ARTIFICIAL SATELLITES, Vol. 49, No. 3 – 2014
DOI: 10.2478/arsa-2014-0013

RERS2013: A NEW HIGH-PRECISION RIGID EARTH ROTATION SERIES

V.V. Pashkevich
Central (Pulkovo) Astronomical Observatory of RAS
Pulkovskoe shosse, 65/1, 196140, St.Petersburg, Russia
e-mail: pashvladvit@yandex.ru

ABSTRACT. In the previous investigation (Pashkevich, 2013) the high-precision Rigid Earth Rotation Series (designated RERS2012) dynamically adequate to the JPL DE406/LE406 (Standish, 1998) ephemeris over 2000 and 6000 years were constructed. The main aim of present research is improvement of the Rigid Earth Rotation Series RERS2012 by using the JPL DE422/LE422 (Folkner, 2011) ephemeris, and as a result is produced construction of the new high-precision Rigid Earth Rotation Series dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 and 6000 years. The discrepancies in Euler angles between the high-precision numerical solutions and the semi-analytical solutions of the rigid Earth rotation problem are investigated by least squares and spectral analysis methods using the iterative algorithm (Pashkevich, 2013). In order to demonstrate the good convergence of this iterative algorithm are constructed additional solutions of the rigid Earth rotation dynamically adequate to the JPL DE422/LE422 over 2000 years by improvement solutions SMART97 (Bretagnon et al., 1998) and S9000 (Pashkevich and Eroshkin, 2005a). As the results of this investigation, the new improved high-precision Rigid Earth Rotation Series RERS2013 dynamically adequate to the DE422/LE422 ephemeris over 2000 and 6000 years have been constructed. The discrepancies in Euler angles between the numerical solution and RERS2013 do not surpass: 4 μas over 2000 years, 1 mas over 6000 years. The RERS2013 series is more accurate than the RERS2012 series, which is dynamically adequate to the DE406/LE406 ephemeris. The good convergence of the iterative algorithm of this study has been confirmed.

Keywords: the rigid Earth rotation, ephemeris, iterative algorithm, least-squares method, spectral analysis method.

1. INTRODUCTION

The present research is the continuation of Pashkevich (2013) investigation of the rigid Earth rotation at long time intervals. In the previous investigation (Pashkevich, 2013) the high-precision Rigid Earth Rotation Series (designated RERS2012A/B and RERS2012C/D) dynamically adequate to the JPL DE406/LE406 ephemeris over 2000 and 6000 years, respectively, were constructed. The discrepancies in Euler angles between the numerical solution and RERS2012 do not surpass: 10 μas over 2000 years, 2 mas over 6000 years. The purposes of these studies are the construction of the new high-precision Rigid Earth Rotation Series (RERS2013A /B /C and RERS2013D) dynamically adequate to the JPL DE422/LE422 (was reported by Folkner (2011)) over 2000 and 6000 years, respectively, the demonstration of the good convergence of the iterative algorithm of this research and the comparison of new solution RERS2013 with the previous Pashkevich (2013) solution RERS2012. The series indexes A, B, C and D refer to the improving initial semi-analytical solutions of the rigid Earth rotation SMART97 (Bretagnon et al., 1998), S9000 (Pashkevich and Eroshkin, 2005a), RERS2012B and RERS2012D (Pashkevich, 2013), respectively. The dynamics of the rotational motion of the rigid Earth is studied numerically by using Rodrigues-Hamilton parameters over 2000 and 6000 years. The numerical solution of the problem is obtained by solving the Lagrange differential equations of the second kind for the rigid Earth rotation with respect to the fixed ecliptic and equinox of epoch J2000 (Pashkevich, 2013). The initial epoch of the numerical integration is 1 January 2000 (JD=2451545.0). The orbital motions of the disturbing celestial bodies are defined by the DE422/LE422 ephemeris. This investigation is carried out for the relativistic (Kinematical) case, in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account. The mathematical model of the problem is described in detail by Pashkevich (2013).

2. ITERATIVE ALGORITHM OF THE PROBLEM

The results of the numerical solutions of the problem in Euler angles are compared with the semi-analytical solutions of the rigid Earth rotation. The residuals of these comparisons are studied by means the iterative algorithm:

1. Numerical solution of the rigid Earth rotation is implemented with the quadruple precision of calculations. The initial conditions are computed by the semi-analytical solution of the rigid Earth rotation (SMART97 or S9000 or RERS2012). Discrepancies between the numerical solution and the semi-analytical solution are obtained in Euler angles over all investigation time intervals with one-day spacing (presented in Figures 1, 5, 9 and 13). The expressions for these discrepancies are as follows

$$\left.\begin{aligned}\Delta\psi &= \sum_{k=0}^{6}\psi_k\ t^k + \sum_j\sum_{k=0}^{4}[\psi_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\psi_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k\\ \Delta\theta &= \sum_{k=0}^{6}\theta_k\ t^k + \sum_j\sum_{k=0}^{4}[\theta_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\theta_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k\\ \Delta\varphi &= \sum_{k=0}^{6}\varphi_k\ t^k + \sum_j\sum_{k=0}^{4}[\varphi_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\varphi_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k\end{aligned}\right\}, \qquad (1)$$

   where ψ is the longitude of the ascending node of the Earth's dynamical equator on the fixed ecliptic J2000; θ is the angle of the inclination of the Earth's dynamical equator to the fixed ecliptic J2000; φ is the proper rotation angle of the Earth between the ascending node of the Earth's dynamical equator and the principal axis of the minimum moment of inertia; ν_{j0}, ν_{j1} are the phases and the frequencies of the corresponding semi-analytical solutions, respectively; $j = 1,\dots, 4113$; t is the time in the Julian days; ψ_k, θ_k, φ_k are the coefficients of the secular terms; ψ_{sjk}, θ_{sjk}, φ_{sjk}, ψ_{cjk}, θ_{cjk}, φ_{cjk} are the coefficients of the periodic and Poisson terms.

2. Investigation of the discrepancies is carried out by the least squares method and by the spectral analysis method (described by Pashkevich and Eroshkin, (2005b)). The set of the frequencies of the semi-analytical solution is used without change. Only the coefficients of the periodical terms and the coefficients of the Poisson terms are improved. The secular, periodic and Poisson terms representing the new high-

precision rigid Earth rotation series RERS2013X_i (where i is the number of iteration X=A, B, C, D is the index of the series) are determined:

$$\left.\begin{aligned} \psi_{\mathrm{RERS2013X\_i}} &= \Delta\psi_{\mathrm{i-1}} + \psi_{\mathrm{RERS2013X\_i-1}} \\ \theta_{\mathrm{RERS2013X\_i}} &= \Delta\theta_{\mathrm{i-1}} + \theta_{\mathrm{RERS2013X\_i-1}} \\ \varphi_{\mathrm{RERS2013X\_i}} &= \Delta\varphi_{\mathrm{i-1}} + \varphi_{\mathrm{RERS2013X\_i-1}} \end{aligned}\right\}, \tag{2}$$

where $\psi_{\mathrm{RERS2013X\_0}} = \psi_{\mathrm{Y}}$, $\theta_{\mathrm{RERS2013X\_0}} = \theta_{\mathrm{Y}}$, $\varphi_{\mathrm{RERS2013X\_0}} = \varphi_{\mathrm{Y}}$, Y=SMART97, S9000, RERS2012 is the index of the initial series. The series with index A, B and C are constructed over 2000 years from series SMART97, S9000 and RERS2012B, respectively. The series with index D is constructed from series RERS2012D over 6000 years.

3. Numerical solution of the rigid Earth rotation is constructed anew with the new initial conditions, which are calculated by RERS2013X_i.
4. Steps 2 and 3 are repeated till the assumed convergence level has been achieved.

3. RESULTS

At first this investigation is carried out on 2000 years with used the semi-analytical solution SMART97, which is dynamically adequate to the DE403/LE403 ephemeris (Standish et al., 1995).

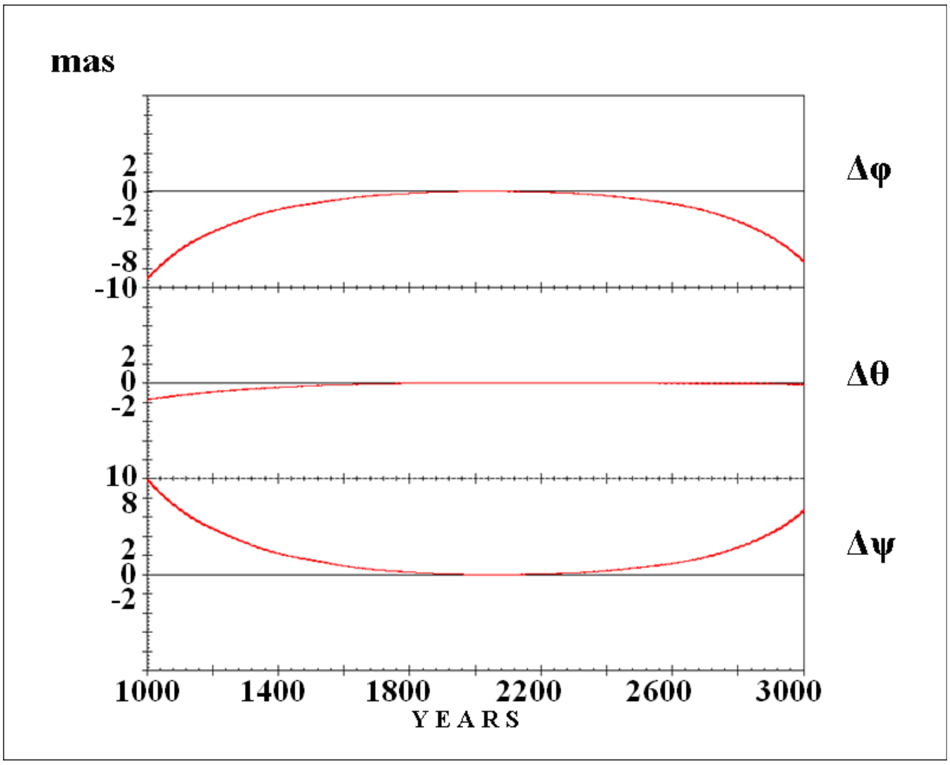

Fig. 1. Discrepancies between the numerical and SMART97 semi-analytical solutions of the Earth rotation (dynamically adequate to the DE403/LE403 ephemeris) over 2000 years

Figure 1 presents the discrepancies between the numerical and SMART97. The residuals have the secular trends, which do not exceed 2 mas over 2000 years for θ and 10 mas over 2000 years for ψ and for φ.

After application of the third iteration of the iterative algorithm, the convergence level was achieved and the process of the iterative algorithm was finished at this step. As a result, the Rigid Earth Rotation Series (RERS2013A) was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 2000 years. The discrepancies in Euler angles between the new numerical solutions and the semi-analytical solutions of RERS2013A do not surpass 4 μas over 2000 year time interval (presented in Figure 2).

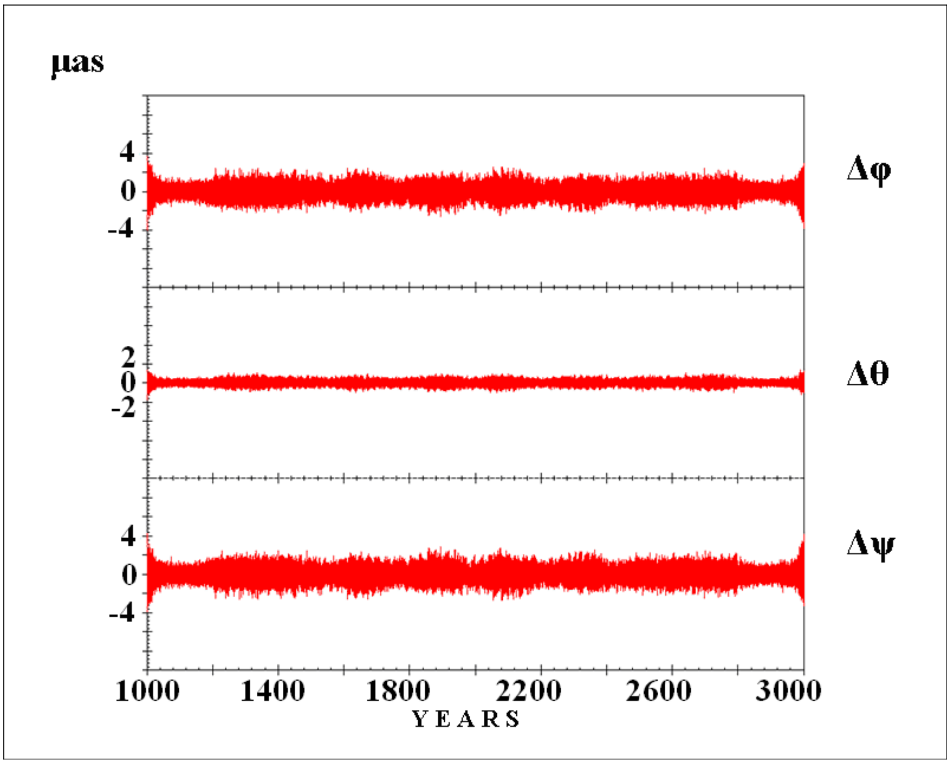

Fig. 2. Discrepancies between the numerical and RERS2013A semi-analytical solutions of the Earth rotation (dynamically adequate to the DE422/LE422 ephemeris) over 2000 years after applied 3rd iterations of the iterative algorithm

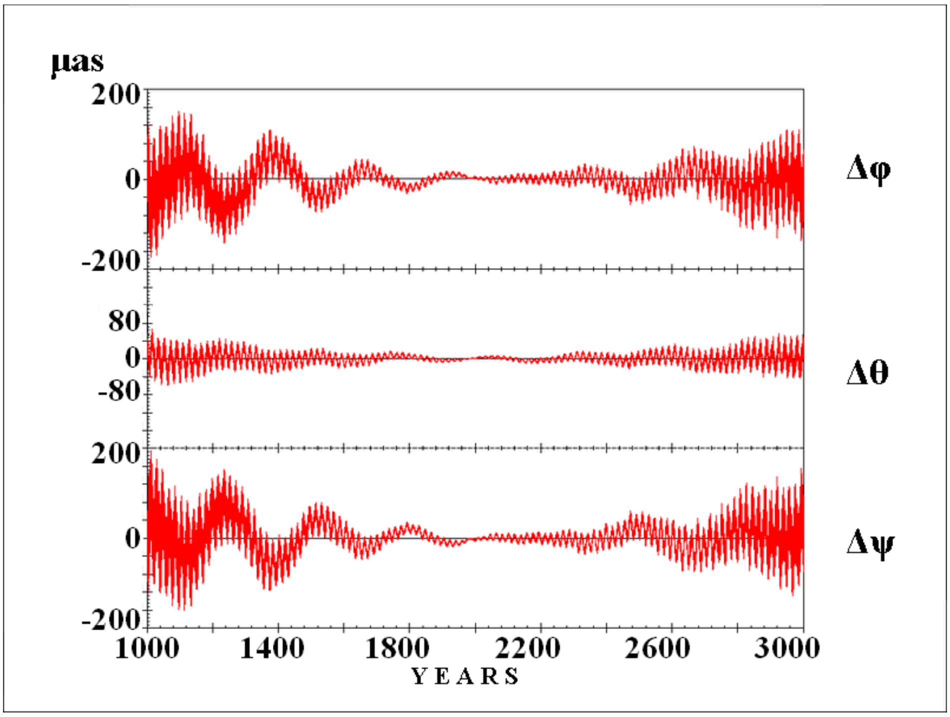

Fig. 3. Discrepancies between the semi-analytical solutions of the Earth rotation RERS2013A (dynamically adequate to the DE422/LE422 ephemeris) and SMART97 (dynamically adequate to the DE403/LE403 ephemeris) over 2000 years after removal of the secular trends

The comparison between the semi-analytical solutions of the Earth rotation SMART97 (dynamically adequate to the DE403/LE403 ephemeris) and RERS2013A (dynamically adequate to the DE422/LE422 ephemeris) is given in Figure 3. The discrepancies in Euler angles between RERS2013A and SMART97 after removal of the secular trends do not surpass 200 µas over 2000 year time interval.

Figure 4 demonstrates the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013A and SMART97. The power spectra for angles $\Delta\psi$ and $\Delta\varphi$ are very similar. Table 1 presents the main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013A and SMART97.

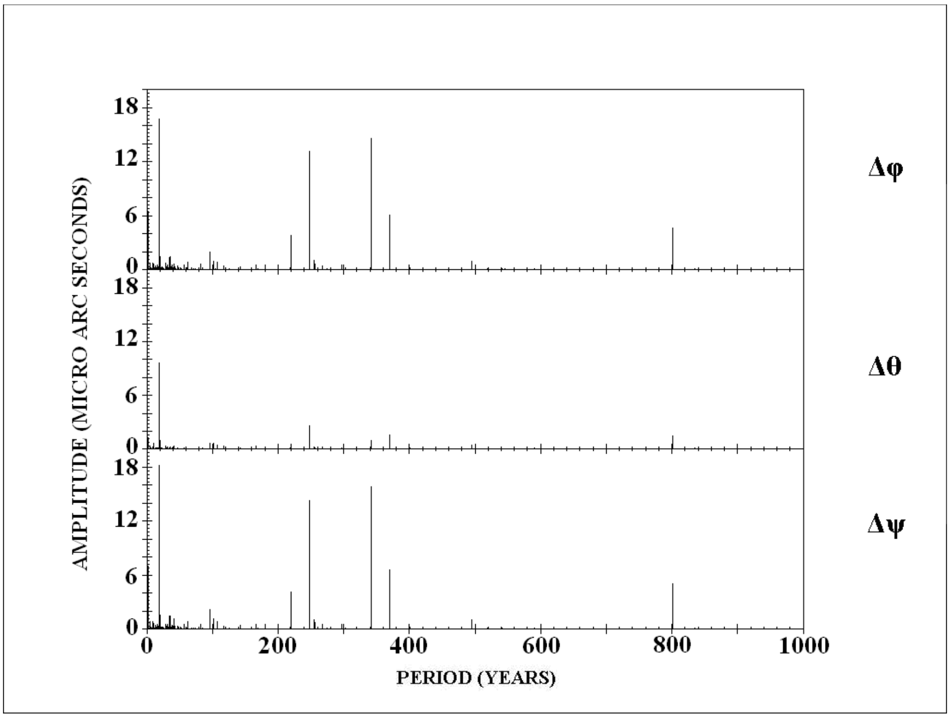

Fig. 4. Spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013A and SMART97

Table 1. The main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013A and SMART97

No	Period (years)	Planetary mean longitudes								Delauney arguments				Amplitude (μas)		
		λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	D	F	l	φ	Δψ	Δθ	Δφ
1	18.37863752	0	5	-7	-4	0	0	0	0	-1	1	0	0	18.17	9.64	16.72
2	18.85333253	0	5	-5	-4	0	0	0	0	1	-1	0	0		4.48	
	341.38619905	0	0	11	-21	2	0	0	0	0	0	0	0	15.81		14.53
3	247.46285681	0	8	-5	-16	4	5	0	0	0	0	0	0	14.31	2.65	13.14
4	18.19022804	0	0	1	0	2	-5	0	0	1	-1	0	0		2.43	
	18.85333253	0	5	-5	-4	0	0	0	0	1	-1	0	0	8.17		7.49
5	0.03739946	0	0	2	0	-2	5	0	0	2	0	0	0		1.61	
	0.99939197	0	0	1	0	2	-6	3	0	0	0	0	0	7.00		6.44
6	369.41503599	0	3	-3	-4	3	0	0	0	0	0	0	0	6.59	1.53	6.02
7	800.92081471	0	0	0	0	2	-5	0	0	0	0	0	0	5.05	1.45	4.56

Here and further λ_1, λ_2, λ_3, λ_4, λ_5, λ_6, λ_7, λ_8 are the mean longitudes of the planets (Mercury, Venus, the Earth, Mars, Jupiter, Saturn, Uranus and Neptune, respectively); D is the difference between the mean longitudes of the Moon and the Sun; F is the mean argument of the Moon's latitude; l is the mean anomaly of the Moon. The harmonics in all Tables are arranged in decreasing order from the harmonic with the maximum amplitude of the power spectrum.

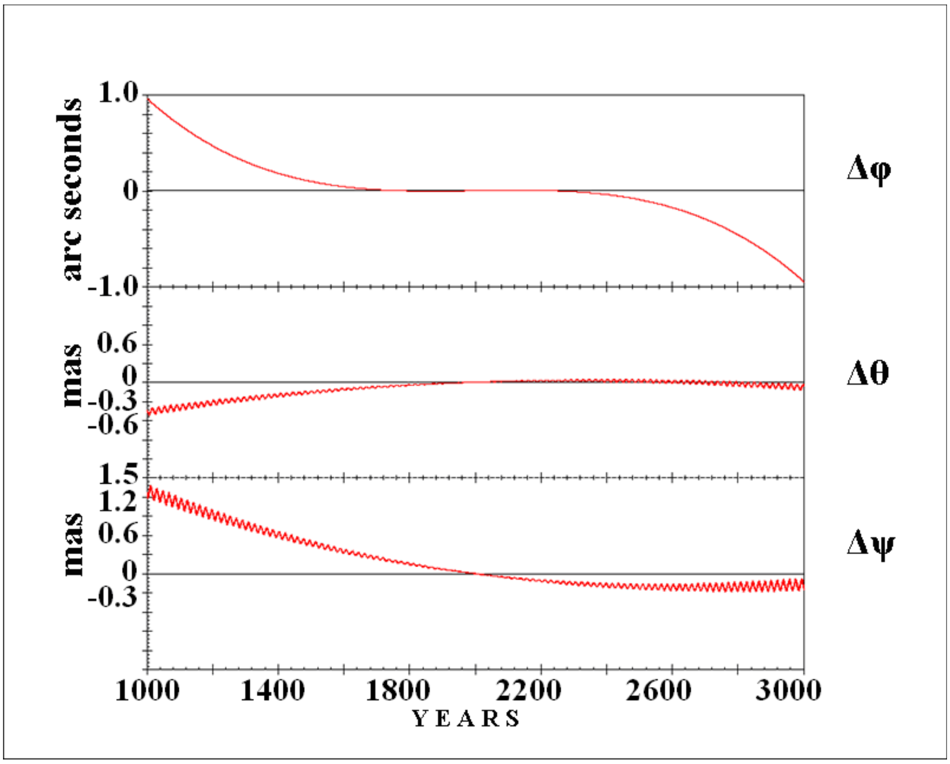

Fig. 5. Discrepancies between the numerical and S9000 semi-analytical solutions of the Earth rotation (dynamically adequate to the DE404/LE404 ephemeris) over 2000 years

This investigation is continued on same time interval with used the semi-analytical solution S9000, which is dynamically adequate to the DE404/LE404 ephemeris (Standish et al., 1995).

Figure 5 presents the discrepancies between the numerical and S9000. The secular trend of the residuals does not surpass 1.5 mas over 2000 years for ψ, 0.6 mas over 2000 years for θ and 1 arc second over 2000 years for φ.

The process of the iterative algorithm was finished after application of the third iteration of the iterative algorithm, when the convergence level has been achieved. As a result, the Rigid Earth Rotation Series (RERS2013B) was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 2000 years. The discrepancies in Euler angles between the new numerical solutions and the semi-analytical solutions of RERS2013B do not surpass 4 μas over 2000 year time interval (presented in Figure 6).

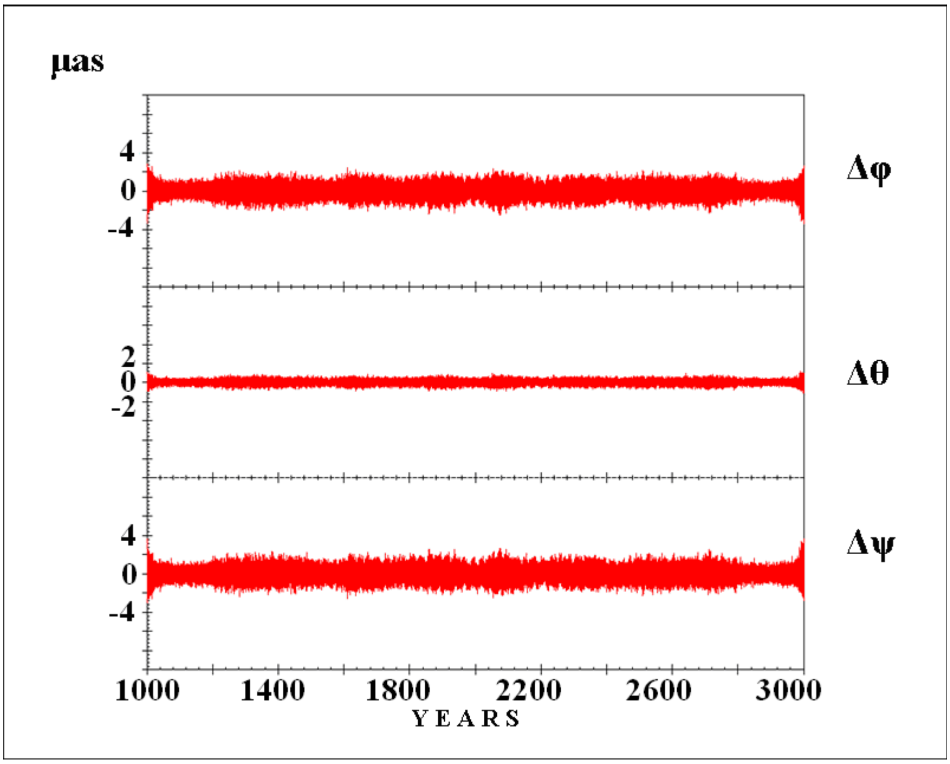

Fig. 6. Discrepancies between the numerical and RERS2013B semi-analytical solutions of the Earth rotation (dynamically adequate to the DE422/LE422 ephemeris) over 2000 years after applied 3rd iterations of the iterative algorithm

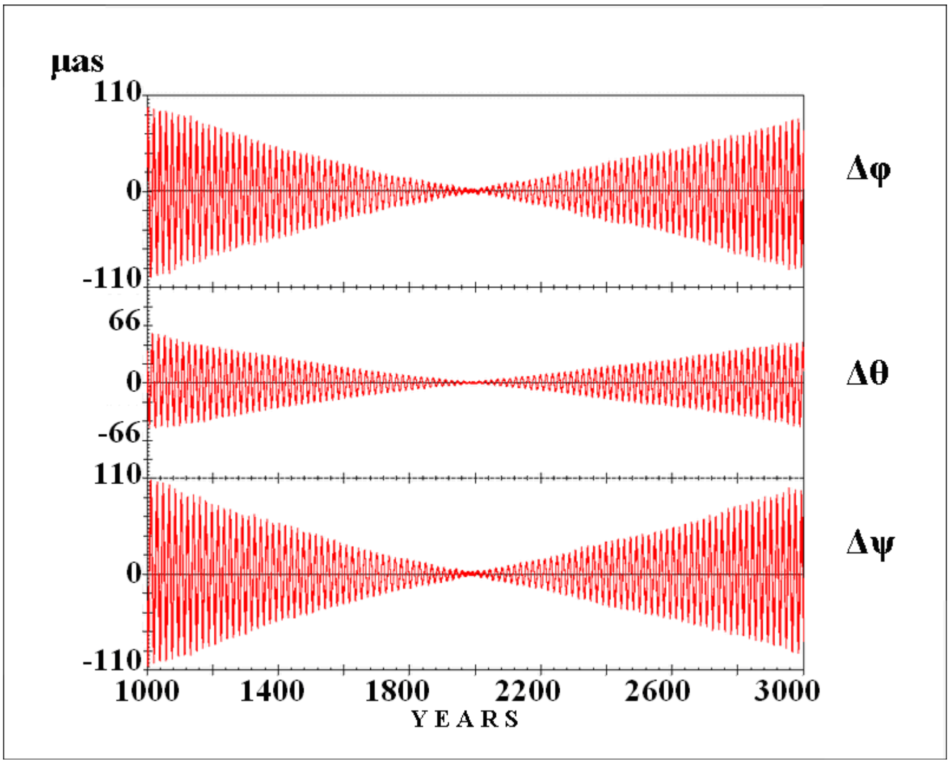

Fig. 7. Discrepancies between the semi-analytical solutions of the Earth rotation RERS2013B (dynamically adequate to the DE422/LE422 ephemeris) and S9000 (dynamically adequate to the DE404/LE404 ephemeris) over 2000 years after removal of the secular trends

The comparison between the semi-analytical solutions of the Earth rotation S9000 (dynamically adequate to the DE404/LE404 ephemeris) and RERS2013B (dynamically adequate to the DE422/LE422 ephemeris) is given in Figure 7. The discrepancies in Euler angles between RERS2013B and S9000 after removal of the secular trends do not surpass 110 µas over 2000 year time interval.

Figure 8 demonstrates the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013B and S9000. Table 2 presents the main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013B and S9000. The first five frequencies of the main harmonics contributions of the power spectrum are same for all Euler angles (presented in Figure 8 and Table 2).

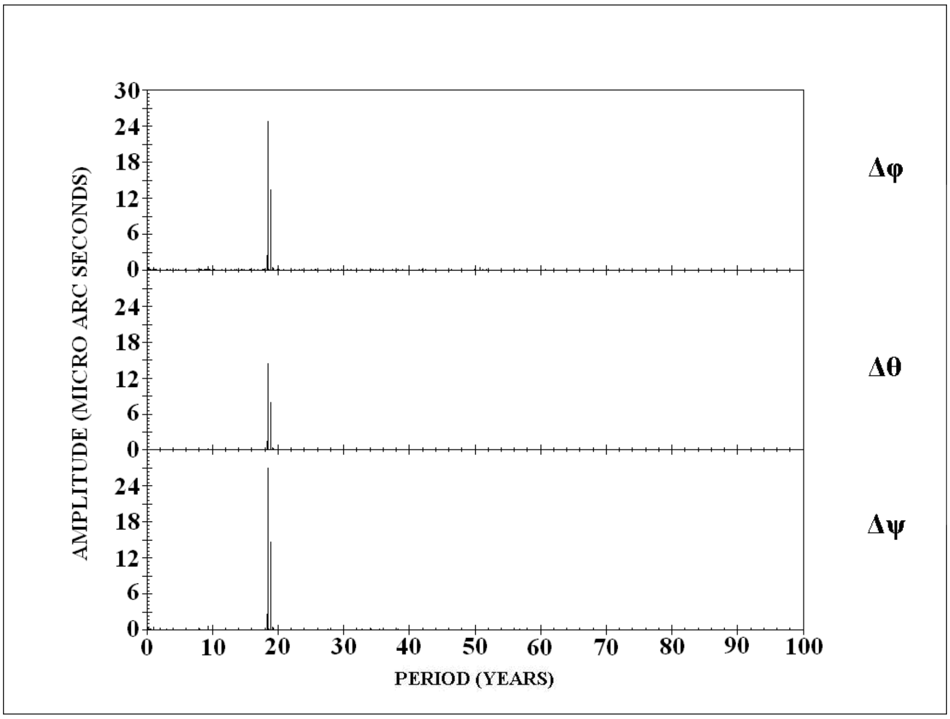

Fig. 8. Spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013B and S9000

Table 2. The main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013B and S9000

No	Period (years)	Planetary mean longitudes								Delauney arguments			φ	Amplitude (µas)		
		λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	D	F	l		$\Delta\psi$	$\Delta\theta$	$\Delta\varphi$
1	18.37863752	0	5	-7	-4	0	0	0	0	-1	1	0	0	26.94	14.41	24.71
2	18.85333253	0	5	-5	-4	0	0	0	0	1	-1	0	0	14.64	7.87	13.43
3	18.39078811	0	0	3	-8	1	5	0	0	-1	1	0	0	4.15	2.23	3.81
4	0.03740295	0	0	2	0	2	-5	0	0	2	0	0	0	3.58	1.53	3.28
5	0.03739946	0	0	2	0	-2	5	0	0	2	0	0	0	3.42	1.47	3.14
6	0.03740358	0	0	5	-7	0	0	0	0	1	-1	2	0	2.66		2.44
	18.19022804	0	0	1	0	2	-5	0	0	1	-1	0	0		1.38	
7	18.19022804	0	0	1	0	2	-5	0	0	1	-1	0	0	2.58		2.34
	18.84056327	0	0	5	-8	1	5	0	0	1	-1	0	0		1.22	
8	18.84056327	0	0	5	-8	1	5	0	0	1	-1	0	0	2.25		2.07
	0.03740358	0	0	5	-7	0	0	0	0	1	-1	2	0		1.13	

This investigation is continued on same time interval with used the semi-analytical solution RERS2012B, which is dynamically adequate to the DE406/LE406 ephemeris (Standish, 1998).

Figure 9 presents the discrepancies between the numerical and RERS2012B. The secular trend of the residuals does not surpass 600 μas over 2000 years for ψ and 160 mas over 2000 years for φ. The behavior of Δθ residuals are only periodic character and do not exceed 80 μas over 2000 years.

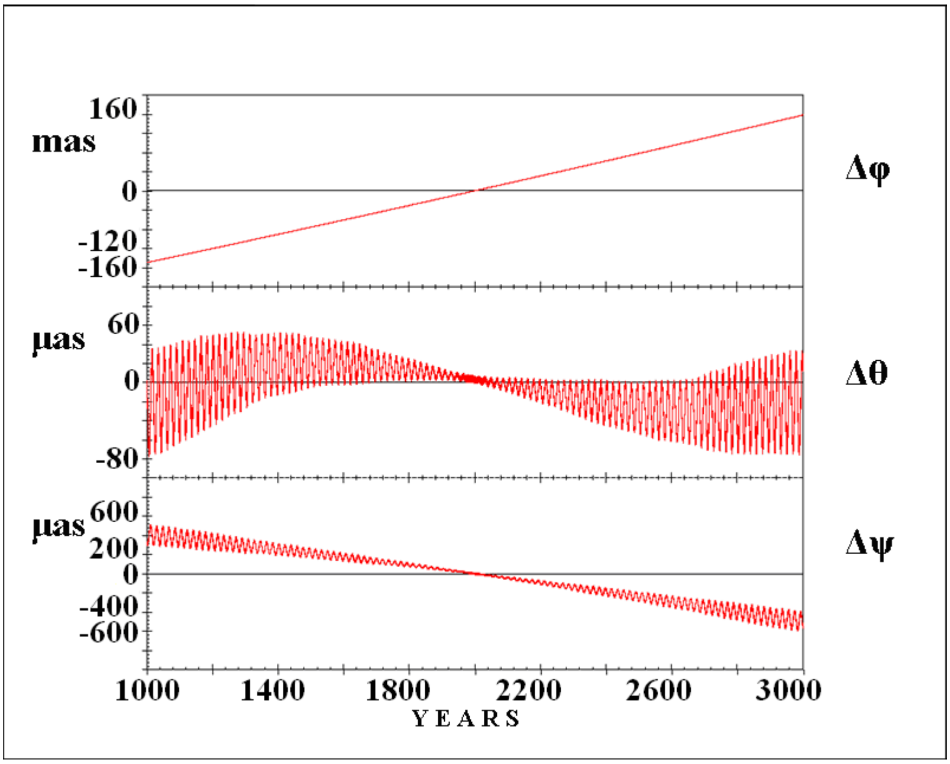

Fig. 9. Discrepancies between the numerical and RERS2012B semi-analytical solutions of the Earth rotation (dynamically adequate to the DE406/LE406 ephemeris) over 2000 years

The convergence level was achieved after application of the third iteration of the iterative algorithm. Thus, the process of the iterative algorithm was finished at this step. As a result, the Rigid Earth Rotation Series (RERS2013C) was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 2000 years. The discrepancies in Euler angles between the new numerical solutions and the semi-analytical solutions of RERS2013C do not surpass 4 μas over 2000 year time interval (presented in Figure 10).

The comparison between the semi-analytical solutions of the Earth rotation RERS2012B (dynamically adequate to the DE406/LE406 ephemeris) and RERS2013C (dynamically adequate to the DE422/LE422 ephemeris) is given in Figure 11. The discrepancies in Euler angles between RERS2013C and RERS2012B after removal of the secular trends do not surpass 110 μas over 2000 year time interval.

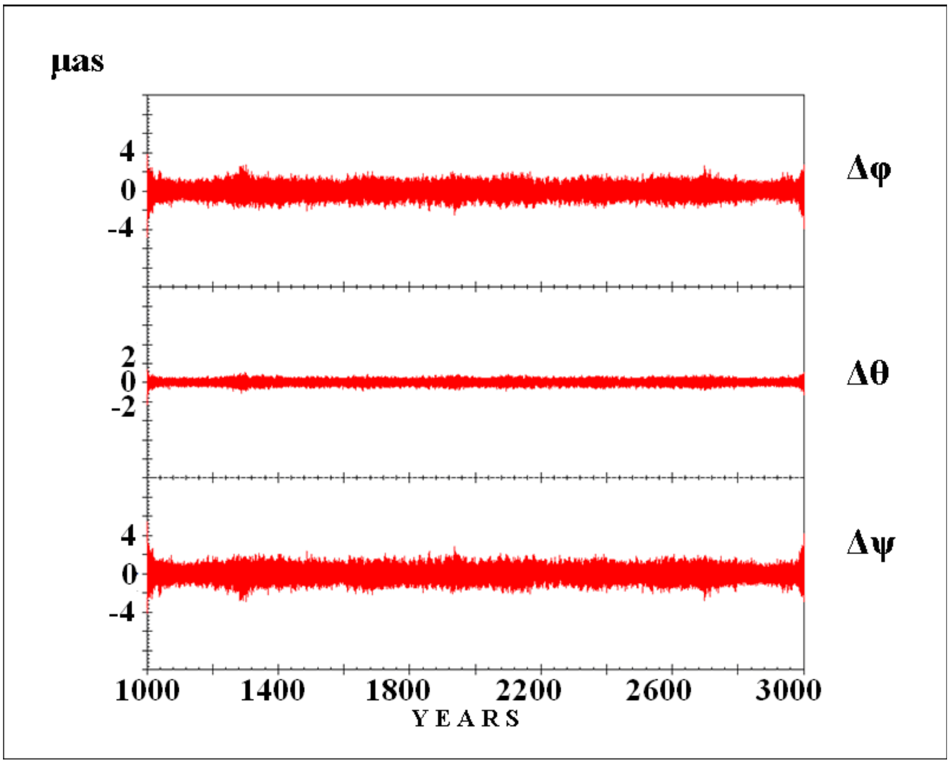

Fig. 10. Discrepancies between the numerical and RERS2013C semi-analytical solutions of the Earth rotation (dynamically adequate to the DE422/LE422 ephemeris) over 2000 years after applied 3rd iterations of the iterative algorithm

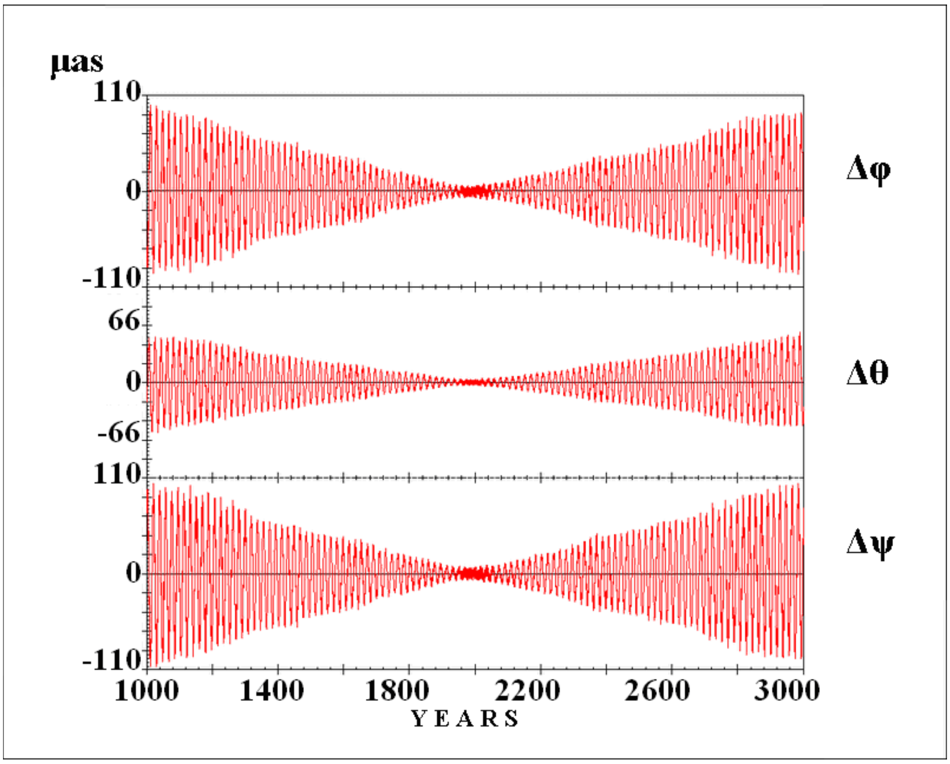

Fig. 11. Discrepancies between the semi-analytical solutions of the Earth rotation RERS2013C (dynamically adequate to the DE422/LE422 ephemeris) and RERS2012B (dynamically adequate to the DE406/LE406 ephemeris) over 2000 after removal of the secular trends

Figure 12 demonstrates the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013C and RERS2012B. The power spectra presented in Figures 8 and 12 are very similar. Table 3 presents the main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013C and RERS2012B. The first three frequencies of the main harmonics contributions of the power spectrum are same for all Euler angles and same that presented in Table 2.

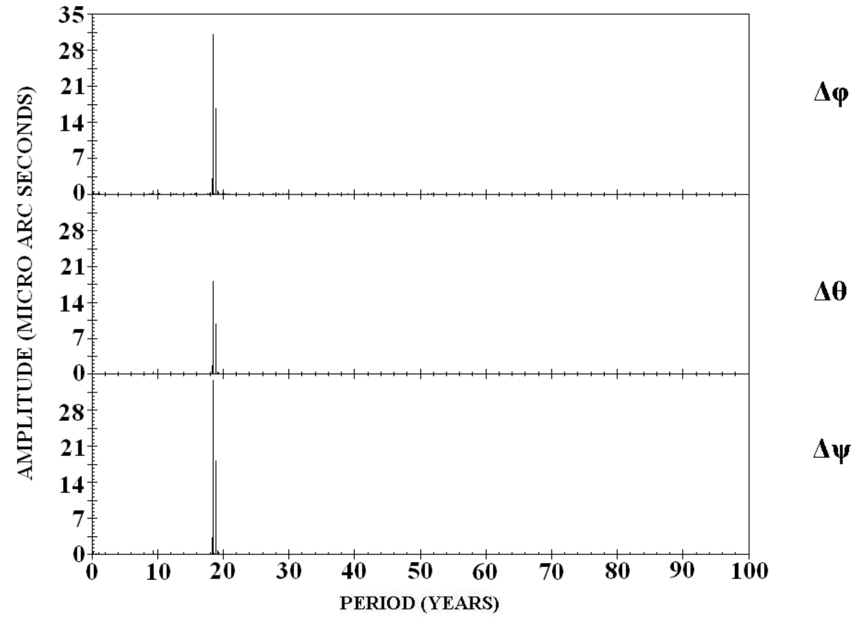

Fig. 12. Spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013C and RERS2012B

Table 3. The main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013C and RERS2012B.

No	Period (years)	Planetary mean longitudes								Delauney arguments			φ	Amplitude (μas)		
		λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	D	F	l		Δψ	Δθ	Δφ
1	18.37863752	0	5	-7	-4	0	0	0	0	-1	1	0	0	33.81	18.00	31.02
2	18.85333253	0	5	-5	-4	0	0	0	0	1	-1	0	0	18.21	9.73	16.70
3	18.39078811	0	0	3	-8	1	5	0	0	-1	1	0	0	5.23	2.79	4.80
4	0.03740037	0	0	2	-8	3	0	0	0	-2	0	0	0	3.60		3.30
	18.19022804	0	0	1	0	2	-5	0	0	1	-1	0	0		1.71	
5	18.19022804	0	0	1	0	2	-5	0	0	1	-1	0	0	3.19		2.91
	0.03740037	0	0	2	-8	3	0	0	0	-2	0	0	0		1.54	
6	18.84056327	0	0	5	-8	1	5	0	0	1	-1	0	0	2.84	1.53	2.60
7	18.40755089	0	0	5	-8	3	0	0	0	1	-1	0	0	1.48	0.80	1.37

The residuals between the new numerical solutions and the semi-analytical solutions of RERS2013A/B/C are very close to each other (presented in Figures 2, 6 and 10, respectively) by this confirms the good convergence of the iterative algorithm. The last previous version of the extended integration time ephemeris is the DE406/LE406 ephemeris. Thus, this investigation is finished at 6000 years time interval only with used semi-analytical solution

RERS2012D, which is dynamically adequate to the DE406/LE406 ephemeris. In Figure 13 the discrepancies are depicted between the numerical and RERS2012D over 6000 years. The secular trend in all Euler angles does not surpass 6 arc seconds over 6000 years.

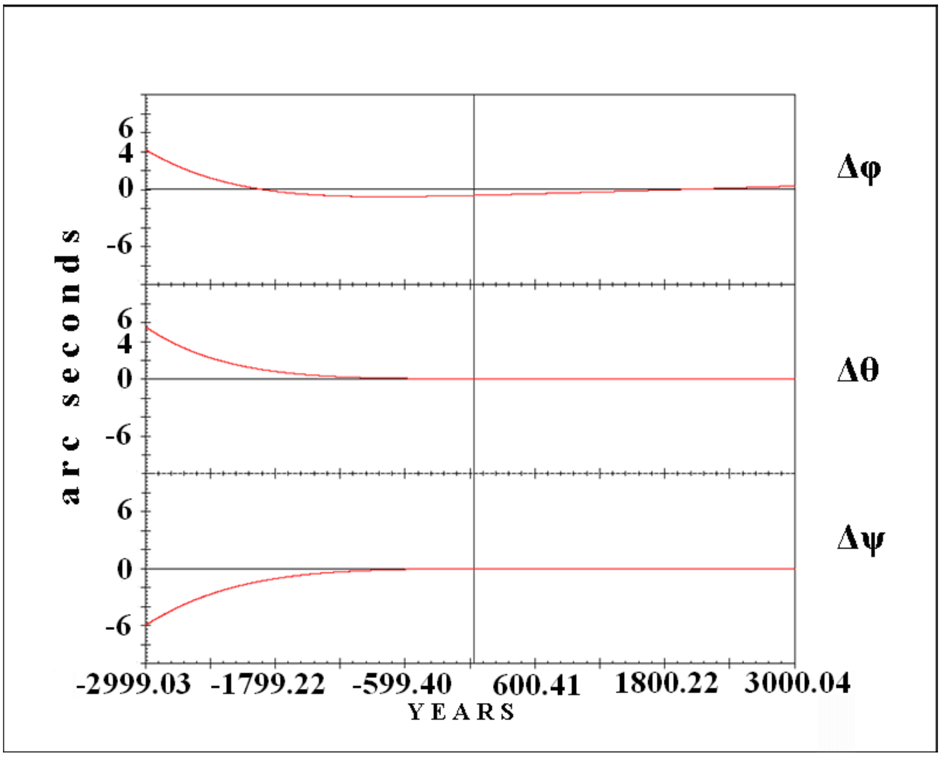

Fig. 13. Discrepancies between the numerical and RERS2012D semi-analytical solutions of the Earth rotation (dynamically adequate to the DE406/LE406 ephemeris) over 6000 years

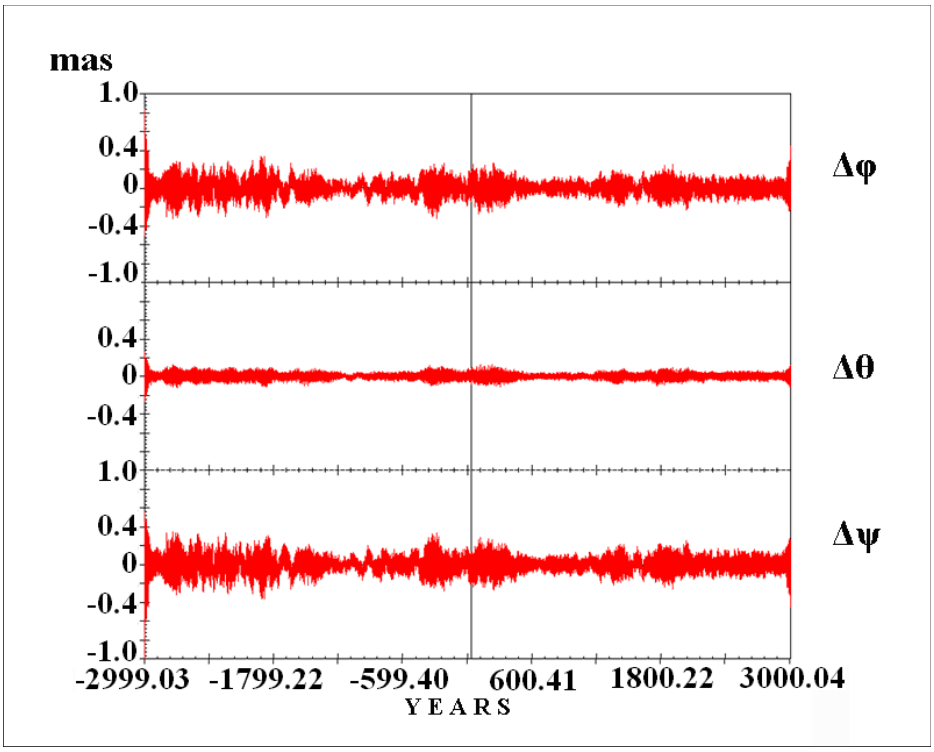

Fig. 14. Discrepancies between the numerical and RERS2013D semi-analytical solutions of the Earth rotation (dynamically adequate to the DE422/LE422 ephemeris) over 6000 years after applied 3rd iterations of the iterative algorithm

After application of the third iteration of the iterative algorithm, the convergence level was achieved and the process of the iterative algorithm was finished at this step. As a result, the Rigid Earth Rotation Series (RERS2013D) was constructed, which is dynamically adequate to the DE422/LE422 ephemeris over 6000 years. The discrepancies in Euler angles between the new numerical solutions and the semi-analytical solutions of RERS2013D do not surpass 1 mas over 6000 year time interval (presented in Figure 14).

Thus, the result of the comparison on 2000 and 6000 years demonstrates a good consistency of RERS2013 series with the DE422/LE422 ephemeris and that the RERS2013 series is more accurate than the RERS2012 series.

The comparison between the semi-analytical solutions of the Earth rotation RERS2012D (dynamically adequate to the DE406/LE406 ephemeris) and RERS2013D (dynamically adequate to the DE422/LE422 ephemeris) is given in Figure 15. The discrepancies in Euler angles between RERS2013D and RERS2012D after removal of the secular trends do not surpass 2 mas over 6000 year time interval.

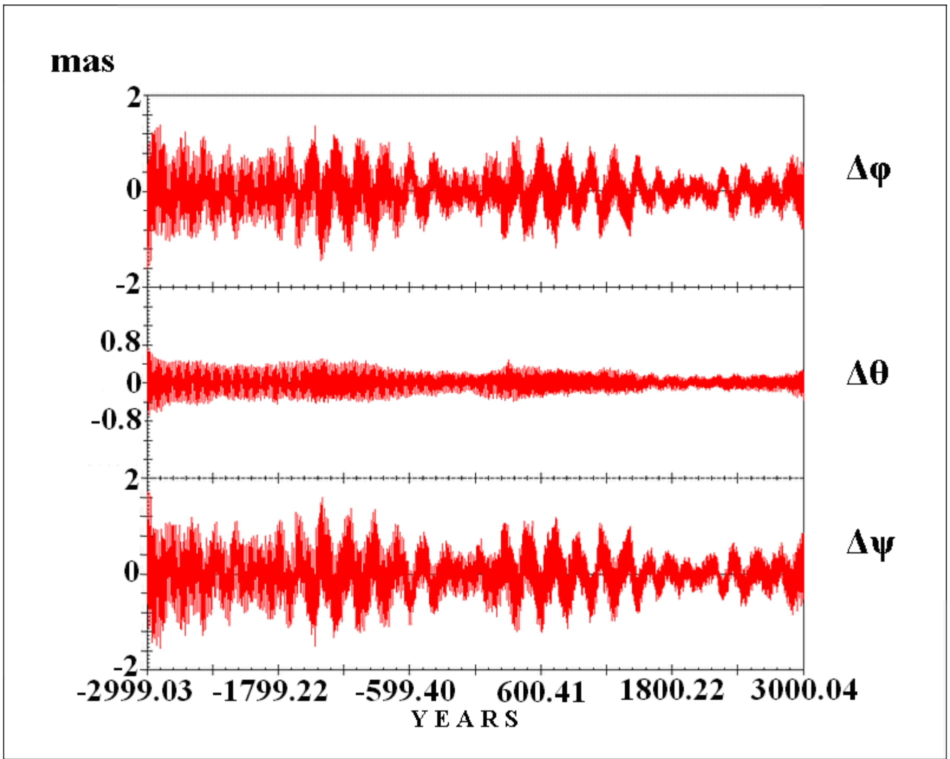

Fig. 15. Discrepancies between the semi-analytical solutions of the Earth rotation RERS2013D (dynamically adequate to the DE422/LE422 ephemeris) and RERS2012D (dynamically adequate to the DE406/LE406 ephemeris) over 6000 years after removal of the secular trends

Figure 16 demonstrates the the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013D and RERS2012D. The power spectra for angles $\Delta\psi$ and $\Delta\varphi$ are very similar. Table 4 presents the main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013D and RERS2012D.

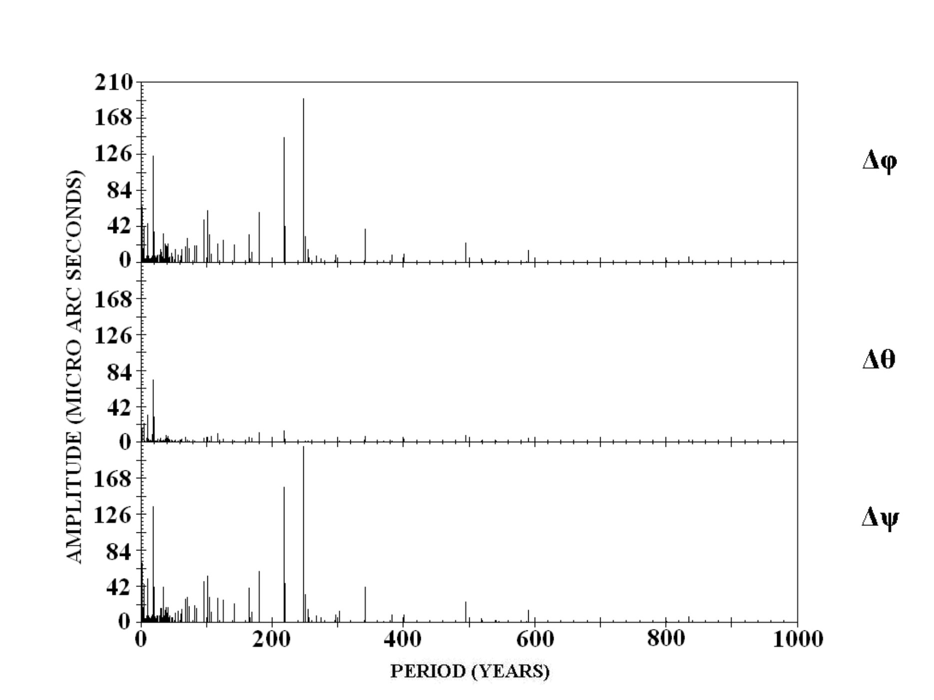

Fig. 16. Spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013D and RERS2012D

Table 4 (part one). The main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013D and RERS2012D

No	Period (years)	Planetary mean longitudes								Delauney arguments				Amplitude (μas)		
		λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	D	F	l	φ	Δψ	Δθ	Δφ
1	247.46285681	0	8	-5	-16	4	5	0	0	0	0	0	0	205.38		190.09
	18.70047146	0	10	-4	0	0	0	0	0	-1	1	-1	0		72.61	
2	217.81665206	0	8	-17	8	-3	0	0	0	0	0	0	0	157.50		144.86
	18.52626163	0	10	-2	0	0	0	0	0	1	-1	-1	0		54.41	
3	18.70047146	0	10	-4	0	0	0	0	0	-1	1	-1	0	135.16		124.16
	9.30647946	0	0	2	0	0	0	0	0	2	-2	0	0		31.50	
4	18.52626163	0	10	-2	0	0	0	0	0	1	-1	-1	0	98.51		90.23
	0.24995719	0	0	0	8	-3	0	0	0	0	0	0	0		30.35	
5	0.24995719	0	0	0	8	-3	0	0	0	0	0	0	0	70.70		65.00
	19.05580535	0	0	1	0	-2	5	0	0	1	-1	0	0		29.79	
6	0.99933003	0	0	5	-8	1	5	0	0	0	0	0	0	68.98		64.13
	0.24999156	0	0	12	-16	4	5	0	0	0	0	0	0		28.49	
7	0.24999156	0	0	12	-16	4	5	0	0	0	0	0	0	66.30		60.97
	9.25484253	0	0	6	-8	3	0	0	0	2	-2	0	0		27.95	
8	179.33557350	0	0	3	0	0	0	0	0	3	-2	-1	0	59.52		
	100.99634395	0	0	9	-11	0	0	0	0	3	-3	0	0			60.58
	18.61295892	0	0	1	0	0	0	0	0	1	-1	0	0		24.10	
9	0.25003213	0	0	8	-8	3	0	0	0	0	0	0	0	56.01	24.05	
	179.33557350	0	0	3	0	0	0	0	0	3	-2	-1	0			57.48

Table 4 (part two). The main harmonics contributions of the power spectrum of the discrepancies between the semi-analytical solutions of the Earth rotation RERS2013D and RERS2012D

No	Period (years)	Planetary mean longitudes								Delauney arguments				Amplitude (μas)		
		λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	D	F	l	φ	Δψ	Δθ	Δφ
	100.99634395	0	0	9	-11	0	0	0	0	3	-3	0	0	54.04		
10	0.25003213	0	0	8	-8	3	0	0	0	0	0	0	0			51.51
	3.57420723	0	0	1	0	0	0	0	0	1	1	-2	0		22.13	
	9.30647946	0	0	2	0	0	0	0	0	2	-2	0	0	50.17		
11	95.44074048	0	3	-4	0	0	0	0	0	1	0	-1	0			49.57
	3.56656472	0	0	3	-8	3	0	0	0	-1	-1	2	0		20.80	
	9.25484253	0	0	6	-8	3	0	0	0	2	-2	0	0	48.68		
12	9.30647946	0	0	2	0	0	0	0	0	2	-2	0	0			45.43
	18.78166562	0	2	-12	15	0	0	0	0	-1	-1	2	0		19.90	
	95.44074048	0	3	-4	0	0	0	0	0	1	0	-1	0	47.89		
13	9.25484253	0	0	6	-8	3	0	0	0	2	-2	0	0			44.39
	18.85333253	0	5	-5	-4	0	0	0	0	1	-1	0	0		19.34	
14	218.97980127	0	0	1	-2	0	2	0	0	0	0	0	0	45.25		41.35
	0.24999775	0	0	4	-16	4	5	0	0	0	0	0	0		18.42	
15	18.61295892	0	0	1	0	0	0	0	0	1	-1	0	0	45.17		41.32
	3.58188256	0	0	5	-8	3	0	0	0	1	1	-2	0		17.67	
16	3.57420723	0	0	1	0	0	0	0	0	1	1	-2	0	43.80		40.26
	0.99933007	0	0	5	-8	1	5	0	0	0	0	0	0		16.71	
17	0.24999775	0	0	4	-16	4	5	0	0	0	0	0	0	42.83		39.40
	217.81384076	0	8	-17	8	-3	0	0	0	0	0	0	0		13.20	
	19.05580535	0	0	1	0	-2	5	0	0	1	-1	0	0	40.94		
18	341.38619905	0	0	11	-21	2	0	0	0	0	0	0	0			38.49
	9.32567236	0	0	7	-9	0	0	0	0	2	-2	0	0		13.13	
19	18.78166562	0	2	-12	15	0	0	0	0	-1	-1	2	0	40.88		36.74
	1.05742243	0	0	6	-8	3	0	0	0	1	-1	0	0		11.46	
	34.17436627	0	0	9	-15	0	0	0	0	1	-1	0	0	40.70		
20	3.56656472	0	0	3	-8	3	0	0	0	-1	-1	2	0			36.44
	179.33557350	0	0	3	0	0	0	0	0	3	-2	-1	0		11.16	
	341.38619905	0	0	11	-21	2	0	0	0	0	0	0	0	40.58		
21	19.05580535	0	0	1	0	-2	5	0	0	1	-1	0	0			35.19
	1.00122869	0	0	1	0	2	-5	0	0	0	0	0	0		10.45	

Note that the power spectra of the discrepancies between the previous and new versions of the semi-analytical solutions of the Earth rotation (presented in Figures 4, 8, 12 and 16) for angles Δψ and Δφ are very similar.

4. CONCLUSIONS

As the results of this investigation, the new improved high-precision Rigid Earth Rotation Series RERS2013 dynamically adequate to the DE422/LE422 ephemeris over 2000 and 6000 years have been constructed. The series RERS2013 include about 4113 periodical and Poisson terms (without attempt to estimate new sub-diurnal and diurnal periodical and Poisson terms). The sub-diurnal and diurnal periodical and Poisson terms have not been investigated in this study. Therefore, they entered into new solutions RERS2013 without change from RERS2012. The discrepancies in Euler angles between the numerical solution and RERS2013 do not surpass: 4 μas over 2000 years, 1 mas over 6000 years. It means a good consistency of the RERS2013 series with the DE422/LE422 ephemeris. The RERS2013 series is more accurate than the RERS2012 series, which is dynamically adequate to the DE406/LE406 ephemeris. The good convergence of the iterative algorithm of this study has been confirmed.

Acknowledgements. The investigation was carried out at the Central (Pulkovo) Astronomical Observatory of the Russian Academy of Science and the Space Research Centre of the Polish Academy of Science, under a financial support of the Cooperation between the Polish and Russian Academies of Sciences, Theme No 34.

REFERENCES

Standish E.M., Newhall X X, Williams J.G. and Folkner W.M. (1995) JPL Planetary and Lunar Ephemerides, DE403/LE403, JPL IOM 314.10-127.

Bretagnon P., Francou G., Rocher P., Simon J.L. (1998) SMART97: a new solution for the rotation of the rigid Earth, *Astron. Astrophys.*, Vol. 329**,** 329–338.

Standish E.M. (1998) JPL Planetary and Lunar Ephemerides, DE405/LE405, JPL IOM 312.F-98-048.

Pashkevich V. V., Eroshkin G. I. (2005a) Application of the spectral analysis for the mathematical modelling of the rigid Earth rotation, *Artificial Satellites*, Vol. 40, No. 4, 251–259.

Pashkevich V.V., Eroshkin G.I. (2005b) Choice of the optimal spectral analysis scheme for the investigation of the Earth rotation problem, *Proc. Journées Systèmes de Référence Spatio-Temporels 2005,* eds. Brzeziński A., Capitaine N., Kolaczek B.,Warsaw 2005, 105–109.

Folkner W.F. (2011) JPL Planetary and Lunar Ephemerides : Export Information http://iau-comm4.jpl.nasa.gov/README.html

Pashkevich V.V. (2013) Construction of the numerical and semi-analytical solutions of the rigid Earth rotation at a long time intervals, Journal of Planetary Geodesy, A*rtificial Satellites,* Vol. 48, No. 1, (DOI: 10.2478/arsa-2013-0003), 25–37.

Received: 2013-12-13,
Reviewed: 2014-06-02 and 2014-08-18, by V. Zharov,
Accepted: 2014-08-29.