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COCOS: A NEW SINEX COMBINATION SOFTWARE

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ABSTRACT. A simple and flexible algorithm for combination of space geodesy observations is presented. The CoCoS software that implements this algorithm is described in details. The combined GPS–VLBI solution for the CONT02 VLBI experiment data and observations from co–located GPS sites is obtained and discussed.

Keywords: GPS, VLBI, IERS, combined solution.

1. INTRODUCTION

Modern space geodesy techniques allow to estimate positions of observing stations (realization of Terrestrial Reference Frame), radio sources positions (realization of Celestial Reference Frame), and Earth orientation parameters (EOPs) — transformation parameters between Terrestrial and Celestial Frames. Each technique gives own realization of reference frames, EOP estimations and has some advantages and weaknesses. It is considered the most reliable estimation can be achieved combining observation information from the various space geodesy techniques and on–ground geodetic surveys (Rothacher, 2002).

Currently there are some combined products maintained by International Earth Rotation and Reference Systems Service (IERS, http://www.iers.org), such as International Terrestrial Reference Frame (ITRF, Boucher et al., 2004), EOPs series. The only product that obtained using full variance–covariance information is ITRF (Gambis, 2004; Bizouard, 2008).

Recently at the Main Astronomical Observatory of the National Academy of Sciences of Ukraine (MAO) an alternative combination algorithm was developed. This algorithm was implemented in CoCoS (**Co**nstruct **Co**mbined **S**olution) software.

2. ALGORITHM DESCRIPTION

Assume that we have n initial solutions from Data Analysis Centers in SINEX format (http://www.iers.org/documents/ac/sinex/sinex_v202.pdf), and local tie information for sites with collocated techniques. The combined solution can be obtained using the following steps.

1. The constraints imposed in an initial solution should be removed:

$$\mathbf{N}_i^{free} = \mathbf{N}_i - \mathbf{N}_i^{constr},$$

where $\mathbf{N}_i^{free}$ — constraint free normal equations matrix for i–th solution,
$\mathbf{N}_i$ — initial solution normal equations matrix for i–th solution,
$\mathbf{N}_i^{constr}$ — constraints normal equation matrix for i–th solution,
$i = 1, \ldots, n$.

2. Unconstrained systems should be brought to the equal a priori values for common parameters. The following unknowns types can be taked as common parameters: station positions, station velocities and EOPs (pole coordinates and their rates, length of day, UT1 estimations). At this step it is possible to accomplish pre–elimination of uninterested parameters to reduce normal equations systems dimensions (Dach et al., 2007).

   Let us obtain main equation for this step. The observation equations for initial solution can be expressed as:

$$\begin{array}{rcccl} \mathbf{y_1} & = & \mathbf{A}_1\mathbf{x_1} & + & \mathbf{e_1}; \\ \mathbf{y_2} & = & \mathbf{A}_2\mathbf{x_2} & + & \mathbf{e_2}; \\ & & \vdots & & \\ \mathbf{y_n} & = & \mathbf{A}_n\mathbf{x_n} & + & \mathbf{e_n}, \end{array} \tag{1}$$

where $\mathbf{y_1},\ \mathbf{y_2},\ \ldots,\ \mathbf{y_n}$ — observations vectors,
$\mathbf{A}_1,\ \mathbf{A}_2,\ \ldots,\ \mathbf{A}_n$ — design matrices for observation equations system,
$\mathbf{x_1},\ \mathbf{x_2},\ \ldots,\ \mathbf{x_n}$ — vectors of estimated parameters,
$\mathbf{e_1},\ \mathbf{e_2},\ \ldots,\ \mathbf{e_n}$ — vectors of observation errors.

Solving observation (1) equations by least–square method we can obtain normal equation systems:

$$\begin{array}{rcl} \mathbf{N}_1\Delta\mathbf{x_1} & = & \mathbf{b_1}; \\ \mathbf{N}_2\Delta\mathbf{x_2} & = & \mathbf{b_2}; \\ & \vdots & \\ \mathbf{N}_n\Delta\mathbf{x_n} & = & \mathbf{b_n}, \end{array} \tag{2}$$

where $\mathbf{N}_1,\ \mathbf{N}_2,\ \ldots,\ \mathbf{N}_n$ — normal equations design matrices,
$\mathbf{\Delta x_1}, \mathbf{\Delta x_2}, \ldots, \mathbf{\Delta x_n}$ — estimated corrections to a priori values,
$\mathbf{b_1},\ \mathbf{b_2},\ \ldots,\ \mathbf{b_n}$ — right hand side vector of normal equations system.

The equation systems (2) and (1) are connected each other by the following formulae:

$$\begin{array}{rcl} \mathbf{N}_i & = & \mathbf{A}_i^T\mathbf{P}_i\mathbf{A}_i; \\ \mathbf{b_i} & = & \mathbf{A}_i^T\mathbf{P}_i\mathbf{y_i}, \end{array} \tag{3}$$

where $\mathbf{P}_i$ — weight matrices,
i =1, $\ldots, n$,

The combination has the sense if there are some common parameters in vectors $\mathbf{x_i}$. In this case, we can write the following connection equations:

$$\mathbf{x_i} = \mathbf{B_i}\mathbf{x_0} + \mathbf{c_i}, \tag{4}$$

where $\mathbf{x_0}$ — observables vector that contains common and uncommon parameters from the vectors $\mathbf{x_i}$. Matrix $\mathbf{B}_i$ changes parameter order and vector $\mathbf{c}_i$ accounts for differences in a priori values for common parameters or local ties for station positions. For more details about forming $\mathbf{x_0}$, $\mathbf{B}_i$ and $\mathbf{c_i}$ cf. (Lytvyn, 2008).

Standard deviations for local ties should be incorporated in standard deviations for corresponding parameters:

$$\sigma = \sqrt{\sigma^2_{tie} + \sigma^2_{initial}}$$

Substituting (4) into (1) we can obtain normal equations system with $\mathbf{x_0}$.

$$\begin{array}{ccccccc} \mathbf{y_1} & - & \mathbf{A}_1\mathbf{c_1} & = & \mathbf{A}_1\mathbf{B}_1\mathbf{x_0} & + & \mathbf{e_1}; \\ \mathbf{y_2} & - & \mathbf{A}_2\mathbf{c_2} & = & \mathbf{A}_2\mathbf{B}_2\mathbf{x_0} & + & \mathbf{e_2}; \\ & & & \vdots & & & \\ \mathbf{y_n} & - & \mathbf{A}_i\mathbf{c_n} & = & \mathbf{A}_n\mathbf{B}_n\mathbf{x_0} & + & \mathbf{e_n}. \end{array} \tag{5}$$

The new observation equations system (5) can be rewritten in the following form:

$$\begin{pmatrix} \mathbf{y_1} - \mathbf{A_1 c_1} \\ \mathbf{y_2} - \mathbf{A_2 c_2} \\ \vdots \\ \mathbf{y_n} - \mathbf{A_n c_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1\mathbf{B}_1 \\ \mathbf{A}_2\mathbf{B}_2 \\ \vdots \\ \mathbf{A}_n\mathbf{B}_n \end{pmatrix} \mathbf{x_0} + \begin{pmatrix} \mathbf{e_1} \\ \mathbf{e_2} \\ \vdots \\ \mathbf{e_n} \end{pmatrix}. \tag{6}$$

Solving equations (6) we can obtain a normal equations system:

$$\mathbf{N}'\Delta\mathbf{x_0} = \mathbf{b}', \tag{7}$$

where

$$\mathbf{N}' = \begin{pmatrix} \mathbf{A}_1\mathbf{B}_1 \\ \mathbf{A}_2\mathbf{B}_2 \\ \mathbf{A}_3\mathbf{B}_3 \\ \vdots \\ \mathbf{A}_n\mathbf{B}_n \end{pmatrix}^T \begin{pmatrix} \mathbf{P}_1 & 0 & 0 & \dots & 0 \\ 0 & \mathbf{P}_2 & 0 & \dots & 0 \\ 0 & 0 & \mathbf{P}_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \mathbf{P}_n \end{pmatrix} \begin{pmatrix} \mathbf{A}_1\mathbf{B}_1 \\ \mathbf{A}_2\mathbf{B}_2 \\ \mathbf{A}_3\mathbf{B}_3 \\ \vdots \\ \mathbf{A}_n\mathbf{B}_n \end{pmatrix}, \tag{8}$$

$$\mathbf{b}' = \begin{pmatrix} \mathbf{A}_1\mathbf{B}_1 \\ \mathbf{A}_2\mathbf{B}_2 \\ \mathbf{A}_3\mathbf{B}_3 \\ \vdots \\ \mathbf{A}_n\mathbf{B}_n \end{pmatrix}^{\mathbf{T}} \begin{pmatrix} \mathbf{P}_1 & 0 & 0 & \dots & 0 \\ 0 & \mathbf{P}_2 & 0 & \dots & 0 \\ 0 & 0 & \mathbf{P}_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \mathbf{P}_n \end{pmatrix} \begin{pmatrix} \mathbf{y_1} - \mathbf{A_1 c_1} \\ \mathbf{y_2} - \mathbf{A_2 c_2} \\ \vdots \\ \mathbf{y_n} - \mathbf{A_n c_n} \end{pmatrix}. \tag{9}$$

Accomplishing multiplication in (8)–(9) and accounting for (3) we are coming to the following equations:

$$\mathbf{N}' = \sum_{i=1}^{n} \mathbf{B}_i^T \mathbf{N}_i \mathbf{B}_i; \tag{10}$$

$$\mathbf{b}' = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{B_i^T b_i} - \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{B_i^T N_i c_i}. \tag{11}$$

And now we can obtain common normal equations system

$$\left(\sum_{i=1}^{n} \mathbf{B}_i^T \mathbf{N}_i \mathbf{B}_i\right) \Delta\mathbf{x_0} = \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{B_i^T b_i} - \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{B_i^T N_i c_i}. \tag{12}$$

3. Impose constraints to fix solution reference frame.

   There are several ways to impose constraints. The simple one is to set positions of two stations a priori. This approach is used in SLR. The reference frame can be also fixed by setting a priori standard deviations for set of stations positions. This approach is used for processing GPS data from regional networks. And in the case of global networks it is possible to use No–Net–conditions (No–Net translation and No–Net–rotation) (Brockmann, 1997). This method is widely used in processing VLBI data.

   Imposing constraints means adding fake observations to normal equation system:

$$\mathbf{N}^{fixed} = \mathbf{N}^{free} + \mathbf{N}^{constr}; \tag{13}$$

$$\mathbf{b^{fixed}} = \mathbf{b^{free}} + \mathbf{b^{constr}}. \tag{14}$$

2. SOFTWARE DESCRIPTION

CoCoS software is an implementation of algorithm described above. This software uses solutions in SINEX format provided by IERS Analysis Centers and local tie information (local ties and their standard deviations) for station with collocated techniques.

The main features of CoCoS software are:

— support SINEX formats: 2.01 (IERS official standard), 2.10 (Goddart Space Flight Center (GSFC) modification);

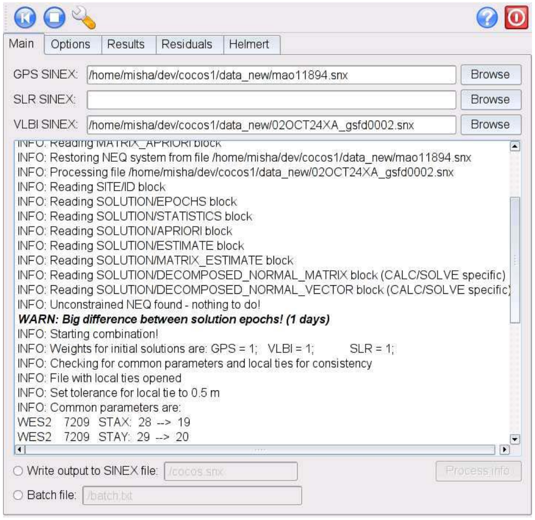
— graphical user interface (GUI) written using Qt3 widgets library (http://trolltech.com/products/qt/);

— implementation of linear algebra algorithms: matrix object with dynamic memory allocation, LU-decomposition for determinant computation and matrix inverse, Householder transformation for solving least square problems, basic matrix operations;

— local tie verification using tolerance interval;

— possibility to exclude/include any parameter type from the common parameters list.

CoCoS works under Linux operation system but after porting GUI part to Qt4 software can be compiled under Windows and MacOS. CoCoS was written using C++ programming language. This allowed to create modular and easily extendable system. The software implements fast and low cost algorithms for matrix computations with dynamic memory allocation. That means that CoCoS need not to be recompiled to dealt with very large equation systems. The amount of PC memory is only limit for a maximum unknown parameters number. All elementary matrix operations are optimized for parallel computing using OpenMP programming interface (http://openmp.org). It is especially useful at multiprocessor systems such as clusters or grids.



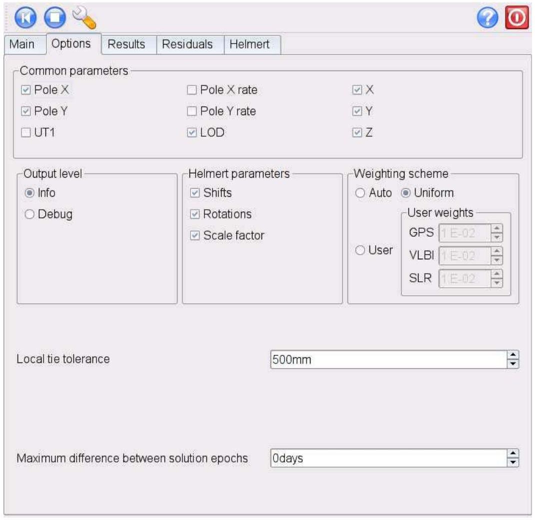
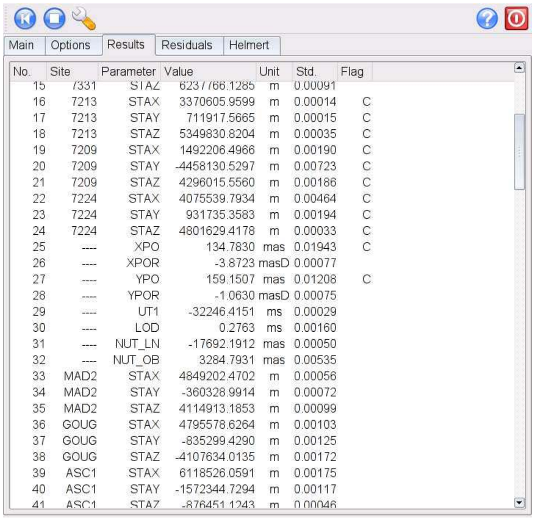


Fig. 1. CoCoS main window **Fig. 2.** CoCoS options tab



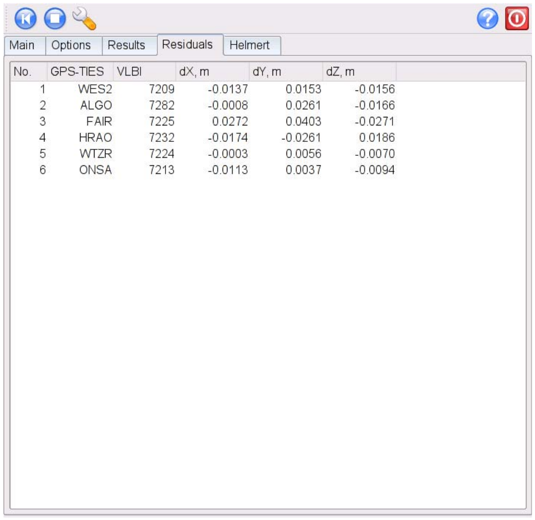


Fig. 3. CoCoS results tab **Fig. 4.** CoCoS residuals tab

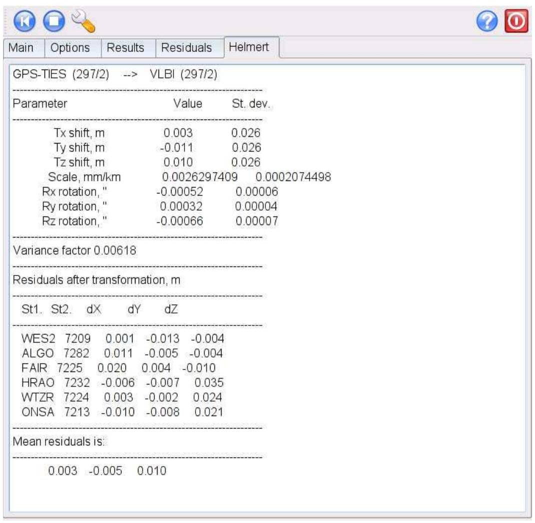


Fig. 5. CoCoS Helmert parameters tab

3. USING COCOS

To create combined solution with CoCoS user needs to have two or more initial solutions with some common parameters at the same epoch. It is desirable (but not necessary) to have local ties and their standard deviations for sites with collocated techniques. User should put names of input SINEX files into according lines at CoCoS main window (Fig. 1). If user needs to process multiple files, there is a possibility to make batch processing. After that user should choose desirable parameters in options tab (Fig. 2). When all about is done processing can be started by pushing start button from toolbar. Once processing has been finished, the result can be seen at results tab (Fig. 3). User can check initial solutions agreement by reviewing the residuals and Helmert transformation parameters in according tabs (Fig. 4–5). Finally combined solution can be stored in SINEX format.

COMBINED SOLUTION FOR CONT02 EXPERIMENT

For testing CoCoS software it was decided to obtain GPS–VLBI combined solution using VLBI observations from CONT02 geodetic experiment (http://ivs.nict.go.jp/mirror/program/cont02/) and GPS observations from collocated sites. The eight VLBI stations participated in CONT02: Ny Alesund, Kokee Park, Gilmore Creek, Algonquin, Westford, Wettzell, HartRAO, Onsala. The last six of them are the stations with known local ties to GPS stations (Boucher et al., 2004).

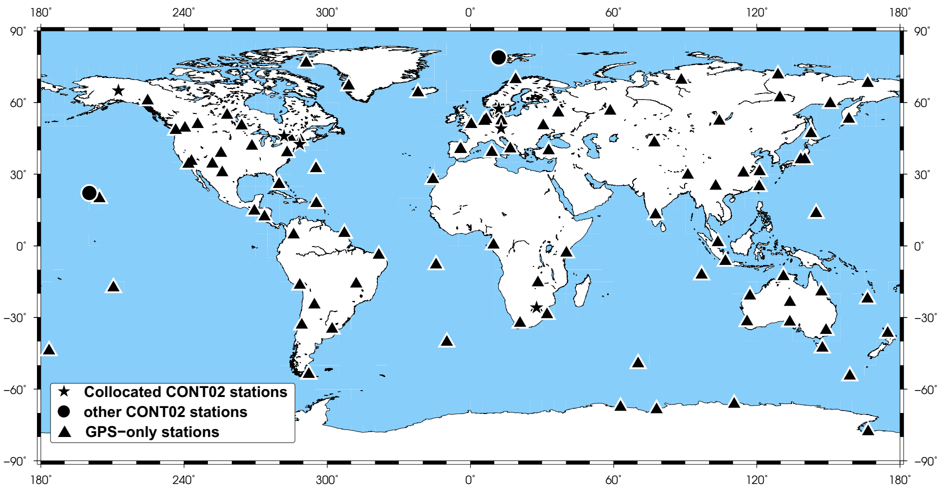


Fig. 6. Observing network

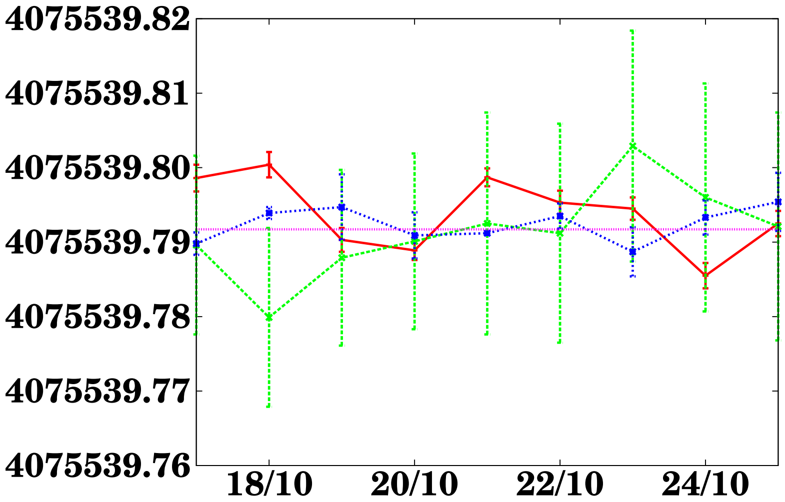


Fig. 7. Time series for Wettzell X coordinate

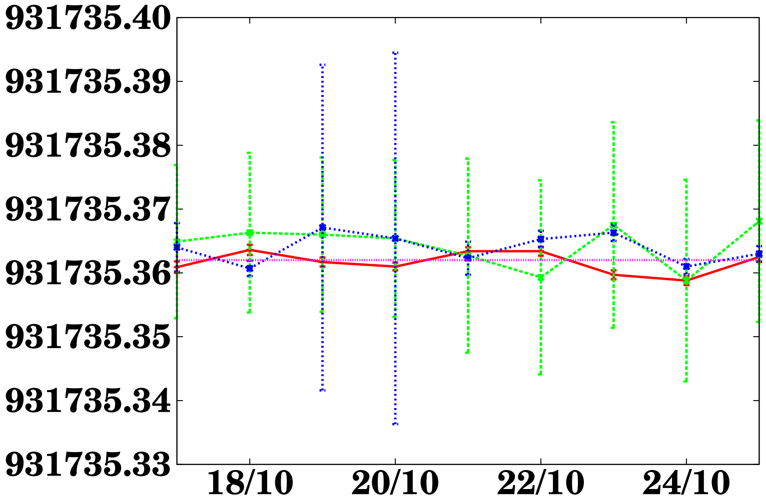


Fig. 8. Time series for Wettzell Y coordinate

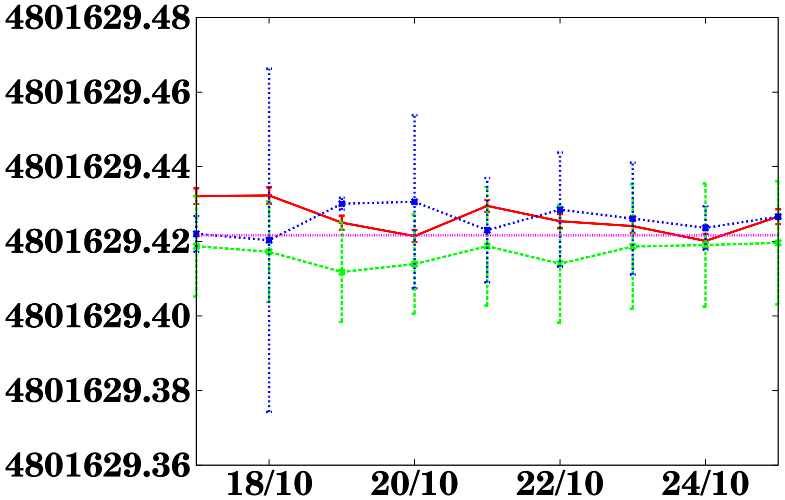


Fig. 9. Time series for Wettzell Z coordinate

Solution provided by GSFC was taken as initial solution. The initial GPS solution was obtained at MAO using GAMIT GPS processing software ver.10.1 (King & Bock, 2003). At first solution for eight GPS stations collocated with the CONT02 VLBI stations was obtained. Processing strategy is described in details in (Lytvyn, 2005). To fix reference frame for GPS solution it was combined with global solution of Scripps Institution of Oceanography Analysis Center (SIO) applying No–Net–conditions. The resulting stations configuration in GPS solution can be found at Fig. 6. The combined solution was calculated for every day of the CONT02 experiment. Estimated coordinates of station Wettzell are shown at Fig. 7–9. Analyzing coordinate time series it can be concluded that all solutions are agree each other at their standard deviations level.

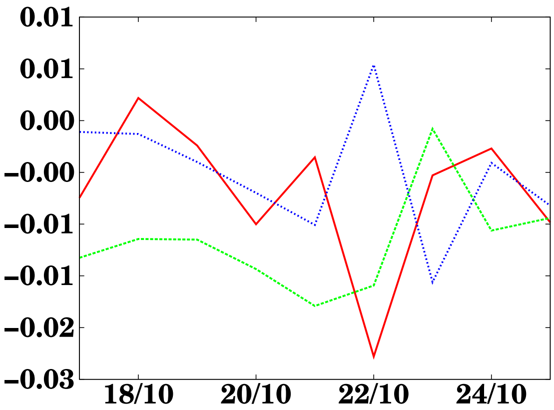


Fig. 10. Shifts between initial solutions[1], m

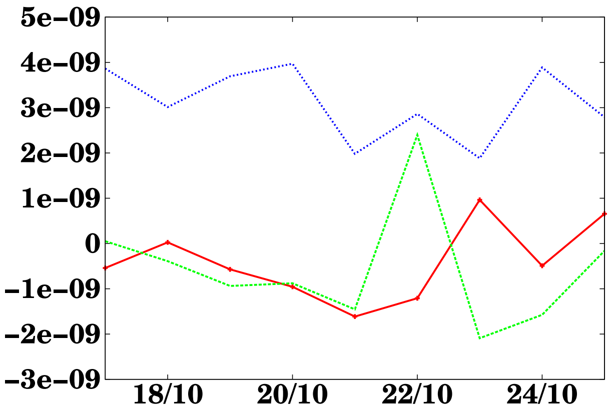


Fig. 11. Rotations between initial solutions[2], rad

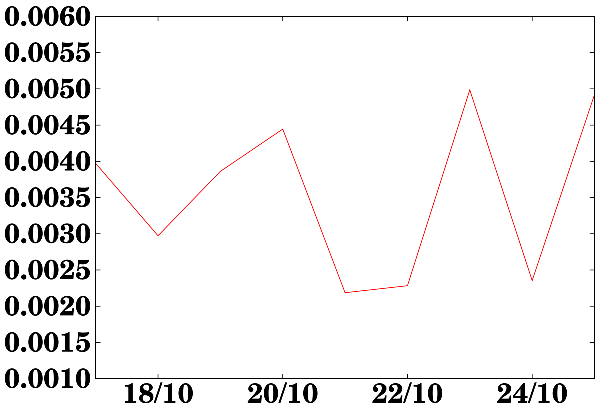


Fig. 12. Scale factor between initial solutions, mm/km

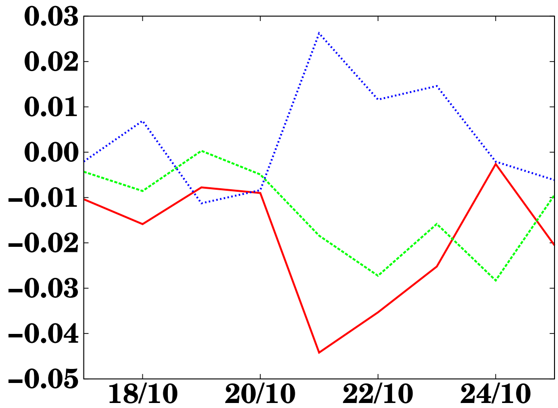


Fig. 13. Shift between VLBI and Combined solutions[1], m

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[1]red solid line is OX shift, green dashed line — OY shift, blue dotted line — OZ shift

[2]red solid line is OX rotation, green dashed line — OY rotation, blue dotted line — OZ rotation

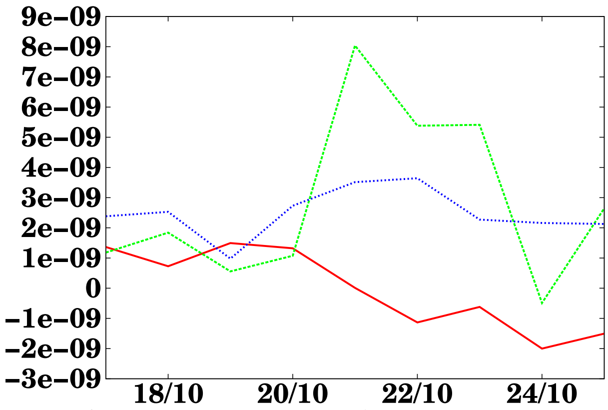

Fig. 14. Rotations between VLBI and Combined solutions[2], rad

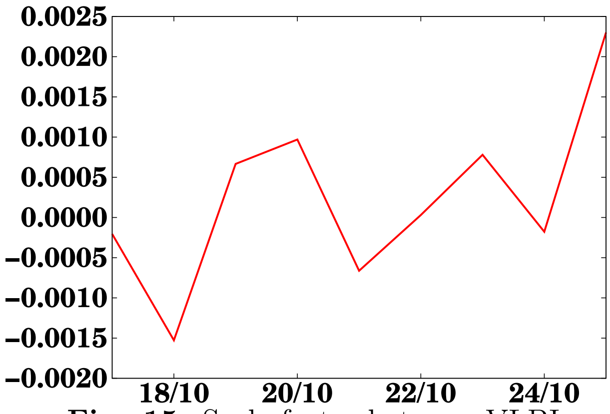

Fig. 15. Scale factor between VLBI and Combined solutions, mm/km

Seven Helmert transformation parameters were computed to estimate differences between initial and combined solutions. These parameters are shown at Fig. 10–15. The translations between initial systems are at the same degree as local ties standard deviations. Rotations between systems are rather small and scale factor change for the case "VLBI — Combined system" is smaller than for the case "VLBI — GPS - local ties".

5. CONCLUSIONS

This paper presents the simple and flexible algorithm for obtaining combined solution using observations of space geodesy techniques. Also algorithm implementing software described in details. The proposed approach has some advances and limitations.

The main advances of algorithm are:

— ability to obtain combined solution even if there are no any available local ties;

— possibility to include/exclude any type of unknown parameters (station coordinates, pole coordinates and their rates, length of day, and UT1 estimations) in common parameters list;

— this method rather formal and can be used for combination any equation systems with common parameters.

Besides, method used for ITRF computation requires full variance–covariance information for local ties surveys in SINEX format. And according to (Altamimi, 2005) there are proper local tie SINEX files only for all Australian co–location sites. Proposed method needs only local tie values and their standard deviations.

From the other hand there are some limitations of proposed algorithm:

— the method is oriented on combination of simultaneous observations;

— there is no reliable weighting procedure for initial solutions.

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