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ON THE PROBABILITY DISTRIBUTION OF EARTH ORIENTATION PARAMETERS DATA

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ABSTRACT. Earth Orientation Parameters (EOPs), i.e. pole coordinates (x_p, y_p), Universal Time (UT1-UTC), and celestial pole offsets (dX, dY), are the transformation parameters between the International Terrestrial Reference Frame (ITRF) and the International Celestial Reference Frame (ICRF). It is customarily assumed that each of the EOP time series follows the normal distribution. The normality assumption has been used specifically in EOP prediction studies. The objective of this paper is to investigate the normality hypothesis in detail. We analysed the daily time series of x_p, y_p, UT1-UTC, length-of-day (Δ), dX, and dY in the time interval from 01.01.1962 to 31.12.2008. The UT1-UTC data were transformed to UT1R-TAI by removing leap seconds and the tidal signal using the IERS model. The tidal effects $\delta\Delta$ were also removed from the Δ time series and $\Delta - \delta\Delta$ data were obtained. Furthermore, we constructed the residuals of these time series using least-squares fit. We evaluated the skewness and kurtosis and tested their statistical significance by the D'Agostino and the Anscombe-Glynn tests, respectively. In addition, the Anderson-Darling test for the normal distribution was applied. It was found that the x_p, y_p time series and their residuals slightly depart from the normal distribution, but this departure is rather due to marginal flattening/narrowing of the probability density function than due to extreme values. The UT1R-TAI time series and its residuals were also classified as non-Gaussian, however, the deviations from the normal distribution are again slight. The similar results hold for the $\Delta - \delta\Delta$ data, but some of its residuals were found to be Gaussian. We noticed that the celestial pole offsets, dX and dY, tend to deviate from the Gaussian distribution. In addition, we examined the determination errors of EOP data and found them to depart significantly from the normal distribution.

Keywords: Earth Rotation Parameters, probability distribution, modelling, time series.

1. INTRODUCTION

Variations of Earth's rotation rate occur over a wide range of time scales from a few hours to centuries. Such variations of seasonal nature were first detected using the pendulum clock by Stoyko (1937). This finding was later confirmed using quartz and atomic clocks (Eubanks, 1993). The length of day (LOD; or Δ), which is the difference between observed duration of the day and 86400 s, is often used to describe the variations in Earth's rotation rate. The Universal Time UT1-UTC (i.e. the integral of Δ with leap seconds corrections), together with the pole coordinates (x_p, y_p) and celestial pole offsets (dX, dY) are called Earth Orientation Parameters (EOPs). The EOPs are required to perform the time-varying transformation between the terrestrial reference frame (TRF) and the celestial reference frame (CRF). The accuracy of this transformation in real time is important for the purpose of navigation and tracking objects in space, e.g. interplanetary spacecrafts and Earth orbiting satellites (Schuh et al., 2002; Kalarus and Kosek, 2004). The short-term predictions of EOPs are also applied for navigation and tracking of interplanetary spacecrafts using the Deep Space Network (DSN).

The EOP predictions are being computed using different methods, e.g. least-squares extrapolation (Kosek et al., 2005), autocovariance methods (Kosek et al., 1998), Kalman filtering (Freedman et al., 1994; Petrov et al., 1995), autoregressive and moving average forecasting (Kosek et al., 2005), uni- and multivariate autoregressive prediction (Niedzielski and Kosek, 2008), artificial neural networks (Schuh et al., 2002), and fuzzy inference systems (Akyilmaz and Kutterer, 2004). Several of these techniques customarily assume that the data themselves or their residuals are normally distributed. However, no systematic probabilistic assessment of EOP data distribution has been provided so far.

The purpose of this paper is to examine the empirical probability distributions of the EOP time series and their residuals, and determine if these data sets are normally distributed. In addition, determination errors of these EOPs are analysed to assess the nature of their probability distributions.

2. DATA AND THEIR LEAST-SQUARES RESIDUALS

We examined the EOP time series in the time interval from 01.01.1962 to 31.12.2008. The data are provided by the International Earth Rotation and Reference System Service (IERS) in the file eopc04_IAU2000.61-now (http://hpiers/obspm/fr/iers/eop/eopc04_05/). The temporal resolution of these time series is equal to one day. The leap seconds were removed from the UT1-UTC data and hence the UT1-TAI time series was obtained. The tidal signals were removed from the UT1-TAI and Δ data using the IERS models (McCarthy and Petit, 2004) in order to obtain UT1R-TAI and $\Delta - \delta\Delta$ time series.

In addition, we analysed the residual time series. The least-squares (LS) residuals were computed as the difference between the original EOP data and their LS polynomial-harmonic models, which were fit to 1,2,...,10,15,20,...,40 years of these EOP data. In the case of pole coordinate data, the model consisted of the Chandler circle, annual and semiannual ellipses, and a trend. In the case of $\Delta - \delta\Delta$ or UT1R-TAI data, the model consisted of a linear trend, 18.6-, 9.3-, 1- and 0.5-year oscillations. In the case of dX, dY celestial pole offsets residuals the model consisted of 430-day oscillation corresponding to the free core nutation period.

The merged residuals for the entire time series were constructed by advancing each segment by one day in a stepwise fashion and using the trapezoidal weighting function for each time segment. The 1-year and 10-years residuals of the various EOP time series are plotted in Fig. 1 B,C,H,I and Fig. 2 B,C,H.

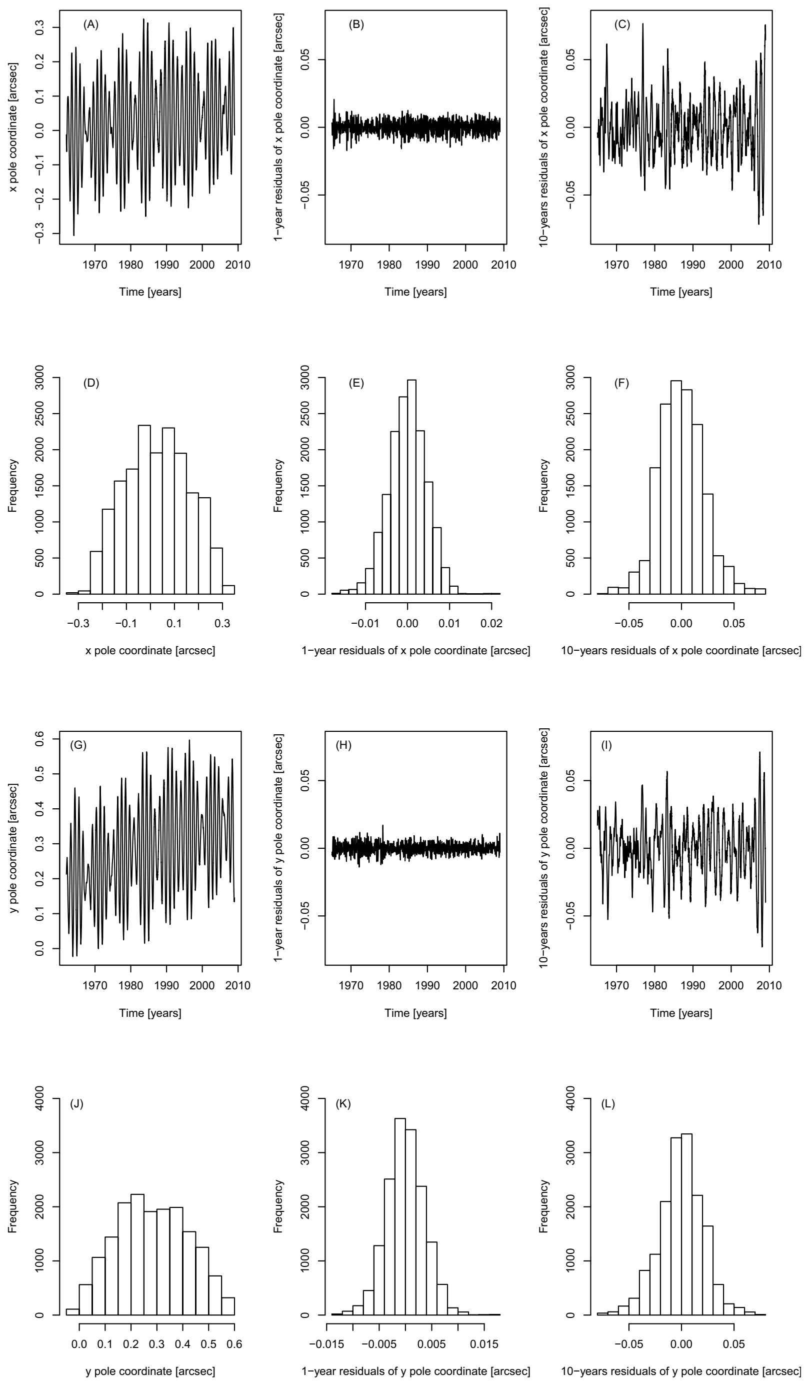

Fig. 1. x_p pole coordinate time series (A), its 1- and 10-years residuals (B,C) and the corresponding histograms (D,E,F); y_p pole coordinate time series (G), its 1- and 10-years residuals (H,I) and the corresponding histograms (J,K,L).

3. RESULTS AND DISCUSSION

The time series and the histograms for x_p, y_p, UT1R-TAI, $\Delta - \delta\Delta$, together with their residuals are shown in Fig. 1 and Fig. 2. The time series for the celestial pole offsets, dX and dY, and their histograms are depicted in Fig. 3. We have also calculated the skewness (S) and kurtosis (K) values of the empirical distributions of these parameters. In addition, we applied the test by D'Agostino (1970) to evaluate the statistically significant deviations of skewness from 0. The test by Anscombe and Glynn (1983) was utilized to assess the statistically significant departures of kurtosis from 3. The normality hypothesis was verified using the Anderson-Darling test. The results are juxtaposed in Tab. 1.

Tab. 1. Skewness (S) and kurtosis (K) of EOP data, their residuals, and determination errors. Numbers in bold indicate significant (significance level (α) of 0.01) deviations from 0 and 3 for S (D'Agostino test) and K (Anscombe-Glynn test), respectively. If the Anderson-Darling test suggests normality at $\alpha = 0.01$, values of S and K are underlined.

	x_p		y_p		UT1R-TAI	
	S	K	S	K	S	K
Original EOP data	0.00	**2.19**	0.05	**2.21**	**0.40**	**1.85**
Determination error	**8.53**[1]	**109.37**[1]	**8.45**[1]	**113.05**[1]	**15.80**[1]	**372.42**[1]
1-year residuals	**-0.14**[2]	**3.49**[2]	0.02[2]	**3.64**[2]	0.01[2]	**2.89**[2]
2-years residuals	-0.02[2]	**2.26**[2]	0.07[2]	**2.27**[2]	0.01[2]	3.03[2]
3-years residuals	**0.16**[2]	2.92[2]	**-0.12**[2]	**2.71**[2]	0.05[2]	3.02[2]
4-years residuals	**0.17**[2]	**3.30**[2]	-0.04[2]	**3.23**[2]	0.07[2]	3.08[2]
5-years residuals	**0.22**[2]	3.07[2]	**0.13**[2]	**3.74**[2]	**0.13**[2]	**3.17**[2]
6-years residuals	**0.21**[2]	3.08[2]	**0.15**[2]	**3.45**[2]	**0.20**[2]	**3.31**[2]
7-years residuals	**0.41**[2]	**4.04**[2]	**0.13**[2]	**3.55**[2]	**0.22**[2]	**3.26**[2]
8-years residuals	**0.67**[2]	**5.18**[2]	0.01[2]	**3.70**[2]	**0.21**[2]	**3.13**[2]
9-years residuals	**0.63**[2]	**4.69**[2]	**-0.14**[2]	**3.58**[2]	**0.14**[2]	3.04[2]
10-years residuals	**0.24**[2]	**3.66**[2]	**-0.17**[2]	**3.55**[2]	-0.05[2]	2.99[2]

	$\Delta - \delta\Delta$		dX		dY	
	S	K	S	K	S	K
Original EOP data	**-0.27**	**2.26**	0.04	**7.57**	-0.02	**6.32**
Determination error	**11.52**[1]	**248.10**[1]	**7.48**[1]	**93.75**[1]	**8.59**[1]	**119.64**[1]
1-year residuals	0.00	3.05	**0.20**[1]	**10.76**[1]	**0.29**[1]	**7.93**[1]
2-years residuals	0.01	3.00	0.06[1]	**10.82**[1]	**0.35**[1]	**8.01**[1]
3-years residuals	0.03	**3.12**	0.07[1]	**10.87**[1]	**0.39**[1]	**8.12**[1]
4-years residuals	0.06	**3.22**	0.07[1]	**10.93**[1]	**0.39**[1]	**8.02**[1]
5-years residuals	**0.08**	**3.27**	0.10[1]	**10.95**[1]	**0.36**[1]	**7.92**[1]
6-years residuals	**0.08**	**3.31**	0.07[1]	**10.67**[1]	**0.33**[1]	**7.85**[1]
7-years residuals	**0.09**	**3.36**	0.03[1]	**10.45**[1]	**0.31**[1]	**7.73**[1]
8-years residuals	**0.12**	**3.42**	0.02[1]	**10.26**[1]	**0.29**[1]	**7.59**[1]
9-years residuals	**0.16**	**3.46**	-0.02[1]	**10.05**[1]	**0.25**[1]	**7.49**[1]
10-years residuals	**0.19**	**3.47**	-0.02[1]	**9.93**[1]	**0.23**[1]	**7.41**[1]
15-years residuals	**0.15**	**3.33**	-0.02[1]	**9.13**[1]	**0.19**[1]	**7.03**[1]
20-years residuals	**0.60**[2]	**3.91**[2]	**0.12**[1]	**8.25**[1]	0.06[1]	**6.51**[1]
25-years residuals	**0.32**[2]	**3.66**[2]	-	-	-	-
30-years residuals	**0.15**[2]	**3.11**[2]	-	-	-	-
35-years residuals	**0.10**[2]	**3.27**[2]	-	-	-	-
40-years residuals	**0.30**[2]	**3.63**[2]	-	-	-	-

(1) Truncated data, 01.01.1984-31.12.2008, because before 01.01.1984 data were constant
(2) Truncated data, 01.01.1965-31.12.2008, to avoid misfit of the least squares model

For the Gaussian probability distribution $S = 0$, and $K = 3$. If $S < 0$, the probability density function (PDF) is skewed to the left; if $S > 0$, the PDF is right-skewed. If $K < 3$, the PDF is more flat in comparison to the normal distribution; in contrast, if $K > 3$, the PDF is more peaked than the Gaussian probability curve.

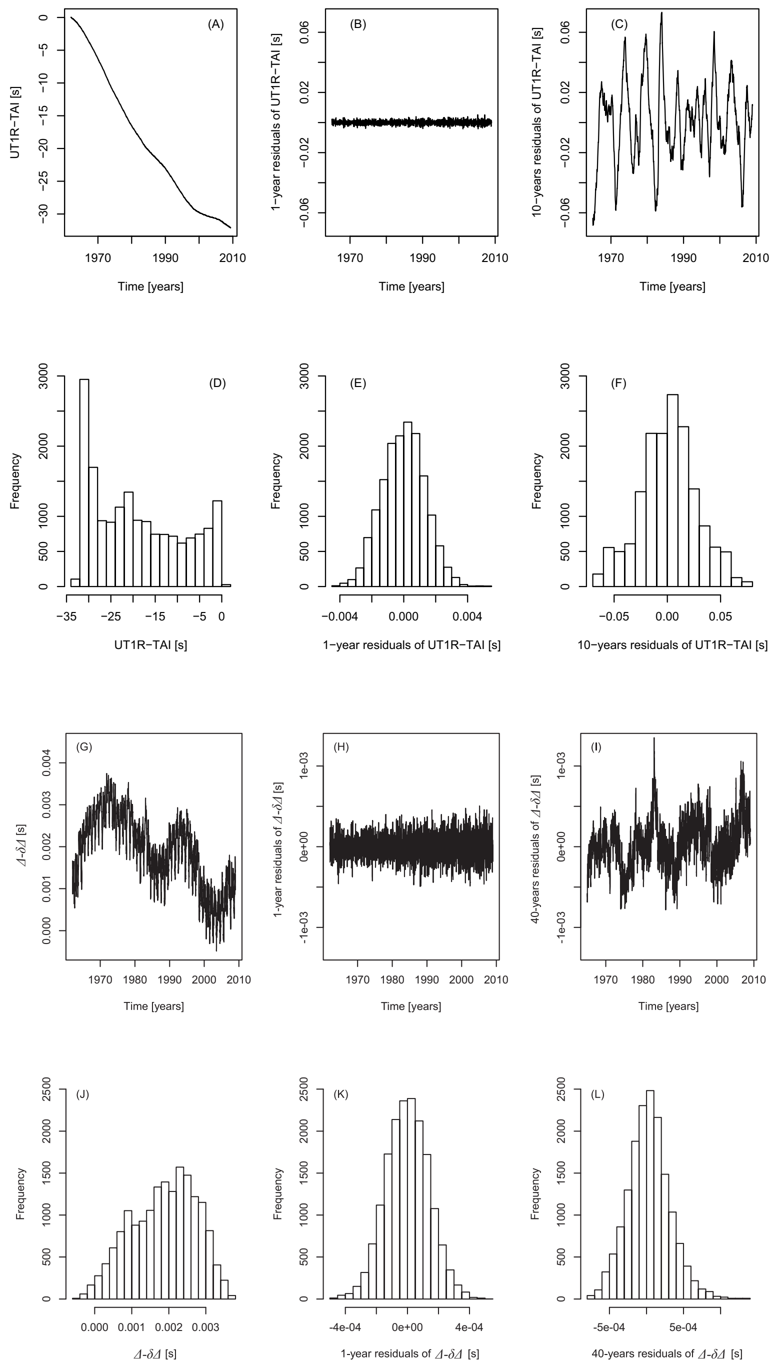

Fig. 2. UT1R-TAI time series (A), its 1- and 10-years residuals (B,C) and their histograms (D,E,F); $\Delta - \delta\Delta$ time series (G), its 1- and 40-years residuals (H,I) and the corresponding histograms (J,K,L).

The x_p, y_p time series and their residuals were found to slightly depart from the Gaussian distribution. This can be linked to marginal flattening/narrowing of the probability density function. The impact of extreme values on the probability law is rather negligible. The UT1R-TAI data and its residuals were also found to be non-Gaussian, but the deviation from normality is again marginal. The $\Delta - \delta\Delta$ time series was classified as slightly non-Gaussian, but the selected residuals were approximately normally distributed. The time series of the celestial pole offsets, dX and dY, and their residuals tend to considerably deviate from the Gaussian distribution.

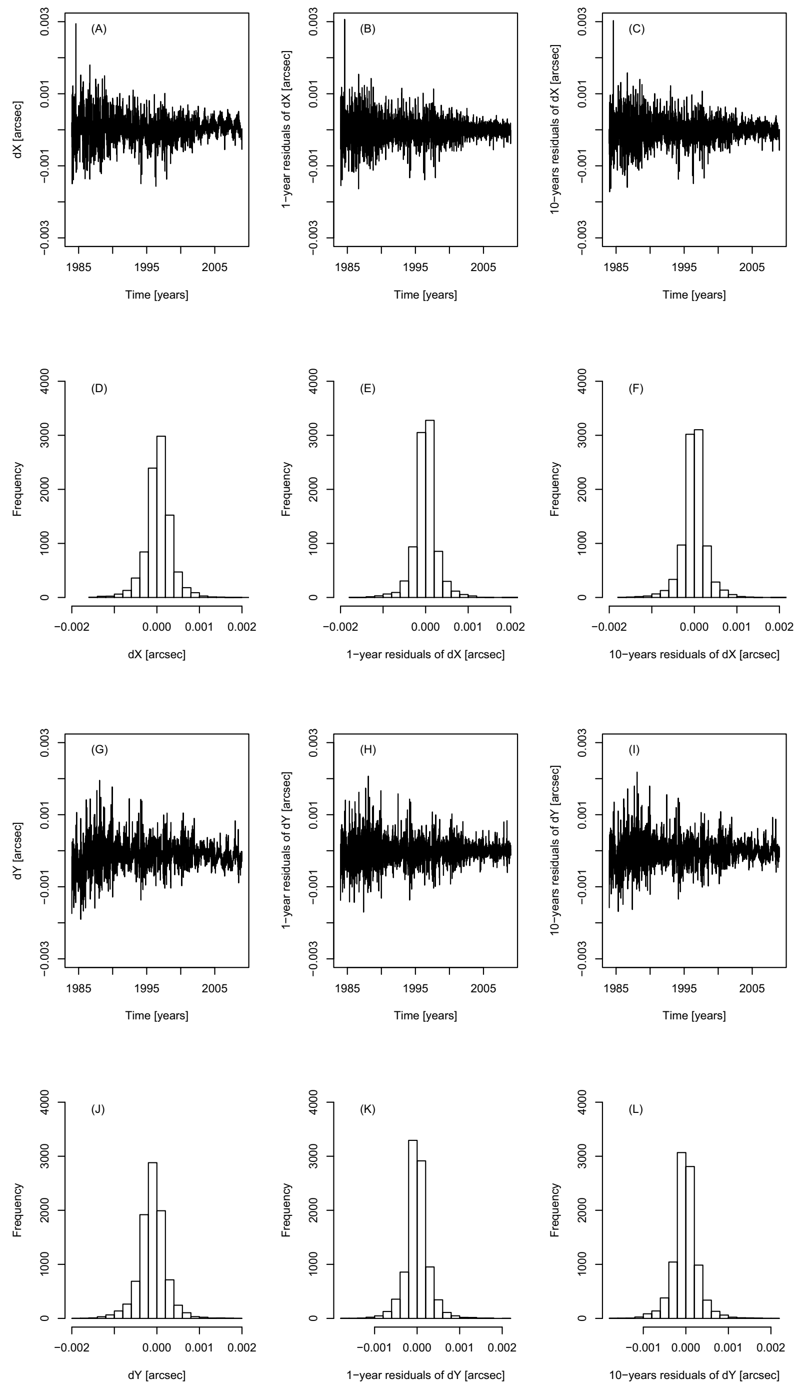

Fig. 3. dX data (A), its 1- and 10-years residuals (B,C) and their histograms (D,E,F); dY data (G), its 1- and 10-years residuals (H,I) and their histograms (J,K,L).

The data and the histograms of the determination errors of the various EOPs are depicted in Fig. 4. The skewness and kurtosis values of these time series are listed in Tab. 1. It is apparent from the shapes of the histograms as well as the values of skewness and kurtosis that the empirical distributions of the determination errors deviate significantly from the normal distribution. The heavy-tailed shapes of the histograms are consistent with the fact that the temporal variations of the errors are intermittent (Consolini and De Michelis, 1998).

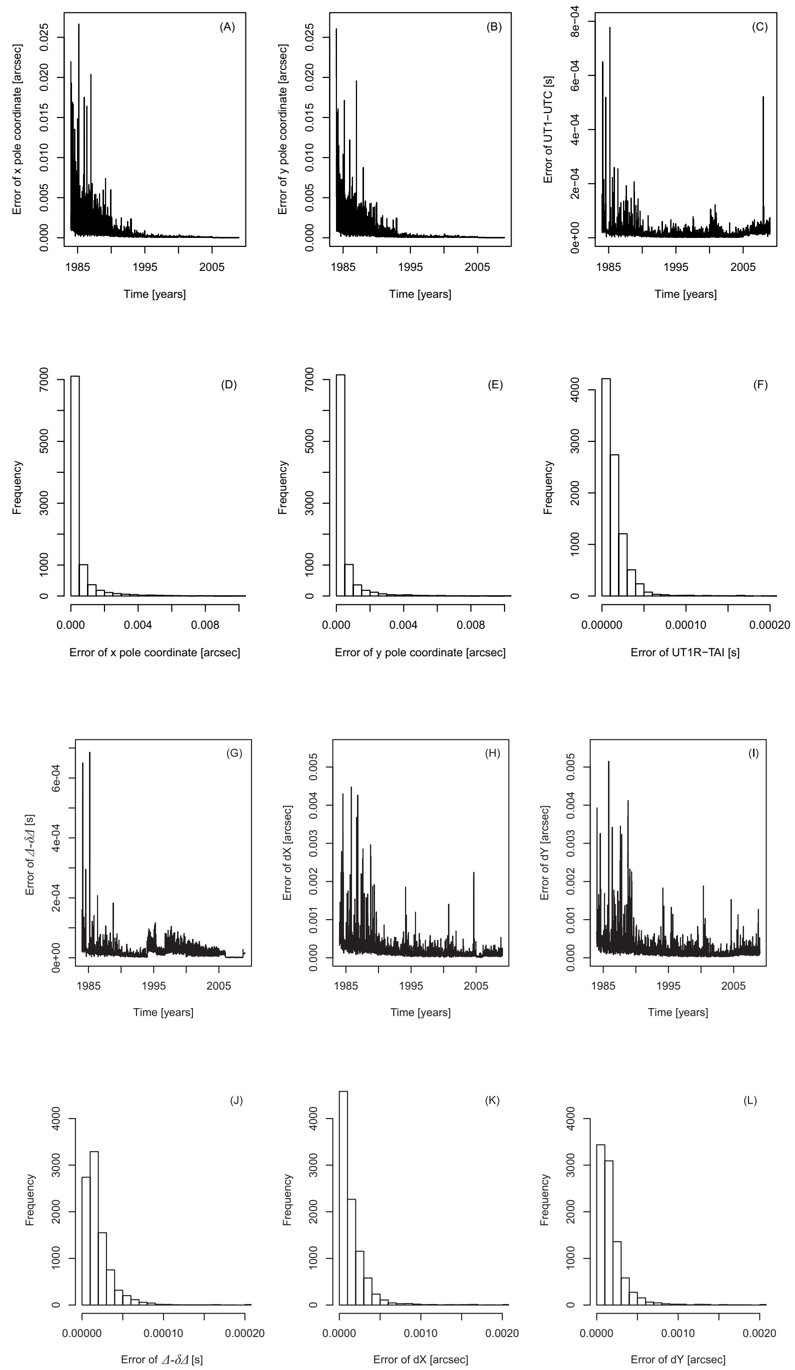

Fig. 4. Time series of determination errors of the EOPs (A,B,C,G,H,I) and the corresponding histograms (D,E,F,J,K,L).

4. CONCLUSIONS

The x_p, y_p, UT1R-TAI, $\Delta - \delta\Delta$ time series and their residuals distributions tend to marginally deviate from the normal distribution. However, the strength of the departure from normality varies. The stronger evidence for the non-Gaussian behaviour is noticed in the case of the celestial pole offsets, dX and dY. In addition, the determination errors of EOP data deviate very significantly from the Gaussian distribution.

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