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CONTRIBUTION OF 1st -3rd ORDER TERMS OF A BINOMIAL EXPANSION OF TOPOGRAPHIC HEIGHTS IN TOPOGRAPHIC AND ATMOSPHERIC EFFECTS ON SATELLITE GRAVITY GRADIOMETRIC DATA

M. Eshagh
Division of Geodesy, Royal Institute of Technology, Stockholm, Sweden
e-mail: eshagh@kth.se

ABSTRACT. In mathematical modeling of the topographic and atmospheric potentials in spherical harmonics, the topographic heights can binomially be expanded a certain order, usually to the third order. Some studies have been done on the effect of each order on geoid and gravity anomaly. However similar study on the satellite gravity gradiometric data is missed yet. This paper will investigate this matter globally. It presents that the contribution of the second- and third-order topographic terms is within 0.08 E and 2 mE, respectively on satellite gravity gradiometric data at 250 km level. Also the contribution of these terms is within 0.5 mE and 0.08 mE for the atmospheric effect.

Keywords: Spherical harmonics, potential, binomial expansion, atmospheric density model, global topographic height model

1. INTRODUCTION

The satellite gravity gradiometric data are affected by the masses of topography and the atmosphere. The topographic effect (TE) has been considered by several geodesists (e.g. Martinec et al. 1993, Martinec and Vaníček 1994, Sjöberg 1998a, Sjöberg and Nahavandchi 1999, Tsoulis 2001, Heck 2003, Seitz and Heck 2003, Sjöberg 2000 and 2007). The main goal of these efforts was to compute the TEs on geoid and terrestrial gravimetric data and also considering terrain correction. Wild and Heck (2004a and 2004b) have considered the TE on satellite gradiometry observations. Makhloof and Ilk (2005 and 2006) worked on the topographic-isostatic effects on airborne gravimetry, satellite gravimetry and gradiometry data. More details about their work can be found in Makhloof (2007). Novák and Grafarend (2006) presented a method for computing the TE and AE in satellite gravimetry and gradiometry. Novák and Grafarend (2006) and Tenzer et al. (2006) used a simple second-order polynomial as an atmospheric density model for computing the atmospheric effect (AE) on spaceborne data and gravity anomaly, respectively. Eshagh and Sjöberg (2008 and 2009a) investigated the TE and AE in Iran and Fennoscandia. Eshagh and Sjöberg (2009b) investigated further Novák's polynomial and proposed a combination of the polynomial and a power model as a density model which we call it the Eshagh/Sjöberg model (ESM) in this paper. This power model was originally proposed by Sjöberg (1993) and used by Sjöberg (1998b, 1999, 2001 and 2006), Nahavandchi (2004) and Sjöberg and Nahavandchi (1999 and

2000). The Sjöberg model (SM) and the exponential model (EM) of the atmospheric density were investigated and compared in Eshagh (2009a and 2009b).

The above studies miss the contribution of the topographic terms (TTs) obtained based of a binomial expansion of the topographic heights in formulating either the TE or AE on the satellite gradiometric data. This paper will investigate this matter. Some studies has been done in convergence of the binomial expansion of the topographic heights by Sun and Sjöberg (2001) and they concluded that truncation of the binomial expansion to the third order is a good approximation as along as the degree of the spherical harmonic expansion is not so high. Here we take advantage of the real topographic data and we study the contribution of each term of the binomial expansion in the TE and AE on the satellite gravity gradiometric data at 250 km level, which is a new study in the scope of satellite gravity gradiometry.

2. TOPOGRAPHIC AND ATMOSPHERIC POTENTIALS IN SPHERICAL HARMONICS

The topographic or atmospheric potentials can be expressed by the following spherical harmonics series:

$$V^{\mathrm{t,a}}(P) = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n} v_{\mathrm{nm}}^{\mathrm{t,a}} Y_{nm}(P), \tag{1a}$$

where $GM \approx 0.3986004418 \times 10^{15}\ m^3/s^2$ is the geocentric gravitational constant, R = 6378137 m is the semi-major axis of the reference ellipsoid, r is the geocentric distance of the point P, $v_{\mathrm{nm}}^{\mathrm{t,a}}$ is the spherical harmonic coefficient of either topographic or atmospheric potential with degree n and order m, and $Y_{nm}(P)$ is the fully-normalized spherical harmonics with following orthogonality property (Heiskanen and Moritz 1967, p. 31):

$$\iint_{\sigma} Y_{nm}(P) Y_{n'm'}(P) d\sigma = 4\pi\delta_{nn'}\delta_{mm'}, \tag{1b}$$

where σ is the unit sphere, $d\sigma$ is the surface integration element and δ stands for Kronecker's delta.

2.1 HARMONICS OF THE TOPOGRAPHIC POTENTIAL

If in formulation of the harmonic of topographic potential the binomial expansion of the topographic heights is truncated to third-order (before spherical harmonic analysis), the external harmonics will have the following forms (Eshagh 2009a):

$$v_{\mathrm{nm}}^{\mathrm{t}} \approx \frac{3\rho^{\mathrm{t}}}{(2n+1)\rho^{\mathrm{e}}}\left[\frac{H_{nm}}{R} + (n+2)\frac{H^2_{nm}}{2R^2} + (n+2)(n+1)\frac{H^3_{nm}}{6R^3}\right], \tag{2}$$

where $\rho^{\mathrm{t}} \approx 2667\ \mathrm{kg/m^3}$ and $\rho^{\mathrm{e}} \approx 5500\ \mathrm{kg/m^3}$ are the densities of the topographic and the mean Earth's masses, respectively, H_{nm}, H^2_{nm} and H^3_{nm} are the spherical harmonic coefficients of H, H^2 and H^3, respectively and H stands for the topographic height.

2.2. ATMOSPHERIC DENSITY MODELS AND THE HARMONICS OF THE ATMOSPHERIC POTENTIAL

Formulation of the harmonics of the atmospheric potentials is not as simple as that of topography, because the atmospheric density changes by altitude. The mathematical model of the harmonics depends on the atmospheric density model which is used. Different analytical models were proposed for the atmospheric density, which are summarized in the following subsection. There are few analytical density models for the atmospheric masses such as the EM (Lambeck 1988, p. 154), SM (Sjöberg 1993) and ESM (Eshagh and Sjöberg 2009b). In the following we briefly review these models as well as their corresponding harmonic coefficients.

2.2.1 The EM

Ecker and Mittermayer (1969) considered an exponential function for the atmospheric density. Since the atmospheric density decreases fast by altitude, considering such a model will not be so far from reality. Ecker and Mittermayer's (1969) exponential function was based on an ellipsoidal Earth and they used the ellipsoidal height in their investigations. However in a spherical approximation the EM for the atmospheric density will be (see e.g. Lambeck 1988, p. 154):

$$\rho^{\mathrm{a}}(r) = \rho_0 e^{-\alpha(r-R)} = \rho_0 e^{\alpha R} e^{-\alpha r}, \tag{3a}$$

where $\rho^{\mathrm{a}}(r)$ is the atmospheric density, $\rho_0 \approx 1.2227 \ \mathrm{kg/m^3}$ the atmospheric density at sea level, R is the mean radius of the Earth, r is the radial distance of any point inside the atmosphere and $\alpha \approx 1.3886 \times 10^{-4}$ is a constant.

The harmonics of the atmospheric potential based on the EM are (Eshagh 2009a and 2009b):

$$v_{\mathrm{nm}}^{\mathrm{a}} \approx \frac{3\rho_0}{(2n+1)\rho_e}\left\{M\delta_{n0} - \left[\frac{H_{nm}}{R} + \frac{n+2-\alpha R}{2R^2}H_{nm}^2 + \frac{(n+2)(n+1-2\alpha R)+\alpha^2R^2}{6R^3}H_{nm}^3\right\},\right. \tag{3b}$$

where

$$M = \left[\left(1-Le^{-\alpha Z}\right)\left(\frac{2+\alpha(R+Z)}{\alpha^2R^2}\right) + \frac{2\left(1-e^{-\alpha Z}\right)}{\alpha^3R^3} - \frac{Z}{R^2\alpha}\right], \tag{3c}$$

$$L = 1 + \frac{Z}{R}. \tag{3d}$$

In formulating the atmospheric potential it is assumed that the atmospheric masses are bounded to a certain altitude above sea level, say to Z = 250 km. The topographic masses are replaced by the atmospheric masses which are subtracted from the atmospheric shell from sea level to the upper bound of the atmospheric masses.

2.2.2 The SM

In order to take the atmospheric potential in spatial domain and to use integral formulas for computing the AE, the EM will not be a suitable model in formulation of the AE. Sjöberg

(1993) selected a simple power model which is considerably simpler than the EM. The SM is (Sjöberg 1993):

$$\rho^{\mathrm{a}}(r)=\rho_0\left(\frac{R}{r}\right)^{\nu}\ ,\ \text{where}\ R\le r\,. \tag{4a}$$

The constant ν was derived by a simple fitting to the atmospheric density in logarithmic scale at different altitudes. The Sjöberg (1993) fitting was based on the model presented by the Reference Atmosphere Committee in 1961 (Reference Atmosphere Committee 1961) and this model was updated based on the United State Standard Atmosphere (1976), by Eshagh and Sjöberg (2009b). In the former case the exponent ν = 850 was derived, but in the latter (updated) model ν = 930 was achieved.

The harmonics of atmospheric potential based on the SM have the following mathematical forms (Eshagh 2009a and 2009b):

$$V_{\mathrm{nm}}^{\mathrm{a}}\approx\frac{3\rho_0}{(2n+1)\rho^e}\left\{\frac{\left(L^{3-\nu}-1\right)\delta_{n0}}{3-\nu}-\frac{H_{nm}}{R}-\frac{n+2-\nu}{2R^2}H^2_{nm}-\frac{(n+2-\nu)(n+1-\nu)}{6R^3}H^3_{nm}\right\}. \tag{4b}$$

2.2.3 The ESM

The atmospheric densities generated based on the EM and SM are more or less the same; see Eshagh (2009a and 2009b). These models cannot express the densities below 10 km, which are the most massive part of the atmospheric masses. Novák (2000) proposed a second-order polynomial for the massive part of the atmosphere and he used the United States Standard Atmosphere (1976) model to estimate the densities above 10 km. In this case, the atmospheric masses above 10 km should be divided into several shells with different densities. The summation of potentials of the shells will be the total potential of the atmospheric masses above 10 km; see Novák (2000), Eshagh (2009a) or Eshagh and Sjöberg (2008). Eshagh and Sjöberg (2009b) proposed a mathematical model for the densities above 10 km height. The ESM has the following mathematical expression:

$$\rho^a(r)=\begin{cases}\rho_0\left[1+\alpha'(r-R)+\beta(r-R)^2\right], & 0\le(r-R)\le H_0\\ \rho^a(H_0)\left(\dfrac{R+H_0}{r}\right)^{\nu''}, & H_0\le(r-R)\le Z\end{cases}\ , \tag{5a}$$

where H_0=10 km, $\rho^a(H_0)$=0.4127 kg/m^3, α' = -7.6495$\times10^{-5}$m^{-1}, β = 2.2781 $\times10^{-9}$m^{-2} and ν''=890.

Formulation of harmonics based on the ESM is rather complicated and its harmonics have the following form (Eshagh and Sjöberg 2008 and 2009b, Eshagh 2009a):

$$v_{nm}^{\mathrm{a}}\approx\frac{3}{(2n+1)\rho^{\mathrm{e}}}\left\{\rho_0\left[\left(\frac{H_0}{R}(2-\alpha'R)\frac{H_0^2}{2R^2}+2\left(1-2\alpha'R+\beta R^2\right)\frac{H_0^3}{6R^3}\right)+\right.\right.$$

$$+\frac{\rho(H_0)\left(K^{\nu}L^{3-\nu}-K^3\right)}{3-\nu}\Bigg]\delta_{n0}-\rho_0\left[\frac{H_{nm}}{R}+\left(n+2-\alpha'R\right)\frac{H^2{}_{nm}}{2R^2}+\right.$$

$$\left.+\left((n+2)\left(n+1-2\alpha'R\right)+2\beta R^2\right)\frac{H^3{}_{nm}}{6R^3}\right]\Bigg\}, \tag{5b}$$

where

$$K=1+\frac{H_0}{R}. \tag{5c}$$

3. TE AND AE ON SATELLITE GRAVITY GRADIOMETRIC DATA

The gravitational gradients can be expressed in terms of spherical harmonics. Formulation of the gradients depends on the frame which is used. In satellite gradiometry the main frames are: the geocentric, local north oriented and orbital frames. For definition of these frames the reader is referred to e.g. Koop (1993), Mueller (2003), Petrovskaya and Vershkov (2006), Eshagh (2008). It should be stated that the gravitational gradients are measured in none of these frames. As Mueller (2003) mentioned the best frame is the frame which is as close as possible to the gradiometer reference frame. Here we prefer to use the local north-oriented reference frame because the consequence of our study in this paper does not dependent on the choice of the frame.

In order to compute the TE and AE it suffices to insert the harmonic coefficients of the topographic and atmospheric potentials (Eqs. 2, 3b, 4b and 5b) into the spherical harmonic expansion of the gravitational gradients. Here we take the advantage of the presented formulas by Eshagh (2009a) for their suitability in numerical computations. The interested readers are referred to the references to see the formulas.

4. INVESTIGATION OF TTS OF THE TE AND AE ON SATELLITE GRAVITY GRADIOMETRIC DATA

So far the investigations of the TE and AE are either based on the harmonics to the second or third TTs. It is not clear how big the contribution of each term is. Is it significant to consider the third TT in the formulation or not? Sun and Sjöberg (2001) stated that as long as the maximum degree of spherical harmonic expansion is not so high and for the highest elevation in the world (9 km) the effect of higher order terms of the binomial expansion of the topographic height remains below 1% of total contribution. In order to investigate the effect of each TT we generate the topographic and atmospheric harmonics using each TT separately. Therefore we will have three sets of the spherical harmonic coefficients for either the TE or the AE and each set considers only one TT. In order to do better comparison the effect by considering all the TTs are also computed and presented. We have generated the spherical harmonics H_{nm}, $H^2{}_{nm}$ and $H^3{}_{nm}$ using the shuttle radar topographic model presented by Wieczorek (2007). The harmonics of each TT are generated to the degree and order of 720 corresponding with $15'\times15'$ resolution. Each set of harmonics is inserted to the spherical harmonic expression of the gravitational gradients at 250 km altitude to generate the effect of the TTs.

Table 1 shows the TE considering all TTs and the first TT separately. According to the selected resolution for synthesizing the effects the maximum and minimum effects (in magnitude point of view) are about 7 E and -1.45 E in a global point of view on V_{zz}^{t}. In average, V_{zz}^{t} is more affected by the topographic masses than the other gradients. Now, if we just consider the first TT of Eq. (2) to generate the TE, the result will be very similar to previous case in which all the TTs were considered. It means that the TEs generated by the first TT can express most of the TE.

Table 1. TE considering all TTs and first TT. Unit: 1 E.

	TE with all TTs				TE with the first TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{t}	2.27	-0.09	-5.26	± 0.56	2.23	-0.09	-5.20	± 0.56
V_{yy}^{t}	1.53	-0.18	-4.98	± 0.54	1.51	-0.18	-4.92	± 0.54
V_{zz}^{t}	6.95	0.28	-1.45	± 0.97	6.88	0.28	-1.43	± 0.97
V_{xy}^{t}	2.33	0.00	-1.93	± 0.22	2.30	0.00	-1.90	± 0.22
V_{xz}^{t}	4.86	0.15	-6.71	± 0.66	4.80	0.15	-6.63	± 0.66
V_{yz}^{t}	4.33	0.00	-4.27	± 0.57	4.27	0.00	-4.22	± 0.57

Table 2 presents the TE with second and third TTs. The maximum and minimum effects are 0.08 E on V_{zz}^{t} and 0.02 E on V_{yy}^{t} (in magnitude point of view), respectively when the second TT is considered for generating the effect. In general we can say that the TE based on second and third TTs are about 1 cE and 1 mE levels.

Table 2. TE considering the second and third TT. Unit: 1 mE

	TE with the second TT				TE with the third TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{t}	42.81	-0.05	-62.46	± 3.24	42.81	-0.05	-62.46	± 3.24
V_{yy}^{t}	25.54	-0.24	-58.56	± 2.72	58.56	0.24	-25.54	± 2.72
V_{zz}^{t}	80.27	0.29	-50.48	± 5.02	80.27	0.29	-50.48	± 5.02
V_{xy}^{t}	33.46	0.00	-27.55	± 1.54	33.46	0.00	-27.55	± 1.54
V_{xz}^{t}	84.14	0.27	-77.97	± 3.82	1.31	0.00	-1.03	± 0.03
V_{yz}^{t}	58.98	0.00	-51.39	± 3.29	0.79	0.00	-0.71	± 0.03

Table 3. AE based on EM considering all TTs and first TT. Unit: 1 mE.

	AE with all TTs				AE with the first TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{a}	0.70	-0.95	-1.74	± 0.21	1.39	-0.95	-2.02	± 0.26
V_{yy}^{a}	0.75	-0.92	-1.52	± 0.20	1.26	-0.95	-1.68	± 0.25
V_{zz}^{a}	2.50	1.88	0.27	± 0.37	2.54	1.86	-1.17	± 0.44
V_{xy}^{a}	0.07	0.00	-0.75	± 0.08	0.87	0.00	-1.05	± 0.10
V_{xz}^{a}	2.25	0.06	-1.63	± 0.25	3.04	-0.07	-2.20	± 0.30
V_{yz}^{a}	1.44	0.00	1.49	± 0.22	1.93	0.00	-1.96	± 0.26

Now, the AE based on the EM of density for the atmospheric masses is investigated. Table 3 shows the effects considering all and the first TTs. As the table shows that difference between the effects when the first TT is considered differs with that is considered based on all the TTs. The difference is less than 1 mE level. The maximum and minimum effects are related with V_{zz}^{t} and V_{xy}^{t}, respectively. Table 4 illustrates that the effect of the second and third TTs are below 1 mE except for V_{zz}^{a}. As the table shows that effect of the third TT is smaller than the second one.

Table 4. AE based on EM considering the second and third TTs. Unit: 1 mE.

	AE with the second TT				AE with the third TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{a}	-0.63	-0.99	-1.89	± 0.06	-0.77	-0.99	-1.07	± 0.01
V_{yy}^{a}	-0.77	-1.01	-1.62	± 0.05	-0.88	-0.99	1.04	± 0.01
V_{zz}^{a}	3.17	2.01	1.79	± 0.10	2.03	1.98	1.70	± 0.02
V_{xy}^{a}	0.40	0.00	-0.25	± 0.02	0.05	0.00	-0.09	± 0.00
V_{xz}^{a}	0.91	0.01	-1.03	± 0.07	0.23	-0.00	-0.22	± 0.01
V_{yz}^{a}	0.73	0.00	-0.63	± 0.06	0.13	0.00	-0.17	± 0.01

Tables 5 and 6 show the statistics of the AEs based on the SM. Table 5 states that the maximum and minimum effects are 2.68 mE and -0.12 mE on V_{zz}^{a}, respectively when all three TTs of Eq. (4b) are used. If the first TT is used these values will be 2.81 mE and -0.98 mE and again on V_{zz}^{a}, respectively. Again the maximum effect of the second and third topographic terms is 3.29 mE and 2.20 mE on V_{zz}^{a}, respectively. The minimum effects of the second TT are -0.24 mE and 0.05 mE for the third term on V_{xy}^{a}.

The AEs based on the EM and SM are very similar. If we see Tables 1 and 2 we will conclude that the effect of higher order TTs is smaller than the lowers in the TE. However, we observed a reverse situation for the AE based on these models. The reason is the appearance of Bouguer's shell effect in the formulas of the harmonics. Bouguer's shell potential is constant and contributes just to the zero-degree harmonic. When we subtract the first TT the value of the zero-degree harmonic is reduced. The second TT is smaller than the first one; therefore the value of the zero-degree harmonic will be larger by considering just the second TT. Similar interpretation can also be made for the third TT. This is the reason of obtaining larger values for effect of higher TTs. However the consequence is to have smaller AE by considering all the TTs. Tables 3, 4, 5 and 6 confirm this statement as the means of the effects are very close to zero except for V_{zz}^{a}, V_{xx}^{a} and V_{yy}^{a}. The reason is that these gradients include the zero-degree harmonic in its spherical harmonic expansion.

The ESM differs with the EM and SM as it expresses the most massive part of the atmosphere well, which is below 10 km altitude from sea level. The atmospheric masses below this level do not increase linearly with respect to height and this model considers this non-linearity in the formulation. In this model the atmospheric masses are divided into two parts. The part below 10 km and involves the atmospheric topography and the part above which expresses a constant potential of an atmospheric Bouguer's shell.

Table 5. AE based on SM considering all TTs and first TT. Unit: 1 mE.

	AE with all TTs				AE with the first TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{a}	0.64	-1.04	-1.83	$\pm$0.21	1.30	-1.03	-2.10	$\pm$0.53
V_{yy}^{a}	0.68	-1.01	-1.61	$\pm$0.21	1.18	-0.99	-1.76	$\pm$0.25
V_{zz}^{a}	2.68	2.05	-0.12	$\pm$0.37	2.81	2.03	-0.98	$\pm$0.44
V_{xy}^{a}	0.68	0.00	-0.76	$\pm$0.08	0.87	0.00	-1.05	$\pm$0.10
V_{xz}^{a}	2.27	-0.06	-1.64	$\pm$0.25	3.04	-0.07	-2.20	$\pm$0.30
V_{yz}^{a}	1.45	0.00	-1.50	$\pm$0.22	1.93	0.00	-1.96	$\pm$0.26

Table 6. AE based on SM considering the second and third TTs. Unit: 1 mE.

	AE with the second TT				AE with the third TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{a}	-0.73	-1.08	-1.94	$\pm$0.06	-0.88	-1.08	-1.15	$\pm$0.01
V_{yy}^{a}	-0.87	-1.09	-1.68	$\pm$0.05	-0.97	-1.08	-1.12	$\pm$0.01
V_{zz}^{a}	3.29	2.18	1.97	$\pm$0.10	2.20	2.15	1.89	$\pm$0.02
V_{xy}^{a}	0.38	0.00	-0.24	$\pm$0.02	0.05	0.00	-0.08	$\pm$0.00
V_{xz}^{a}	0.88	0.01	-0.99	$\pm$0.07	0.21	-0.00	-0.20	$\pm$0.01
V_{yz}^{a}	0.70	0.00	-0.60	$\pm$0.06	0.12	0.00	-0.16	$\pm$0.01

The same investigation process will be carried out for the AE based on each TT and the ESM. Table 7 presents the statistics of the AE considering all TTs and first TT. According to this model the maximum and minimum global AE will be about 5.22 mE and -0.26 mE, respectively on V_{zz}^{a} and V_{yy}^{a}. As the table shows, the difference between the case where all TTs are considered and the case where just the first TT is used is below 1 mE level.

Table 7. AE based on ESM considering all TTs and first TT. Unit: 1 mE.

	AE with all TTs				AE with the first TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{a}	-0.58	-2.21	-3.44	$\pm$0.27	-0.08	-2.21	-3.17	$\pm$0.23
V_{yy}^{a}	-0.26	-2.17	-3.03	$\pm$0.25	-0.19	-2.18	-2.87	$\pm$0.22
V_{zz}^{a}	5.22	4.37	0.76	$\pm$0.46	5.09	4.39	1.62	$\pm$0.39
V_{xy}^{a}	0.97	0.00	-1.25	$\pm$0.11	0.79	0.00	-0.96	$\pm$0.09
V_{xz}^{a}	3.51	-0.06	-2.67	$\pm$0.32	0.76	-0.06	-2.00	$\pm$0.27
V_{yz}^{a}	2.26	0.00	-2.31	$\pm$0.28	1.81	0.00	-1.77	$\pm$0.24

Table 8 presents the effect of second and third TTs. As can be observed in the table the means of the effects are very close to zero except for V_{zz}^{a}, V_{xx}^{a} and V_{yy}^{a}. This is related to Bouguer's shell effect which all the other gradients exclude it in their harmonic formulations.

Table 8. AE based on ESM considering the second and third TTs. Unit: 1 mE.

	AE with the second TT				AE with the third TT			
	Max.	Mean	Min.	Std.	Max.	Mean	Min.	Std.
V_{xx}^{a}	-1.65	-2.24	-2.49	± 0.04	-2.17	-2.24	-2.27	± 0.00
V_{yy}^{a}	-1.82	-2.24	-2.40	± 0.03	-2.20	-2.24	-2.26	± 0.00
V_{zz}^{a}	4.63	4.48	3.71	± 0.06	4.51	4.49	4.40	± 0.00
V_{xy}^{a}	0.17	0.00	0.27	± 0.02	0.02	0.00	-0.03	± 0.00
V_{xz}^{a}	0.68	-0.01	-0.61	± 0.05	0.08	0.00	-0.07	± 0.00
V_{yz}^{a}	0.42	0.00	-0.49	± 0.04	0.04	0.00	-0.06	± 0.00

5. CONCLUSIONS

The main goal of this paper was to numerically investigate the contribution of each topographic term in topographic and atmospheric effects on satellite gradiometric data. The numerical studies were carried out globally and effect of each topographic term was generated on the gradiometric data. The numerical results show that the contribution of the second and third terms of the topographic effects reaches to 80 mE and 1 mE, respectively. The effect of these terms for the atmospheric effect is below 1 mE for the exponential, Sjöberg models and Eshagh/Sjöberg's models and may be negligible. Among the gradients, V_{zz} shows more sensitivity with respect to the topographic terms than the others.

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LIST OF ABBREVIATIONS

Exponential model……………………………………………………………………EM
Sjöberg's model………………………………………………………………………SM
Eshagh/Sjöberg's model………………………………………………………………ESM
Topographic effect……………………………………………………………………TE
Atmospheric effect……………………………………………………………………AE
Topographic term……………………………………………………………………...TT

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