Procedures. *Journal of Geodesy*, **72**(4), 189–199.

Lei, Y., Zhao, D.N. and Cai, H.B. (2015a). Prediction of Length-of-day Using Extreme Learning Machine. *Geodesy and Geodynamics*, **6**(2): 151-159.

Lei, Y., Guo, M., Cai, H.B., Hu, D.D. and Zhao, D.N. (2015b). Prediction of Length-of-day Using Gaussian Process Regression. The Journal of Navigation, **68**(3): 563-175.

Liao, D.C., Wang, Q.J., Zhou, Y.H., Liao, X.H. and Huang, C.L. (2012). Long-term Prediction of the Earth Orientation Parameters by the Artificial Neural Network Technique. *Journal of Geodynamics*, **62**: 87-92.

Malkin, Z and Skurikhina, E. (1996). On Prediction of EOP. *Communications of the Institute of Applied Astronomy RAS*, No.93.

Niedzielski, T. and Kosek, W. (2008). Prediction of UT1–UTC, LOD and AAM χ_3 by Combination of Least-squares and Multivariate Stochastic Methods. *Journal of Geodesy*, **82**(2), 83–92.

Petit, G. and Luzum, B. (2010). IERS Conventions (2010). *IERS Technical Note No. 36*, Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am Main, 123–131.

Schuh, H., Ulrich, M., Egger, D., Müller, J. and Schwegmann, W. (2002). Prediction of Earth Orientation Parameters by Artificial Neural Networks. *Journal of Geodesy*, **76**(5), 247-258.

Xu, X.Q. and Zhou, Y.H. (2015). EOP Prediction Using Least Square Fitting and Autoregressive Filter over Optimized Data Intervals. *Advances in Space Research*, in press.

Zhang, X.H., Wang, Q.J., Zhu, J.J. and Zhang, H. (2012). Application of General Regression Neural Network to the Prediction of LOD Change. *Chinese Astronomy and Astrophysics*, **36**(1), 86-96.