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Report of Meeting

The Eighteenth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities Hajdúszoboszló (Hungary), January 31–February 3, 2018

The Eighteenth Debrecen–Katowice Winter Seminar on Functional Equations and Inequalities was held in Hotel Aurum, Hajdúszoboszló, Hungary, from January 31 to February 3, 2018. It was organized by the Department of Analysis of the Institute of Mathematics of the University of Debrecen.

The Winter Seminar was supported by the following organizations:

— Institute of Mathematics, University of Debrecen;

— Hungarian Scientific Research Fund Grant OTKA K 111651.

14 participants came from the University of Debrecen (Hungary), 12 from the University of Silesia in Katowice (Poland), 2 from the Łódź University of Technology (Poland) and one from each of the universities: Pedagogical University of Cracow (Poland) and University of Miskolc (Hungary).

Professor Zsolt Páles opened the Seminar and welcomed the participants to Hajdúszoboszló.

The scientific talks presented at the Seminar focused on the following topics: equations in a single variable and in several variables, iterative equations, equations on algebraic structures, functional inequalities, Hyers–Ulam stability, functional equations and inequalities involving mean values, generalized convexity, Walsh–Fourier analysis and fixed point theory. Interesting discussions were generated by the talks.

The social program included a Festive Dinner. Furthermore, the participants had the opportunity to take advantage of the use of the thermal bath located in the hotel.

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The closing address was given by Professor Maciej Sablik. His invitation to the Nineteenth Katowice–Debrecen Winter Seminar on Functional Equations and Inequalities in January 2019 in Poland was gratefully accepted.

Summaries of the talks in alphabetic order of the authors follow in the first section and the list of participants in the second section.

1. Abstracts of talks

Roman Badora: *Remarks on the stability of two conditional Cauchy's equations on $C(X)$*

First we consider the stability problem of orthogonally additive functionals on $C(X)$

$$x \cdot y = 0 \Longrightarrow f(x+y) = f(x) + f(y).$$

Next, we discuss a form of a functional f on the algebra $C(X)$ satisfying

$$x \cdot y = 0 \Longrightarrow |f(x+y) - f(x)f(y)| \leq \varepsilon$$

for some $\varepsilon \geq 0$.

Mihály Bessenyei: *Axiomatic and algebraic convexity of regular pairs*

Two dimensional Chebyshev systems, quoted also as regular pairs, induce convex structures both in an axiomatic and in an algebraic way. The aim of this talk is to link these structures by showing that they coincide.

Zoltán Boros: *Functional equation for affine differences*

Let $\mathbb{F}$ denote a subfield of the real number field, X be a linear space over $\mathbb{F}$, and D be an $\mathbb{F}$-convex subset of X. Moreover, for every $n \in \mathbb{N}$, let

$$T_n = \left\{ (t_1, \dots, t_n) \in ([0,1] \cap \mathbb{F})^n \, : \, \sum_{j=1}^{n} t_j = 1 \right\} \text{ and } G_n \colon T_n \times D^n \to \mathbb{R}\,.$$

We establish that there exists a function $\phi \colon D \to \mathbb{R}$ such that

$$\phi\left(\sum_{i=1}^{n} t_i x_i \right) = \sum_{i=1}^{n} t_i \phi(x_i) + G_n(t_1, \dots, t_n; x_1, \dots, x_n)$$

holds for every $n \in \mathbb{N}$, $x_1, \dots, x_n \in D$ and $(t_1, \dots, t_n) \in T_n$ if and only if the sequence (G_n) satisfies the functional equation

$$\begin{aligned} G_n\Bigg(s_1,\dots,s_n\,;\,\sum_{j=1}^{m} t_{1j}x_j\,,\,\dots\,,\,\sum_{j=1}^{m} t_{nj}x_j\Bigg) \\ +\sum_{i=1}^{n} s_i G_m\left(t_{i1}\,,\,\dots\,,\,t_{im}\,;\,x_1\,,\,\dots\,,\,x_m\right) \\ = G_m\left(\sum_{i=1}^{n} s_i t_{i1}\,,\,\dots\,,\,\sum_{i=1}^{n} s_i t_{im}\,;\,x_1\,,\,\dots\,,\,x_m\right) \end{aligned}$$

for every $n, m \in \mathbb{N}$, $x_1, \dots, x_m \in D$, $(s_1, \dots, s_n) \in T_n$ and $(t_{i1}, \dots, t_{im}) \in T_m$ $(i = 1, 2, \dots, n)$.

Pál Burai: *Functional equations involving generalized quasi-arithmetic means* (Joint work with Justyna Jarczyk)

Let $I \subset \mathbb{R}$ be an interval. Given a strictly monotonic, continuous function $\varphi\colon I \to \mathbb{R}$ and a probability Borel measure μ on $[0,1]$, the generalized quasi-arithmetic mean $A_{\varphi,\mu}\colon I^2 \to I$, introduced by Makó and Páles, is defined by

$$A_{\varphi,\mu}(x,y) = \varphi^{-1}\left(\int_0^1 \varphi\left(tx + (1-t)y\right) d\mu(t)\right).$$

The symmetrized version of the previously defined mean is

$$A^*_{\varphi,\mu}(x,y) = \frac{A_{\varphi,\mu}(x,y) + A_{\varphi,\mu}(y,x)}{2}.$$

In this talk we investigate some functional equations involving generalized quasi-arithmetic and symmetrized generalized quasi-arithmetic means.

Borbála Fazekas: *Double Walsh-Fourier series solutions of second order partial differential equations* (Joint work with György Gát)

Walsh functions $\omega_0, \omega_1, \dots$ are step functions with range $\{-1, 1\}$ defined on the interval $[0, 1[$. More precisely, they are given via

$$\omega_n(x) = \prod_{k=0}^{\infty} (-1)^{n_k x_k}\,,$$

with $n \in \mathbb{N}, n = \sum_{k=0}^{\infty} n_k 2^k, x \in [0,1[, x = \sum_{k=0}^{\infty} \frac{x_k}{2^{k+1}}$. The n-th double Walsh-Fourier series approximation of an integrable function u on $[0,1[^2$ is defined by

$$\sum_{i=0}^{n}\sum_{j=0}^{n} \hat{u}(i,j)\omega_i(x)\omega_j(t), \quad \text{with } \hat{u}(i,j) = \int_0^1 \int_0^1 u(x,t)\omega_i(x)\omega_j(t)\, dxdt.$$

One can look for an approximate solution of a two dimensional partial differential equation in the above form by solving a suitably discretised equation. We apply this approach to the initial and boundary value problem for the wave equation, i.e. for the following problem

$$\begin{aligned} u_{tt}(x,t) - u_{xx}(x,t) &= f(x,t), \quad x,t \in]0,1[, \\ u(x,0) &= \varphi(x), \quad x \in [0,1], \\ u_t(x,0) &= \psi(x), \quad x \in [0,1], \\ u(0,t) = u(1,t) &= 0, \quad t \in [0,1]. \end{aligned}$$

References

[1] Gát G., Toledo R., *Estimating the error of the numerical solution of linear differential equations with constant coefficients via Walsh polynomials*, Acta Math. Acad. Paedagog. Nyházi. (N.S.) **31** (2015), no. 2, 309–330.
[2] Shih Y.P., Han J.Y., *Double Walsh series solution of first-order partial differential equations*, Internat. J. Systems Sci. **9** (1978), no. 5, 569–578.

Włodzimierz Fechner: *A Sincov type equation related to fuzzy implications* (Joint work with Michał Baczyński)

A fuzzy implication is a mapping $I\colon [0,1]\times[0,1]\to[0,1]$ such that I is non-increasing with respect to the first variable, non-decreasing with respect to the second variable, $I(0,0) = I(1,1) = 1$ and $I(1,0) = 0$.

We will describe solutions of a Sincov-type equation in the class of fuzzy implications:

$$I(x,y)\cdot I(y,z) = I(x,z), \tag{1}$$

assumed for all $x,y,z \in [0,1]$ such that $1 > x > y > z > 0$. Equation (1) is motivated by recent studies of Sebastia Massanet, Jordi Recasens and Joan Torrens on the so-called T-power based implications [1, 2].

References

[1] Massanet S., Recasens J., Torrens J., *Fuzzy implication functions based on powers of continuous t-norms*, Internat. J. Approx. Reason. **83** (2017), 265–279.
[2] Massanet S., Recasens J., Torrens J., *Some characterizations of T-power based implications.* Preprint.

Żywilla Fechner: *Remarks on spherical functions on affine groups* (Joint work with László Székelyhidi)

We are going to discuss the notion and some properties of spherical functions on special type of hypergroups, namely affine groups. Let V be the n dimensional vector space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, $\mathrm{GL}(V)$ the general linear group of V and K a compact subgroup of $\mathrm{GL}(V)$. Let ω be the normalized Haar measure on K. Recall that the continuous K-invariant function $s\colon V \to \mathbb{C}$ is called a *K-spherical function* on V, if it is non-identically zero and satisfies

$$\int_K s(u + k \cdot v)\, d\omega(k) = s(u)s(v), \quad u, v \in V. \tag{1}$$

Our aim is to describe solutions of (1) for specific compact subgroups of $\mathrm{GL}(V)$.

References

[1] van Dijk G., *Introduction to Harmonic Analysis and Generalized Gelfand Pairs*, de Gruyter Studies in Mathematics, **36**, Walter de Gruyter & Co., Berlin, 2009.
[2] Fechner Ż., Székelyhidi L., *Spherical and moment functions on the affine group $SU(2)$.* Preprint.
[3] Székelyhidi L., *Spherical spectral synthesis*, Acta Math. Hungar. **153** (2017), no. 1, 120–142.

György Gát: *Recent results on convergence of two-dimensonal Walsh–Fourier series*

Let x be an element of the unit interval $I := [0, 1)$. The $\mathbb{N} \ni n$-th Walsh function is

$$\omega_n(x) := (-1)^{\sum_{k=0}^{\infty} n_k x_k} \quad (n = \sum_{k=0}^{\infty} n_k 2^k,\ x = \sum_{k=0}^{\infty} \frac{x_k}{2^{k+1}}).$$

The aim of the talk is to give a short resumé of the recent results with respect to the almost everywhere convergence and divergence issue of two-dimensional Walsh-Fourier series. For a non-negative integer n denote its dyadic variation by $V(n) := \sum_{j=0}^{\infty} |n_j - n_{j+1}| + n_0$. Gát and Goginava recently proved (see [3]) that for a function $f \in L\log L\left(I^2\right)$ under the condition $\sup_{n_A} V\left(n_A\right) < \infty$, the subsequence of quadratic partial sums $S^{\square}_{n_A} f$ of two-dimensional Walsh-Fourier series converges to the function f almost everywhere. Besides, this result is sharp. Namely, for any measurable function $\varphi\colon [0,\infty) \to [0,\infty)$ such that $\varphi(u) = o(u\log u)$ as $u \to \infty$ there exists a sequence $\{n_A : A \geq 1\}$ with condition $\sup_{n_A} V\left(n_A\right) < \infty$ and a function $f \in \varphi(L)(I^2)$ for which $\sup_{n_A} \left|S^{\square}_{n_A} f(x)\right| = \infty$ for almost all $x \in I^2$.

References

[1] Gát G., *Almost everywhere convergence of Fejér means of two-dimensional triangular Walsh-Fourier series*, J. Fourier Anal. Appl. (2017) DOI: 10.1007/s00041-017-9566-2.

[2] Gát G., Goginava U., *Almost Everywhere Convergence of lacunary sequence of triangular partial Sums of Double Walsh-Fourier series.* Preprint.

[3] Gát G., Goginava U., *Almost everywhere convergence of subsequence of quadratic partial sums of two-dimensional Walsh-Fourier series*, Anal. Math. **44** (2018), no. 1, 73–88.

Roman Ger: *On a conditional d'Alembert's equation* (Joint work with Radosław Łukasik)

The following question was asked by a French mathematician Roger Cuculière (*Problem 11998*, The American Mathematical Monthly **124** no. 7 (2017)):
Find all continuous functions $f\colon \mathbb{R} \to \mathbb{R}$ that satisfy $f(z) \leq 1$ for some nonzero real number z and

$$f(x)^2 + f(y)^2 + f(x+y)^2 - 2f(x)f(y)f(x+y) = 1 \tag{1}$$

for all real numbers x and y.

We will present the general continuous solution of the functional equation (1) (without assuming that $f(z) \leq 1$ for some $z \in \mathbb{R}\setminus\{0\}$), a general solution of the corresponding conditional d'Alembert's equation

$$f(x+y) \neq f(x-y) \Longrightarrow f(x+y) + f(x-y) = 2f(x)f(y), \tag{2}$$

as well as an application of the generated ideal method regarding equation (2).

Attila Gilányi: *Bernstein–Doetsch type theorems for set-valued functions* (Joint work with Carlos González, Kazimierz Nikodem and Zsolt Páles)

During the last more than one hundred years, generalizations of the well-known Bernstein–Doetsch Theorem (cf. [1]) were investigated by several authors. In this talk, we consider strongly as well as approximately Jensen convex (and also Jensen concave) set-valued maps and we present Bernstein–Doetsch type theorems with so-called Tabor type error terms for them. In connection with some details of our results, we refer to the paper [2].

References

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Eszter Gselmann: *Pexiderized functional equations with and without spectral methods* (Joint work with Gergely Kiss, Miklós Laczkovich and Csaba Vincze)

In this talk I would like to present those results that were achieved jointly with Gergely Kiss, Miklós Laczkovich and Csaba Vincze in the series of papers [1, 2, 3]. In the first part of the talk we will focus on solving functional equations of the form

$$\sum_{k=1}^{n} x^{p_k} f_k(x^{q_k}) = 0, \quad x \in R,$$

where p_k and q_k $(k = 1, \ldots, n)$ are given nonnegative integers and the unknown functions $f_1, \ldots, f_n \colon R \to R$ are supposed to be additive on the ring R.

After that, we will present some results concerning vectorial spectral analysis and synthesis. This will allow us to investigate equations of the form

$$\sum_{i=1}^{n} f_i(a_i x + b_i y) = 0, \quad x, y \in \mathbb{K},$$

where $\mathbb{K}$ is a finitely generated field over $\mathbb{Q}$.

Finally, in the last part of the talk, we will prove that if the additive functions $f_1, \ldots, f_n \colon \mathbb{K} \to \mathbb{C}$ fulfill functional equation

$$\sum_{i=1}^{n} f^{q_i}\left(x^{p_i}\right) = 0, \quad x \in \mathbb{K},$$

then all the involved functions are linear combinations of homomorphisms from $\mathbb{K}$ to $\mathbb{C}$.

References

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[2] Gselmann E., Kiss G., Vincze C., *On functional equations characterizing derivations: methods and examples*, Results Math. **73** (2018), no. 2, Art. 74, 27 pp.
[3] Gselmann E., Kiss G., Vincze C., *Characterization of field homomorphisms through Pexiderized functional equations*. Preprint 2018, 15 pp.

Tibor Kiss: *On a functional equation involving the arithmetic mean* (Joint work with Zsolt Páles)

Let I be a nonempty open subinterval of $\mathbb{R}$ and consider the functional equation

$$\varphi\Big(\frac{x+y}{2}\Big)(f(x)-f(y))=0, \quad x,y\in I,$$

concerning the unknown functions $\varphi\colon I\to\mathbb{R}$ and $f\colon I\to\mathbb{R}$. The aim of the talk is to determine all solution pairs (φ,f) assuming only that the set of zeros of the function φ is closed in I.

Gábor Lucskai: *Almost everywhere convergence of Cesàro means of Fourier series on the group of* 2*-adic integers* (Joint work with György Gát)

Let x be an element of the unit interval $[0,1)$. The n^{th} element of the character system of the group of 2-adic integers is:

$$v_n := \prod_{n=0}^{\infty} v_{2^j}^{n_j}, \quad \text{where} \quad v_{2^j}(x) := \exp\Big(2\pi\imath\Big(\frac{x_j}{2}+\dots+\frac{x_0}{2^{j+1}}\Big)\Big)$$

and $\imath := \sqrt{-1}$, $n=\sum_{i=0}^{\infty} n_i 2^i$, $x=\sum_{i=0}^{\infty} x_i 2^{-(i+1)}$ $(n_i,x_i\in\{0,1\}, i\in\mathbb{N})$.

There was a question of Taibleson [4] which was unanswered for 25 years. He asked whether the Fejér-Lebesgue theorem, i.e. $\sigma_n^1 f\to f$ almost everywhere holds for all integrable functions f. In 1997 ([2]), Gát answered the question in the affirmative. In 2009, Blahota and Gát ([1]) verified that result for the two-dimensional case.

In 2007 ([3]), he proved the almost everywhere convergence of the Cesàro (C,α) means of integrable functions $\sigma_n^\alpha f\to f$ for $f\in L^1(I)$ and for every $\alpha>0$. As the result of the joint work with Gát, we proved that the Cesàro (C,α) means converge almost everywhere also in the two dimensional case. In the talk, the main steps of the proof will be discussed.

References

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[3] Gát G., *Almost everywhere convergence of Cesàro means of Fourier series on the group of 2-adic integers*, Acta Math. Hungar. **116** (2007), no. 3, 209–221.
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Radosław Łukasik: *Functional equations connected with the Cauchy mean value theorem*

In this talk we describe the solution (f, g) of the equation

$$[f(x) - f(y)]g'(\alpha x + (1-\alpha)y) = [g(x) - g(y)]f'(\alpha x + (1-\alpha)y), \quad x, y \in I,$$

where $I \subset \mathbb{R}$ is an open interval, $f, g \colon I \to \mathbb{R}$ are differentiable, α is a fixed number from $(0,1)$. This equation was considered in [1] for three times differentiable functions on $\mathbb{R}$.

Reference

[1] Balogh Z.M., Ibrogimov O.O., Mityagin B.S., *Functional equations and the Cauchy mean value theorem*, Aequationes Math. **90** (2016), no. 4, 683–697.

Judit Makó: *Some remarks on h-convexity* (Joint work with Attila Házy)

Let D be a nonempty, convex subset of X, $c > 0$ and $\alpha \colon (D - D) \to \mathbb{R}$ be an error function. We say that a function $f \colon D \to \mathbb{R}$ is (c, α)-Jensen convex if, for all $x, y \in D$,

$$f\Big(\frac{x+y}{2}\Big) \le cf(x) + cf(y) + \alpha(x - y). \tag{1}$$

In [1], we examined the functional inequality (1), namely, we introduced Bernstein–Doetsch type results and Hermite–Hadamard type inequalities. In this talk, we give further properties for (c, α)-Jensen convexity and h-convexity.

Reference

[1] Házy A., Makó J., *On (c, α)-Jensen convexity*. Preprint 2018.

Gyula Maksa: *On the alienation of the Pexiderized additive Cauchy equation and the Pexiderized logarithmic equation*

The motivation of this talk is a paper of R. Ger where the functional equation

$$f(x+y)+g(xy)=f(x)+f(y)+g(x)+g(y)$$

was investigated in a general setting and it was proved that the additive and the logarithmic Cauchy equations are alien up to an additive constant.

In this talk, we discuss the alienation of the Pexiderized version of the additive and the logarithmic Cauchy equations on the set of positive real numbers and show that the situation is quite different in this case.

Janusz Morawiec: *Means of iterates* (Joint work with Szymon Draga)

We determine all continuous bijections f, acting on a real interval into itself, whose k-fold iterate is the quasi-arithmetic mean of all its subsequent iterates from f^0 up to f^n (where $0 \le k \le n$). Namely, we prove that such functions are piecewise affine.

Gergő Nagy: *Isometries on structures of linear operators*

In this talk, we collect some known results concerning the general forms of isometries on classical structures of bounded linear operators acting on Hilbert spaces. Beside these statements, we present a general question of Tingley about the extendibility of isometries between unit spheres in normed spaces. This problem has been solved for several normed spaces, e.g. the space $B(H)$ of linear operators on a finite dimensional complex Hilbert space H. We review the theorem which provides the solution, it describes the structure of surjective isometries of the unit sphere in $B(H)$. Finally, we present a new result of ours in which the general form of all isometries of the space formed by the positive operators in that sphere was determined.

Andrzej Olbryś: *On a problem of Tomasz Szostok*

In our talk we present a solution to the problem posed by Tomasz Szostok who asked about the solutions $f, F\colon (a,b) \to \mathbb{R}$ to the system of inequalities

$$f\Big(\frac{x+y}{2}\Big) \le \frac{F(y)-F(x)}{y-x} \le \frac{f(x)+f(y)}{2}.$$

It turns out that f and F are the solutions to the above system of inequalities if and only if f is a convex function and F is primitive of f.

Zsolt Páles: *On derivations with additional properties*

A function $d\colon \mathbb{R} \to \mathbb{R}$ is called a *derivation* if, for all $x, y \in \mathbb{R}$,

$$d(x+y) = d(x) + d(y) \quad \text{and} \quad d(xy) = yd(x) + xd(y).$$

It is a nontrivial fact that, for any non-algebraic number $t \in \mathbb{R}$, there exists a derivation which does not vanish at t. Nonzero derivations have many striking applications in the theory of functional equations and functional inequalities. Derivations derivate many of the elementary functions. For instance, if $f : I \to \mathbb{R}$ is the ratio of two polynomials with algebraic coefficients, then, for every $x \in I$,

$$d(f(x)) = f'(x)d(x).$$

It has been an old problem of the theory of functional equations whether there exists a nonzero derivation which derivates the exponential function or any of the trigonometric functions in the above sense. Our main result shows that the answer to this problem is affirmative.

Maciej Sablik: *A competitive method of solving functional equations*

In the paper P. Carter, D. Lowry-Duda, *On functions whose mean value abscissas are midpoints, with connections to harmonic functions* that appeared in Amer. Math. Monthly **124** (2017), no. 6, 535–542, the authors prove the following result.

Theorem. *Fix a $\lambda \in (0,1)$, and assume that $f\colon \mathbb{R} \to \mathbb{R}$ satisfies*

$$\frac{f(b) - f(a)}{b - a} = f'\left(\lambda a + (1-\lambda)b\right)$$

for all $a < b$. Then f is a quadratic polynomial. Moreover, if $\lambda \neq \frac{1}{2}$ then f is a linear polynomial.

In the proof the authors use the triple differentiability of f which obviously can be omitted. We recall classical results allowing to obtain the above theorem without excessive regularity assumptions and with f' replaced by an arbitrary function.

Ekaterina Shulman: *On polynomial-like scalar functions on non-commutative groups*

Let G be a topological group, R_h the right shift on the space $C(G)$

$$R_h f(x) = f(xh),$$

and let Δ_h be the difference operator on $C(G)$

$$\Delta_h = R_h - I.$$

We investigate relations between two classes of "polynomial like" continuous functions on G: solutions of the Fréchet functional equation

$$\Delta_{h_{n+1}} \Delta_{h_n} \cdots \Delta_{h_1} f = 0, \qquad \text{for any} \ \ h_1, \cdots, h_{n+1} \in G$$

and solutions of the equation for iterated differences

$$\Delta_h^{n+1} f = 0, \qquad \text{for any} \ \ h \in G.$$

Justyna Sikorska: *On an equation of Sophie Germain* (Joint work with Radosław Łukasik and Tomasz Szostok)

We deal with the following functional equation

$$f(x)^2 + 4f(y)^2 = \big(f(x+y) + f(y)\big)\big(f(x-y) + f(y)\big)$$

which is motivated by the well known Sophie Germain identity. Some connections as well as some differences between this equation and the quadratic functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y)$$

are presented.

Patricia Szokol: *Stability of a functional equation on Banach lattices* (Joint work with Nutefe Kwami Agbeko)

We obtained a generalization of the stability of some Banach lattice-valued functional equation with the addition replaced in the Cauchy functional equation by lattice operations and their combinations.

Tomasz Szostok: *Inequalities for Dragomir's mappings*

We examine some known inequalities involving mappings introduced by S.S. Dragomir. We use the recently developed method connected with convex orderings and Stieltjes integral. It turns out that some of these inequalities are optimal whereas other may be substantially improved.

FILIP TUROBOŚ: *Some remarks about semimetrics and Frink's metrization technique* (Joint work with Katarzyna Chrząszcz, Jacek Jachymski)

Recently, semimetric spaces, i.e. spaces fulfilling the axioms of a metric space excluding the triangle inequality became a subject of research of many authors. The natural idea in this topic is to consider some weaker assumptions than the triangle inequality. This leads to the concept of a regular semimetric space introduced by Páles and Bessenyei in [1].

DEFINITION. We call a semimetric space (X, d) *regular* if there exists a function $\Phi\colon [0,\infty)^2 \to [0,\infty]$ such that $\Phi(0,0) = 0$, Φ is symmetric, continuous at the origin, nondecreasing and for all $x, y, z \in X$,

$$d(x,z) \le \Phi(d(x,y), d(y,z)).$$

Firstly, we will briefly recall a characterisation of such spaces given in our previous paper [2] which states, in particular, that every regular semimetric space is uniformly metrizable. Secondly, we will present Frink's metrization technique for some special subclass of class of semimetric spaces called K-quasimetric spaces.

DEFINITION. A semimetric space (X, d) for which there exists $K > 0$ such that

$$\forall_{x,y,z\in X} \quad d(x,z) \leqslant K \cdot \max\{d(x,y),\ d(y,z)\}$$

is called a *K-quasimetric space.*

The method proposed by Frink works properly provided that K does not exceed 2. In fact, Frink in his paper [3] proved that the metric ρ obtained from his procedure applied to a K-quasimetric space (X, d) satisfies the following inequalities:

$$\forall_{x,y\in X} \quad \rho(x,y) \leqslant d(x,y) \leqslant 4 \cdot \rho(x,y).$$

Lastly, the optimisation for the bounds will be provided. We will show that the upper bound in the estimation above can be relaxed to $K^2 \cdot \rho(x, y)$. Alongside, we will provide an example which proves, that the bounds proposed in our version of Frink's theorem cannot be relaxed by any means.

References

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Paweł Wójcik: *The Daugavet equation in some Banach spaces*

Let X be a Banach space. Suppose that $T \in \mathcal{L}(X)$ is a bounded linear operator. The aim of this report is to discuss a necessary and sufficient condition for the Daugavet equation $\|I + T\| = 1 + \|T\|$. We also discuss the notions of (acs) and (luacs).

Amr Zakaria: *Equality and homogeneity of generalized Bajraktarević means* (Joint work with Zsolt Páles)

Given two continuous functions $f, g\colon I \to \mathbb{R}$ such that g is positive and f/g is strictly monotone, a measurable space $(T, \mathcal{A})$, a measurable family of d-variable means $m : I^d \times T \to I$, and a probability measure μ on the measurable sets $\mathcal{A}$, the d-variable mean $M_{f,g,m;\mu} : I^d \to I$ is defined by

$$M_{f,g,m;\mu}(\boldsymbol{x}) := \left(\frac{f}{g}\right)^{-1} \left(\frac{\int_T f\big(m(x_1, \ldots, x_d, t)\big)\,\mathrm{d}\mu(t)}{\int_T g\big(m(x_1, \ldots, x_d, t)\big)\,\mathrm{d}\mu(t)}\right), \qquad \boldsymbol{x} = (x_1, \ldots, x_d) \in I^d.$$

The aim of this paper is to solve the equality and homogeneity problems of these means, i.e., to find conditions for the generating functions (f, g) and (h, k), for the family of means m, and for the measure μ such that the equality

$$M_{f,g,m;\mu}(\boldsymbol{x}) = M_{h,k,m;\mu}(\boldsymbol{x}), \quad \boldsymbol{x} \in I^d,$$

and the homogeneity property

$$M_{f,g,m;\mu}(\lambda\boldsymbol{x}) = \lambda M_{f,g,m;\mu}(\boldsymbol{x}), \qquad \lambda > 0,\ \boldsymbol{x}, \lambda\boldsymbol{x} \in I^d,$$

respectively, are satisfied.

Thomas Zürcher: *On a functional equation by Matkowski–Wesołowski*

Assume that $\varphi\colon [0,1] \to [0,1]$ is a function. Both, Janusz Matkowski and Jacek Wesołowski, were interested in the following functional equation:

$$\varphi(x) = \varphi\left(\frac{x}{2}\right) + \varphi\left(\frac{x+1}{2}\right) - \varphi\left(\frac{1}{2}\right).$$

They formulated their interest differently, but essentially their question boiled down to whether there is a continuous solution φ (different from the identity) that is increasing and keeps 0 and 1 fixed.

My talk is about my journey together with Janusz Morawiec into studying this equation and its generalizations. It turns out that this topic is connected to many other fields.

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(Compiled by Eszter Gselmann)