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## A Research Based On POT-CAViaR Model of Extreme Risk Measure

Baoshuai Zhang, Jun Duan <sup>†</sup>School of Economics and Management, Chongqing Normal University, Chongqing 401331  
China

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**Abstract** In this paper, the CAViaR model was used to measure the HS300, HSI and N225 risk levels and result showed that CAViaR model was able to measure the sample risk value at the 5% confidence level. However, at the 1% confidence level, the CAViaR model severely underestimated the sample risk value. In order to solve this question, this paper introduces the POT model and CAViaR model to construct the extreme risk measure model based on POT-CAViaR model and the test shows that the improved model can effectively improve the accuracy of extreme risk prediction and the model validity is strengthened.

**Keywords:** CAViaR Model; POT Model; Extreme Risk**AMS 2010 codes:** 03C30.

## 1 Introduction

The value at risk (VaR) emerged as the times required in the 1990s to deal with the financial market crisis like the Wall Street stock market crashed in 1987 and European Monetary System collapsed in 1992. Currently, VaR has become reference point of market risk in the risk management. Value at risk is to measure the maximum potential loss of an investment portfolio within a given period under certain probability level [1].

Despite the sample concept of VaR model, it remains a challenging problem to estimate it accurately [2]. By now, various methods have been developed to predicate VaR; however, no method has provided a satisfactory solution yet [3], as the distribution of typical investment portfolio has changed for a long time. The most common method is to capture the dynamic volatility of residuals through the complete parameter time series model, i.e. ARCH or GARCH [4]. The primary weakness of this method is the distribution of the yield residual to be assumed in advance [5]. Both the GARCH and ARCH models are assumed as in normal distribution at the very beginning; afterwards, taking the leptokurtosis and fat-tail characteristics of financial income series into consideration, i.e. to be replaced by Student-t distribution, is widely accepted as effective [6, 7]. However, so far, there is still no unified answer to how to select the optimal fitting residual term of distribution. In term

<sup>†</sup>Corresponding author.Email address: [110048327@qq.com](mailto:110048327@qq.com)

of the non-parameterized method, the most popular method is historical simulation. This method can take the empirical quantile of historical return as the estimation of VaR. Although this method is manageable, it might be unstable to the VaR estimation, especially the VaR of extreme value point.

On the other hand, there emerged a new method of measuring the financial risk through quantile regression in the recent years. This method has been deeply researched by scholars at home and abroad as it considers no distribution of residual sample at the measurement of risk while reflecting the retail characteristics to some extent, which has provided a satisfying statistical method for fitting the financial data with the leptokurtosis and fat-tail characteristics. Chen (2002) improved the traditional quantile regression VaR model [8]. Talyor (2008), Badshah (2013), Westerberg (2014) believe that when applying the quantile regression model to measure risk, the quantile model estimation effect is better [9–11]. Engle and Manganelli proposed the CAViaR model and quantile regression model [12], this model assumed that the conditional quantile was the self-regression process and the sample distribution needn't be assumed at the estimation of VaR but modeled the quantile directly; according to the experimental evidence, compared with other VaR models, the CAViaR model was rather competitive [13, 14]. However, CAViaR model reveals some problems during estimating VaR; for the estimation of high quantile, as a result of the data sparseness in the tail area. The CAViaR model coefficients are unstable and the quantile regression is always not robust, which will lead to difficulties in predicating the extreme value risk.

Summarizing above the analyses, the quantile regression-based CAViaR model is semiparametric model without assuming the yield distribution in advance; it is of favorable statistic characteristics [15, 16]. What's more, no matter from establishment, predication and test, the quantile regression model has rather complete theoretical evidence, while it is difficult for CAViaR model to estimate the high quantile VaR [17]. What's more, compared with the traditional model methods, POT model is more suitable to the predication of high quantile point in fat-tail distribution and its predication result is relatively stable [18]. As a result, it is a good research direction to estimate the risk according to the CAViaR model and POT model; yet there are less research documents on this aspect at domestic and many problems to be solved [19].

In general, the innovation of the article is mainly reflected in the following two aspects: first, in order to avoid the assumption of the overall distribution of the sample and fully reflect the characteristics of the tail of the sample, a risk measurement model based on the CAViaR-POT model is constructed; secondly, the application of CSI 300 index, Hang Seng index and N225 index test the effect of the model.

## 2 Model Construction

### 2.1 VaR Model

This paper assumes value of a financial market at the time of  $t$  is  $V_t$ , the loss on the subsequent period shall be:

$$L_t(l) = V_{t+1} - V_t \quad (1)$$

Assume the cumulative distribution function of random variable  $L_t(l)$  is  $F_L$ , under the probability level  $P$ , this paper defines  $VaR_{1-p}$  as a possible maximum loss:

$$VaR_{1-p} = \inf \{x | F_L(x) \geq 1 - p\} \quad (2)$$

### 2.2 CAViaR Model

Engle and Manganelli proposed the CAViaR method in 2004, the predication effect of VaR was improved somewhat for this method considering no distribution shape but modeled the data directly with the quantile theory [12]. Financial assets income series are represented by  $Y$ , the corresponding distribution function shall be  $F_Y(y) = P_r(Y \leq y)$ ; according to the relationship between quantile and VaR, the opposite number of  $P$  quantile

of the random variable  $Y$  is numerically equal to the VaR under the reliability level  $(1-p)$ , which means,  $Q_{\bar{y}}(P) = VaR_{\bar{y}}(1 - P)$ , so that the CAViaR model in general pattern shall be:

$$Q_t(P) = \beta_1 + \sum_{i=1}^m \beta_{2i} Q_s(P) + \sum_{j=1}^n \beta_{3i}(t-j) \quad (3)$$

In which,  $\theta = (\beta_1, \beta_{21} \cdots \beta_{2i}, \beta_{31} \cdots \beta_{2i})$  is the parameter vector to be estimated,  $L(t-j)$  is the function form of the lag information set; according to the form in (3), Engle and Manganelli [12] proposed the following common CAViaR models:

$$Q_1(P) \begin{cases} \beta_1 + \beta_2 Q_{1-1}(P) + \beta_3 |y_{t-1}| & \text{SAV} \\ \beta_1 + \beta_2 Q_{1-1}(P) + \beta_3 \max(y_{t-1}, 0) + \beta_4 \min(y_{t-1}, 0) & \text{AS} \\ \sqrt{\beta_1 + \beta_2 Q_{t-1}^2(P) + \beta_3 y_{t-1}^2} & \text{IGARCH} \end{cases} \quad (4)$$

The SAV model is also called the absolutely symmetrical model indicating the same impact of good news and bad news to VaR; AS model is also called the non-symmetrical model indicating the difference of bad news to VaR.

For the CAViaR model estimation, the minimum absolute deviation model proposed by Koenker and Bassett [20] and the optimization algorithm proposed by Engle and Manganelli [12] are combined to estimate parameter  $\theta$ . Assume there exists a quantile equation:

$$y_t = f_t(\beta) + \varepsilon_{t\theta} \quad (5)$$

In which  $Q_\theta(\varepsilon_{t\theta}) = 0$ ,  $\varepsilon \sim N(0, 1)$ ,  $Q_\varepsilon$  is the quantile equation, so that the estimation function of the quantile regression equation can be defined as follows:

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{T} \sum_{i=1}^T P - I(y_t < Q_t(P; \theta)) [y_t - Q_t(P; \theta)] \quad (6)$$

In this formula,  $T$  is the total sample, and the corresponding conditional quantile  $Q_t(P; \theta)$  can be estimated after estimating the parameter  $\theta$  to estimate VaR.

### 2.3 POT-CAViaR Model

When  $p$  is not at the extreme quantile level, the quantile level can be obtained by minimizing the objective function; however, when the quantile level is approaching to 1 or 0, the quantile regression will not be robust enough because the tail data, especially the thick-tail distribution is rather sparse. To solve this problem, the POT model of extreme value theory is introduced in this paper to combine the CAViaR and POT model organically through a certain method to construct the POT-CAViaR-based model. The specific process is as follows:

Firstly, the estimated quantile value  $Q_t(p_1)$  under the  $P_1$  quantile point shall be estimated through the CAViaR model and standardized to obtain the residual quantile series of extreme value:

$$Z_t = \frac{\varepsilon_t(p_1)}{Q_t(p_1)} = \frac{y_t - Q_t(p_1)}{Q_t(p_1)} = \frac{y_t}{Q_t(p_1)} - 1 \quad (7)$$

In which,  $\varepsilon_t(p_1)$  refers to the residual of quantile point  $p_1$  at  $t$  time point, and  $y_t$  represents the value of sample series at  $t$  time point.

Based on constructing the quantile residuals of standard extreme values, it is assumed that there exists the quantile point  $p_2$ ,  $p_1 > p_2$ ; the POT model is adopted to estimate it for the corresponding quantile  $Q_t(p_2)$ . According to the related concepts of quantile, it can be obtained that:

$$P_r[y_t \leq Q_t(p_2)] = P_r[y_t \leq Q_t(p_1) - Q_t(p_1) + Q_t(p_2)] = P_r\left(\frac{y_t}{Q_t(p_1)} - 1 \geq \frac{Q_t(p_2)}{Q_t(p_1)} - 1\right) = p_2 \quad (8)$$

It can be obtained through the combination of formula (7) and (8) that:

$$P_r \left[ Z_t \geq \frac{Q_t(p_2)}{Q_t(p_1)} - 1 \right] = p_2 \quad (9)$$

According to the quantile conversion, there will be:

$$Q_{Z_t}(1 - p_2) = \frac{Q_t(p_2)}{Q_t(p_1)} - 1 \quad (10)$$

Formula (10) indicates the quantile level  $Q_{Z_t}(1 - p_2)$  of the standardized extreme value residual series  $Z_t$  under  $1 - p_2$ ; the solution process is also the modelling process of  $Z_t$  through POT model. The key point of POT modelling is to determine the threshold, generally through the ways of Hill plot, excess expected function graph and Du Mouchel 10% principle [21]. Considering Du Mouchel 10 is more frequently applied now, in this paper, the Du Mouchel 10% principle is uniformly selected to determine the threshold  $\mu$ ; the functional distribution of the standardized extreme value residual series of the super-threshold shall be:

$$F_u(y) = P(x - u \leq y | x > u) = \frac{F(u + y) - F(u)}{1 - F(u)} \quad (11)$$

When the threshold is big enough, according to the extreme value theory [22], there exists  $F_u(y) \approx G_{\xi, \sigma}(y)$ :

$$F_u(y) \approx G_{\xi, \sigma}(y) = 1 - \left( 1 - \frac{y\xi}{\sigma} \right)^{-\frac{1}{\xi}} \quad \xi \neq 0 \quad (12)$$

The  $G_{\xi, \sigma}(y)$  function is in GPD distribution, in which  $\sigma$  and  $\xi$  respectively refer to the scale parameter and shape parameter; the larger  $\xi$ , the fatter the tail; the value of  $\sigma$  and  $\xi$  can be obtained through the maximum likelihood estimation. Under the condition of  $F_u(y) \approx G_{\xi, \sigma}(y)$ , it can be obtained as follows through the combination of formula (11) and (12):

$$F(y) = F(u) + G_{\xi, \sigma}(y - u)[1 - F(u)] \quad (13)$$

For the convenient study,  $N$  is used to represent the total sample,  $n$  refers to the number of super-threshold, the threshold distribution function  $F(u)$  is represented as  $\frac{N-n}{N}$ , substituting it into formula (13) in combination with the estimated values of  $\sigma$  and  $\xi$ , there will be:

$$\hat{F}(y) = 1 - \frac{n}{N} \left( 1 + \frac{\hat{\xi}(y - u)}{\hat{\sigma}} \right)^{-1/\hat{\xi}} \quad (14)$$

According to formula (2), the risk value under the reliability level  $p$  can be obtained as:

$$VaR_{1-p}^\theta = \hat{F}^{-1}(y) = u - \frac{\sigma}{\xi} \left\{ 1 - \left[ \frac{N}{n}(1 - p) \right]^{-\xi} \right\} \quad (15)$$

Therefore, according to formula (1)-(15) mentioned above, the extreme value risk measurement model based on the POT-CAViaR model can be obtained as follows:

$$\begin{cases} Q_t(P) = \beta_1 + \sum_{i=1}^m \beta_{2i} Q_{ii}(P) + \sum_{j=1}^n \beta_{3j} L(t - j) \\ \hat{\theta} = \arg \min_{\theta} \frac{1}{T} \sum_{i=1}^T P - I(y_i < Q_i(P; \theta)) [y_i - Q_i(P; \theta)] \\ F_u(y) \approx G_{\xi, \sigma}(y) = 1 - \left( 1 - \frac{y\xi}{\sigma} \right)^{-\frac{1}{\xi}} \quad \xi \neq 0 \\ VaR_{1-p}^\theta = \hat{F}^{-1}(y) = u - \frac{\hat{\sigma}}{\hat{\xi}} \left\{ 1 - \left[ \frac{N}{n}(1 - p) \right]^{\hat{\xi}} \right\} \end{cases} \quad (16)$$

**Table 1** Descriptive statistics of samples

Sample	Mean	Max	Min	SD	Skewness	Kurtosis	J-B
HS300	0.0200	3.8786	-4.4987	0.8460	-0.4424	6.4561	1419*
HSI	0.0093	5.8225	-8.3011	0.7212	-0.3047	18.5013	26833*
N225	0.0081	5.7478	-11.1353	0.7359	-1.6032	27.9836	70742*

Notes: \*represent 1% Significant level.

## 2.4 POT-CAViaR Model Test

To test the effect of POT-CAViaR model, this paper adopts the posteriori test method proposed by Kupiec, the basic concept of which is: to compare the estimated value and actual value measured by the model; it fails if it is larger than the actual loss, the failure rate can be obtained by dividing the total days of observation by the total days of failure and then comparing it with the preset VaR; the closer it is to the actual VaR, the better the effect is. Refer to document [23] for the specific details.

## 3 Empirical Test

### 3.1 Data Sources and Descriptive Statistics

To test the model constructed earlier in this paper, the Shanghai-Shenzhen 300 Index (HS300), Hong Kong Hang Seng Index as well as the Nikkei225 Index (N225) are taken as the research samples; in which HS300 Index represents the emerging capital market while the HSI Index and N225 Index refer to the mature capital market.

As the Shanghai-Shenzhen 300 Index was issued since April 8<sup>th</sup>, 2005, the selective interval of samples in this paper shall be: April 8<sup>th</sup>, 2005-March 19<sup>th</sup>, 2017, approximately 12 years with almost 29000 sample points for each index. During the research, the samples are divided into estimation samples and test samples in this paper, in which the interval of estimation samples is April 8<sup>th</sup>, 2005-December 17<sup>th</sup>, 2014 (2176 samples in total); the interval of test samples is December 18<sup>th</sup>, 2014-March 19<sup>th</sup>, 2017 (500 samples in total). The data are from Dazhahui Software; the daily income yield is defined as follows:

$$X_t = 100 (\ln p_t - \ln p_{t-1}) \quad (17)$$

It can be seen from Table 1 that all sample series are of the left-skewed form (skewness <0) with leptokurtosis and fat-tail characteristics (kurtosis >3); what's more, with the J-B test, all sample series under the 1% significance level are significantly different from the normal distribution. To further test the distribution characteristics of the sample series, QQ graph of various sample series (Fig.2). Taking the leftmost HS300 as an example, the upper and lower tails of series HS300 are obviously derived from the normal distribution and presenting the fat-tail characteristics; it can be concluded that the sample series presenting the typical "leptokurtosis and fat-tail characteristics" characteristics. According to Fig.1, compared with HSI series and N225 series, the HS300 series have more extreme values than the latter; generally, the sample series are of non-symmetric distribution and typical "leptokurtosis and fat-tail characteristics of financial yield in addition to certain explosion and agglomeration characteristics."

### 3.2 CAViaR Model-Based Estimation and Test

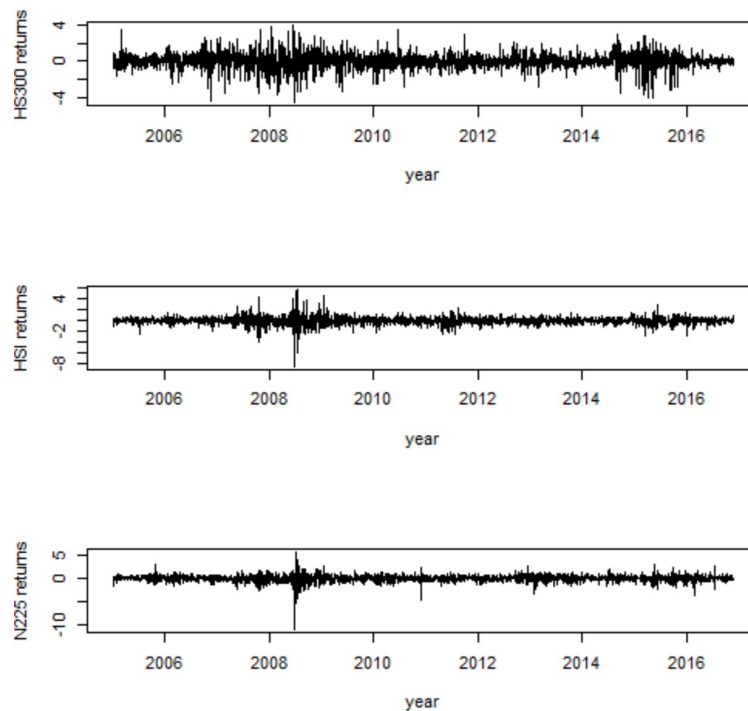
Considering the satisfying characteristics of CAViaR model, the SAV model, AS model and IGARCH model mentioned earlier are adopted in this paper to fit the HS 300, HSI and N225 income yield series; the specific results are shown in Table 2.

According to Table 2,  $\beta_2$  is completely significant, it means that tail quantile of the sample series has ob-

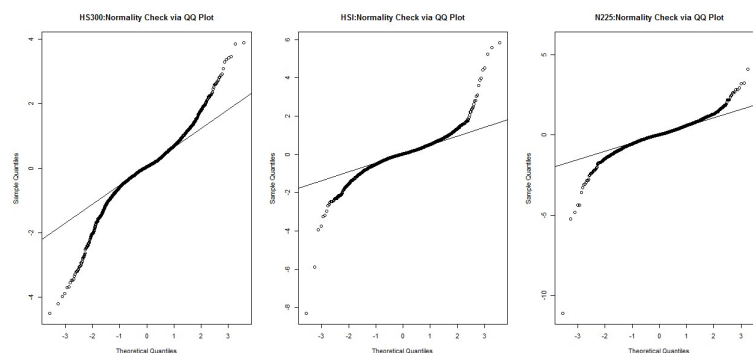
**Table 2** Quantile parameter estimation results of CAViaR class model

		$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
HS300	1%	0.0729** (0.0338)	0.9440* (0.0381)	0.2393* (0.0905)	-
	5%	0.0155* (0.0033)	0.9774* (0.0087)	0.0614* (0.0155)	-
	1%	0.1393** (0.0667)	0.8291* (0.0629)	0.3124** (0.1374)	-
SAV HSI	5%	0.0659* (0.0659)	0.9124* (0.0251)	0.1840* (0.0218)	-
	1%	0.1833* (0.0215)	0.8612* (0.0128)	0.2871* (0.0159)	-
	5%	0.0722* (0.0072)	0.9103* (0.0115)	0.1899* (0.0222)	-
HS300	1%	0.0115*** (0.0585)	0.9464* (0.0791)	0.209 (0.2476)	0.1537*** (0.1513)
	5%	0.0011*** (0.0029)	0.9782* (0.0083)	0.0472*** (0.0172)	0.0514* (0.0195)
	1%	0.0916*** (0.0582)	0.8513* (0.0665)	0.1649 (0.1547)	0.4753* (0.1078)
AS HSI	5%	0.0271* (0.0104)	0.9067* (0.0252)	0.0550*** (0.0323)	0.2449* (0.0722)
	1%	0.1109** (0.0385)	0.8012* (0.0922)	0.2555 (0.2640)	0.6211*** (0.3828)
	5%	0.0314* (0.0070)	0.8939* (0.0152)	0.0466*** (0.0314)	0.2814* (0.0391)
N225	1%	0.0026*** (0.0294)	0.9309* (0.0150)	0.5488 (1.7734)	-
	5%	0.0033*** (0.0030)	0.9811* (0.0031)	0.0583 (0.6437)	-
	1%	0.1635*** (0.1237)	0.7802* (0.0653)	1.1863 (0.4317)	-
IGARCH HSI	5%	0.0128*** (0.0089)	0.9207* (0.0141)	0.1795** (0.9726)	-
	1%	0.2110* (0.0616)	0.7647* (0.0133)	1.0777 (0.9034)	-
	5%	0.0196** (0.0094)	0.9120* (0.0052)	0.1970*** (0.3962)	-

Notes: \*, \*\* and \*\*\* represent 1%, 5% and 10% Significant level respectively, the first number of each row is the parameter estimate, and the standard error in brackets is the standard error.



**Fig. 1** Sequence diagram of the rate of return of the sample



**Fig. 2** QQ diagram of sample sequence

vious volatility clustering characteristics while the meaning stays above 0.85 with obvious short-term memory characteristics. In the meantime,  $\beta_3$  is obvious especially for the AS model and IGARCH model; the quantile predication of model has certain explanatory ability at the external information impact. To further test the predication effect of SAV model, AS model and IGARCH model, the Kupiec test introduced earlier is adopted for analysis. See Table 3 for details.

According to Table 3, in terms of the number of failures, the number of failures of CAViaR model is higher than the standard 25 under the 5% reliability level. Despite the risk underestimations at different levels, the general deviation is small; the minimum P value is 0.2346 and the maximum one is 0.8384, in which the smaller P values are mainly in HS300 Index and AS Index while the larger P values are mainly in the N225 Index and IGARCH model. Generally, although the three models have all underestimated the risk to different extents, under the 5% significance level, the CAViaR model is effective for risk predication of sample, in which the IGARCH model has favorable performances with small differences. HS300 market underestimates risk more obviously, which it is mainly because that compared with the HSI market and N225 market, the HS300 market



**Table 3** The result of the posterior test based on the CAViaR class model

		Out-Sample ( $n = 500$ )					
		SAV		AS		IGARCH	
		1%	5%	1%	5%	1%	5%
HS300	Failure N	15	29	19	31	14	30
	LR Statistics	13.16	0.64	23.13	1.41	10.99	0.3192
	P	0.0001	0.4229	0.0000	0.2346	0.0009	0.4229
HSI	Failure N	16	26	17	28	15	26
	LR Statistics	15.48	0.0416	17.90	0.37	13.16	0.0416
	P	0.0000	0.8384	0.0002	0.5455	0.0003	0.8384
N225	Failure N	16	26	18	28	13	27
	LR Statistics	15.48	0.0416	20.46	0.37	8.97	0.16
	P	0.0000	0.8384	0.0000	0.5455	0.0027	0.6852

starts developing late and the market price fluctuates greatly.

Under the 1% reliability level, with different samples indexes, the number of failures of all quantile models is significantly larger than the theoretical level 5; the corresponding P value is also approaching 0; as the sample risk is seriously underestimated, the model presents obvious incompatibility and invalid predication capacity to risk. The HS 300 index performance is generally weaker than that of HSI and N225 and obviously unstable with risk predication highly deviated from the optimal level. Compared with HSI and N225 Index, as the HS 300 market starts developing later and various systems and mechanisms are incomplete, the current quantile models fail to fit the extreme value risk characteristics effectively in case of extreme risk, hence the predication capacity of the model is invalid.

### 3.3 Extreme Value Risk Estimation Based on CAViaR-POT Model

As the 1% VaR of CAViaR model has been seriously underestimated, the CAViaR model and extreme value POT model are combined in this paper, so that the tail characteristics can be captured while the distribution of initial series is not considered. First, the 5% quantile level is calculated with the CAViaR model, after that, the standardized residual series of extreme value can be obtained according to the standardized quantile residual method to predicate the 1% VaR through the POT model based on standardized extreme value residual series. According to the method in the previous model construction part, the standardized residual series of extreme value shall be calculated first before the POT model is adopted for fitting. The key point for POT model fitting is to select the threshold; as mentioned above, the Du Mouchel 10% principle is adopted to select the threshold of the standardized residual series of extreme value; after which the maximum likelihood estimation is adopted to determine the scale parameter  $\sigma$  and shape parameter  $\xi$  of the POT model. According to Table, the shape parameter  $\xi$  is larger than 0, which means the standardized residual series of extreme value have obvious fat-tail characteristics.

To test the effectiveness of the POT model estimation, the fitting diagnosis diagrams of standardized residual series of extreme value under all models are provided in this paper; taking the HS300 standardized series of extreme value as an example, as shown in Fig. 3, most points have fell on or near the threshold distribution diagram and tail distribution diagram, and only a few points are deviated without influencing the fitting effect. POT model has a favorable fitting effect in general; the same conclusion has been made on the same fitting effect test of the other series, which won't be listed separately due to the limited length.

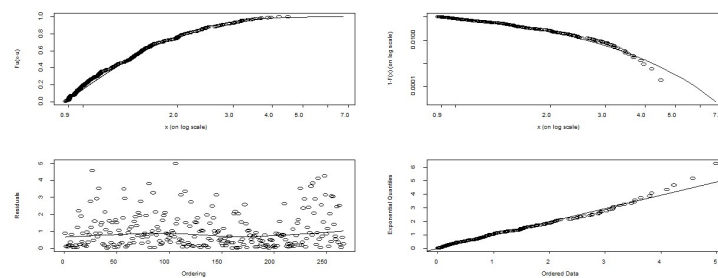
### 3.4 Model Validity Test

To test the risk estimation effect based on the CAViaR-POT, the posteriori test method mentioned earlier is adopted here for test; in the meantime, to make the comparative analysis, the VaR test result of GARCH-POT



**Table 4** POT estimation of the extreme standard residual sequence

		SAV	AS	IGARCH
HS300	$\mu$	0.916	0.851	0.926
	$\xi$	0.342	0.222	0.276
	$\sigma$	0.333	0.450	0.441
HSI	$\mu$	0.731	0.704	0.671
	$\xi$	0.180	0.231	0.172
	$\sigma$	0.473	0.451	0.341
N225	$\mu$	0.767	0.712	0.681
	$\xi$	0.172	0.213	0.351
	$\sigma$	0.365	0.461	0.672

**Fig. 3** POT model diagnosis diagram of HS300 extreme residual sequence

model and POT model under the 1% reliability level is calculated in this paper; see details in Table 5.

**Table 5** Extreme quantile 1% VaR posteriori test results

		Out-Sample( $n = 500$ )				
		POT-SAV	POT-AS	POT-IGARCH	GARCH-POT	POT
HS300	Failure N	9	8	7	9	10
	LR Statistics	2.61	1.54	0.72	2.61	3.91
	P	0.106	0.2149	0.3966	0.106	0.0479
HSI	Failure N	7	8	4	8	9
	LR Statistics	0.72	1.54	0.22	1.54	2.61
	P	0.3966	0.2149	0.6414	0.2149	0.106
N225	Failure N	6	7	3	8	9
	LR Statistics	0.19	0.72	0.94	1.54	2.61
	P	0.6630	0.3966	0.3314	0.2149	0.106

According to the test result of HS300, despite the underestimation in the risk measurement based on CAViaR-POT, the underestimation degree has been greatly reduced. Specifically, the number of failures is significantly lower than that before the model improvement and approaching the ideal level 5; compared with that of the unimproved model, the P value is also greatly improved. Therefore, the improvement of CAViaR model by the POT model improves the accuracy of risk measurement and it can characterize and predicate the extreme risk effectively.

According to the test result of HSI and N 225, the risk measurement effect based on CAViaR-POT model

can be greatly improved and the predication accuracy of risk value is also enhanced; the POT-IGARCH model is comparatively superior to other models. Both the number of failures and P value fully indicate that the extreme risk predication accuracy of the improved model has been significantly improved while the model is strongly accommodative.

Generally, the extreme risk estimation accuracy based on the improved model has been improved at different levels, which means that the improved model is more accommodative with greater effectiveness; the predication accuracy of POT-IGARCH model is the highest. According to the sample market analysis, after the improvement, the extreme risk predication accuracy of the improved model for the relatively mature market HSI and N225 is higher than that for the HS300; despite the certain underestimation in the extreme value risk predication of HS 300, the difference is generally small. This might be related to the maturity of HS300 market. Considering that the HS300 market starts developing later than the HSI market and N255 market, its market price is more easily impacted by policy for system and mechanism issues, so that there will be more extreme value points.

#### 4 Conclusion

Considering that the favorable characteristics of distribution shape and parameter needn't be assumed in the quantile regression model, in this paper the CAViaR model is adopted firstly to measure the risk levels of HS300, HSI and N225; it turns out that under the 5% significance level, the CAViaR model can effectively measure the risk value of samples while under the 1% significance level, the risk value of samples have been seriously underestimated by the CAViaR model.

Consequently, the combination of POT model and CAViaR model is introduced in this paper to construct the extreme value measurement model based on the POT-CAViaR model. According to the posteriori test, the improved model has effectively enhanced the risk measurement accuracy of the extreme value while the model effectiveness has been strengthened; in the meantime, the predication accuracy of extreme value risk of HS 300 market is lower than that of the HSI and N225 market, which is mainly because that the HS 300 market starts developing later than the HSI and N225 market. Owing to the system and mechanism issues, the market price will be more easily impacted by policy, so that there will be more extreme value points. The improved risk measurement model can measure the typical market characteristics effectively and timely to help the investors better understand the financial risk and predication risk; the accurate predication is helpful for the market subjects to avoid risk and provide theoretical support for the governmental decision. Although the improved model has improved its risk predication accuracy of extreme value and strengthened its applicability in this paper, it only makes the exploratory research on the combination of quantile regression and extreme value theory. Considering that quantile model is a huge category, according to the research conclusion of this paper, the performance of IGARCH model is relatively outstanding in the quantile regression model of big category while there are no huge differences among the integral performance in the other quantile regression models. Our research in the future will focus on how to select the models with more outstanding performance to fit the typical characteristics of the financial market. Nowadays, when calculating the threshold, this paper is adopting the Du Mouchel 10% principle popular, there remains no unified standard in the current academic circle of how to select threshold; it will be a new research field as the selection of threshold is directly related to the calculation accuracy of extreme value risk.

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