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Some Results on Generalized Sasakian Space Forms

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Abstract

In the present frame work, we studied the semi generalized recurrent, semi generalized ϕ-recurrent, extended generalized ϕ-recurrent and concircularly locally ϕ-symmetric on generalized Sasakian space forms.

Keywords: Generalized Sasakian space forms, extended generalized ϕ-recurrent, Einstein manifold.
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1 Introduction

The nature of a Riemannian manifold depends on the curvature tensor R of the manifold. It is well known that the sectional curvatures of a manifold determine its curvature tensor completely. A Riemannian manifold with constant sectional curvature c is known as a real space form and its curvature tensor is given by

$$R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}.$$

A Sasakian manifold with constant ϕ-sectional curvature is a Sasakian space form and it has a specific form of its curvature tensor. Similar notion also holds for Kenmotsu and cosymplectic space forms. In order to generalize such space forms in a common frame Alegre, Blair and Carriazo [1] introduced and studied generalized Sasakian space forms. These space forms are defined as follows:

A generalized Sasakian space form is an almost contact metric manifold (M,ϕ,ξ,η,g), whose curvature tensor is given by

$$\begin{aligned} R(X,Y)Z = {} & f_1\{g(Y,Z)X - g(X,Z)Y\} \\ & + f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ & + f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \end{aligned} \tag{1}$$

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The Riemanian curvature tensor of a generalized Sasakian space form $M^{2n+1}(f_1,f_2,f_3)$ is simply given by

$$R = f_1R_1 + f_2R_2 + f_3R_3. \tag{2}$$

where f_1,f_2,f_3 are differential functions on $M^{2n+1}(f_1,f_2,f_3)$ and

$$\begin{aligned}
R_1(X,Y)Z &= g(Y,Z)X - g(X,Z)Y,\\
R_2(X,Y)Z &= g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z,\\
R_3(X,Y)Z &= \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi,
\end{aligned}$$

where $f_1 = \frac{c+3}{4}, f_2 = f_3 = \frac{c-1}{4}$. Where c denotes the constant ϕ-sectional curvature. The properties of generalized Sasakian space form was studied by many geometers such as those mentioned in Refs. [2, 11, 12, 18, 21]. The concept of local symmetry of a Riemanian manifold has been studied by many authors in several ways to a different extent. The locally ϕ-symmetry of Sasakian manifold was introduced by Takahashi in Ref. [26]. De et.al., generalize the notion of ϕ-symmetry and then introduced the notion of ϕ-recurrent Sasakian manifold in Ref. [13]. Further ϕ-recurrent condition was studied on Kenmotsu manifold [10], LP-Sasakian manifold [27] and $(LCS)_n$-manifold [22].

Definition 1. A Riemannian manifold (M^{2n+1},g) is called a semi-generalized recurrent manifold if its curvature tensor R satisfies [6, 9]

$$(\nabla_X R)(Y,Z)W = A(X)R(Y,Z)W + B(X)g(Z,W)Y, \tag{3}$$

where A and B are two 1-forms, B is non-zero, ρ_1 and ρ_2 are two vector fields such that

$$g(X,\rho_1) = A(X), g(X,\rho_2) = B(X),$$

for any vector field X,Y,Z,W and ∇ denotes the operator of covariant differentiation with respect to the metric g.

Definition 2. A Riemannian manifold (M^{2n+1},g) is semi generalized Ricci-recurrent if [6, 9]

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + (2n+1)B(X)g(Y,Z), \tag{4}$$

where A and B are two 1-forms, B is non-zero, ρ_1 and ρ_2 are two vector fields such that

$$g(X,\rho_1) = A(X), g(X,\rho_2) = B(X),$$

Definition 3. A Sasakian manifold $(M^{2n+1},\phi,\xi,\eta,g)$, $n \geq 1$, is said to be an extended generalized ϕ-recurrent Sasakian manifold if its curvature tenor R satisfies the relation

$$\phi^2(\nabla_W R)(X,Y)Z = A(W)\phi^2(R(X,Y)Z) + B(W)\phi^2(G(X,Y)Z) \tag{5}$$

for all vector fields X,Y,Z,W, where A and B are two non-vanishing 1-forms such that $A(X) = g(X,\rho_1), B(X) = g(X,\rho_2)$. Here ρ_1 and ρ_2 are vector fields associated with 1-forms A and B respectively.

Definition 4. A generalized Sasakian space form is said to be locally ϕ-symmetric if

$$\phi^2(\nabla_W R)(X,Y)Z = 0$$

for all vector fields X,Y,Z orthogonal to ξ. This notion was introduced by T. Takahashi for Sasakian manifolds [26].

In 1940, Yano introduce the concircular curvature tensor. A $(2n+1)$ dimensional concircular curvature tensor C is given by [30,31]

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\},$$

where R and r are the Riemannian curvature tensor and scalar curvature tensor, respectively.

Author in Ref. [5] studies the symmetric conditons of generalized Sasakian space forms with concircular curvature tensor such as $C(\xi,X)\cdot C = 0$, $C(\xi,X)\cdot R = 0$, $C(\xi,X)\cdot S = 0$ and $C(\xi,X)\cdot P = 0$. Recently, researcher in Ref. [28] investigate some symmetric condition on generalized Sasakian space forms with W_2-curvature tensor, such as pseudosymmetric, locally symmetric, locally ϕ-symmetric and ϕ-recurrent. Moreover many geometer's studied the generalized Sasakian space forms with different conditions such as those mentioned in Refs. [11–13, 15, 16].

2 Generalized Sasakian space-forms

A $(2n+1)$-dimensional Riemannian manifold is called an almost contact metric manifold if the following result holds [6], [7]:

$$\phi^2 X = -X + \eta(X)\xi, \tag{6}$$
$$\eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad g(X,\xi) = \eta(X), \tag{7}$$
$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{8}$$
$$g(\phi X, Y) = -g(X, \phi Y), \quad g(\phi X, X) = 0 \tag{9}$$
$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y) \tag{10}$$

for all vector field X and Y. On a generalized Sasakian space form $M^{2n+1}(f_1, f_2, f_3)$, we have ([1, 15])

$$(\nabla_X \phi)Y = (f_1 - f_3)(g(X,Y)\xi - \eta(Y)X), \tag{11}$$
$$\nabla_X \xi = -(f_1 - f_3)\phi X. \tag{12}$$

Again, we know that from Ref. [1], $(2n+1)$-dimensional generalized Sasakian space forms holds the following relations:

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y) - (3f_2 + (2n-1)f_3)\eta(X)\eta(Y), \tag{13}$$
$$R(X,Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\}, \tag{14}$$
$$R(\xi,X)Y = (f_1 - f_3)\{g(X,Y)\xi - \eta(Y)X\}, \tag{15}$$
$$\eta(R(X,Y)Z) = (f_1 - f_3)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}, \tag{16}$$
$$S(X,\xi) = 2n(f_1 - f_3)\eta(X). \tag{17}$$

3 Semi generalized recurrent generalized Sasakian space forms

Definition 5. A generalized Sasakian space form (M^{2n+1}, g) is semi-generalized recurrent manifold if

$$(\nabla_X R)(Y,Z)W = A(X)R(Y,Z)W + B(X)g(Z,W)Y, \tag{18}$$

here A and B are two 1-forms, B is non-zero, ρ_1 and ρ_2 are two vector fields such that

$$A(X) = g(X,\rho_1) \quad and \quad B(X) = g(X,\rho_2)$$

Definition 6. A generalized Sasakian space forms (M^{2n+1}, g) is semi generalized Ricci-recurrent if

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + (2n+1)B(X)g(Y,Z). \tag{19}$$

Permutating equation (3) twice with respect to X,Y,Z, adding the three equations and using Bianchi second identity, we have

$$\begin{aligned} &A(X)R(Y,Z)W + B(X)g(Z,W) + A(Y)R(Z,X)W \\ &+B(Y)g(X,W)Z + A(Z)R(X,Y)W + B(Z)g(Y,W) = 0. \end{aligned} \tag{20}$$

Contracting (20) with respect to Y, we get

$$\begin{aligned} &A(X)S(Z,W) + B(X)g(Z,W) - g(R(Z,X)\rho,W) \\ &B(Z)g(X,W) - A(Z)S(X,W) + B(Z)g(X,W) = 0. \end{aligned} \tag{21}$$

Setting $S(Y,Z) = g(QY,Z)$ in (21) and factoring off W, we get

$$A(X)QZ + (2n+1)B(X)Z - R(Z,X)\rho + 2B(Z)X - A(Z)QX = 0. \tag{22}$$

Again contracting with respect to Z and then substitute $X = \xi$ in (22), one can get

$$r = -\frac{1}{\eta(\rho_1)}\{(2n+1)^2 - 2\eta(\rho_2) - 2n(f_1 - f_3)[\eta(\rho_2) + \eta(\rho_1)]\}. \tag{23}$$

Now, we can state the following statement

Theorem 1. *The scalar curvature r of a semi-generalized recurrent generalized Sasakian space forms is related in terms of contact forms $\eta(\rho_1)$ and $\eta(\rho_2)$ is given in (23).*

Next, we prove the semi generalized Ricci-recurrent generalized Sasakian space form, inserting $Z = \xi$ in (19), we have

$$2n(f_1 - f_3)^2 g(W,\phi Y) + (f_1 - f_3)S(Y,\phi W) = A(X)2n(f_1 - f_3)\eta(Y) + (2n+1)B(X)\eta(Y). \tag{24}$$

Again setting $Y = \xi$ in (24), we get

$$A(X)2n(f_1 - f_3) + (2n+1)B(X) = 0. \tag{25}$$

Now, we can state the following theorem

Theorem 2. *A semi-generalized Ricci-recurrent generalized Sasakian space forms, the 1-form A and B holds (25)*

4 Semi generalized ϕ-recurrent generalized Sasakian space forms

Definition 7. A generalized Sasakian space form (M^{2n+1}, g) is called semi-generalized ϕ-recurrent if its curvature tensor R satisfies the condition

$$\phi^2(\nabla_W R)(X,Y)Z = A(W)R(X,Y)Z + B(W)g(Y,Z)X \tag{26}$$

where A and B are two 1-forms, B is non-zero and these are defined by

$$A(W) = (W,\rho_1), \;\; B(W) = (W,\rho_2)$$

and ρ_1 and ρ_2 are vector fields associated with 1-forms A and B respectively.

Let us consider a semi-generalized ϕ-recurrent generalized Sasakian space forms. Then by virtue of (6) and (26), we have

$$\begin{aligned} & -(\nabla_W R)(X,Y)Z+\eta((\nabla_W R)(X,Y)Z)\xi \\ & =A(W)R(X,Y)Z+B(W)g(Y,Z)X. \end{aligned} \tag{27}$$

it follows that

$$\begin{aligned} & -g((\nabla_W R)(X,Y)Z,U)+\eta((\nabla_W R)(X,Y)Z)\eta(U) \\ & =A(W)g(R(X,Y)Z,U)+B(W)g(Y,Z)g(X,U). \end{aligned} \tag{28}$$

Let e_i, $i=1,2,...n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X=U=e_i$ in (28) and taking summation over i, $1\leq i\leq(2n+1)$, we get

$$\begin{aligned} & -(\nabla_W S)(Y,Z)+\sum_{i=1}^{2n}\eta(\nabla_W R)(e_i,Y)Z)\eta(e_i) \\ & =A(W)2n(f_1-f_3)S(Y,Z)+B(W)(2n+1)g(Y,Z). \end{aligned} \tag{29}$$

The second term of left hand side of (29) by putting $Z=\xi$ takes the form $((\nabla_W R)(e_i,Y)Z,\xi)=0$. So, by replacing Z by ξ in (29) and with the help of (7) and (12), we get

$$\begin{aligned} & -2n(f_1-f_3)^2g(W,\phi Y)+(f_1-f_3)S(Y,\phi W) \\ & =A(W)2n(f_1-f_3)\eta(Y)+B(W)(2n+1)g(Y,Z). \end{aligned} \tag{30}$$

Inserting $Y=\xi$ in (30) and using (7), we have

$$-2n(f_1-f_3)A(W)=(2n+1)B(W). \tag{31}$$

In view of (31) and replace Y by ϕY, (30) yields

$$S(Y,W)=2n(f_1-f_3)g(Y,W).$$

Theorem 3. *A semi generalized ϕ-recurrent generalized Sasakian space forms (M^{2n+1},g) is an Einstein manifold and moreover; the 1-forms A and B are related as $-2n(f_1-f_3)A(W)=(2n+1)B(W)$.*

5 Extended generalized ϕ-recurrent generalized Sasakian space forms

According to the definition of extended generalized ϕ-recurrent Sasakian manifolds, we will define the Extended generalized ϕ-recurrent generalized Sasakian space forms

Definition 8. A generalized Sasakian space forms $(M^{2n+1},\phi,\xi,\eta,g)$, $n\geq 1$, is said to be an extended generalized ϕ-recurrent generalized Sasakian space forms if its curvature tenor R satisfies the relation

$$\phi^2(\nabla_W R)(X,Y)Z=A(W)\phi^2(R(X,Y)Z)+B(W)\phi^2(G(X,Y)Z) \tag{32}$$

for all vector fields X,Y,Z,W, where A and B are two non-vanishing 1-forms such that $A(X)=g(X,\rho_1)$, $B(X)=g(X,\rho_2)$. Here ρ_1 and ρ_2 are vector fields associated with 1-forms A and B respectively.

Let us consider an extended generalized ϕ-recurrent generalized Sasakian space forms. Then by virtue of (6), we have

$$\begin{aligned} & -(\nabla_W R)(X,Y)Z+\eta((\nabla_W R)(X,Y)Z)\xi \\ & =A(W)\{-R(X,Y)Z+\eta(R(X,Y)Z)\} \\ & +B(W)\{-G(X,Y)Z+\eta(G(X,Y)Z)\}. \end{aligned} \tag{33}$$

From which it follows that

$$\begin{aligned}
&-g((\nabla_W R)(X,Y)Z,U)+\eta((\nabla_W R)(X,Y)Z)\eta(U)\\
&=A(W)\{-g(R(X,Y)Z,U)+\eta(R(X,Y)Z)\eta(U)\}\\
&+B(W)\{-g(G(X,Y)Z,U)+\eta(G(X,Y)Z)\eta(U)\}.
\end{aligned} \tag{34}$$

Let e_i, $i=1,2,...n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X=U=e_i$ in (34) and taking summation over i, $1\leq i\leq(2n+1)$, and the relation $g((\nabla_W R)(X,Y)Z,U)=-g((\nabla_W R)(X,Y)U,Z)$, we get

$$\begin{aligned}
&-(\nabla_W S)(Y,Z)=A(W)\{-S(Y,Z)+\eta(R(\xi,Y)Z)\}\\
&+B(W)\{-(2n-1)g(Y,Z)-\eta(Y)\eta(Z)\}.
\end{aligned} \tag{35}$$

It follows that,

$$(\nabla_W S)(Y,Z)=A\otimes S(Y,Z)+Kg(Y,Z)+\mu\eta(Y)(Z). \tag{36}$$

where $K=[(2n-1)B(W)-A(W)(f_1-f_3)]$ and $\mu=[(f_1-f_3)A(W)+B(W)]$.
Inserting $Z=\xi$ (35) and using (12), (17) and (7), we get

$$\begin{aligned}
&2n(f_1-f_3)^2g(W,\phi Y)+(f_1-f_3)S(Y,\phi W)\\
&=\{2n(f_1-f_3)A(W)+2nB(W)\}\eta(Y).
\end{aligned} \tag{37}$$

Again inserting $Y=\xi$ and using (7), (37) yields

$$2n(f_1-f_3)A(W)+2nB(W)=0. \tag{38}$$

By taking the account of (38) in (37) and then replace Y by ϕY, we get

$$S(Y,W)=2n(f_1-f_3)g(Y,W).$$

Thus we have the following assertion

Theorem 4. *An extended generalized ϕ-recurrent generalized Sasakian space forms is an Einstein manifold and moreover the associated 1-forms A and B are related by $(f_1-f_3)A+B=0$.*

It is known that a generalized Sasakian space form is Ricci-semisymmetric if and only if it is an Einstein manifold. In fact, by **Theorem 4**, we have the following:

Corollary 5. *An extended generalized ϕ-recurrent generalized Sasakian space forms is Ricci-semisymmetric.*

6 Concircularly locally ϕ-symmetric generalized Sasakian space forms

Definition 9. A $(2n+1)$ dimensional $(n>1)$ generalized Sasakian space form is called concircularly locally ϕ-symmetric if it satisfies [12].

$$\phi^2(\nabla_W C)(X,Y)Z=0.$$

for all vector fields X,Y,Z are orthogonal to ξ and an arbitrary vector field W.

Differentiate covariantly with respect W, we have

$$(\nabla_W C)(X,Y)Z=(\nabla_W R)(X,Y)Z-\frac{dr(W)}{2n(2n+1)}\{g(Y,Z)X-g(X,Z)Y\}. \tag{39}$$

Operate ϕ^2 on both side, we have

$$\phi^2((\nabla_W C)(X,Y)Z) = \phi^2((\nabla_W R)(X,Y)Z) - \frac{dr(W)}{2n(2n+1)}\{g(Y,Z)\phi^2 X - g(X,Z)\phi^2 Y\}. \tag{40}$$

In view of (6), and taking the help of relation (1) with X,Y,Z are orthogonal vector field, one can get

$$\begin{aligned}\phi^2((\nabla_W C)(X,Y)Z) &= df_1(W)\{g(Y,Z)X - g(X,Z)Y\}\\ &+ df_2(W)\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\}\\ &+ f_2\{g(X,\phi Z)(\nabla_W \phi)Y + g(X,(\nabla_W \phi)Z)\phi Y\\ &- g(Y,\phi Z)(\nabla_W \phi)X - g(Y,(\nabla_W \phi)Z)\phi X\\ &+ 2g(X,\phi Y)(\nabla_W \phi)Z + 2g(X,(\nabla_W \phi)Y)\phi Z\}\\ &+ \frac{dr(W)}{2n(2n+1)}\{g(Y,Z)X - g(X,Z)Y\}.\end{aligned} \tag{41}$$

If the manifold is conformally flat then $f_2 = 0$. Therefore, (41) yields

$$\phi^2((\nabla_W C)(X,Y)Z) = \left\{df_1(W) + \frac{dr(W)}{2n(2n+1)}\right\}\{g(Y,Z)X - g(X,Z)Y\}.$$

Hence we can state the following theorem

Theorem 6. *A generalized Sasakian space forms is concircularly locally ϕ-symmetric if and only if f_1 and the scalar curvature are constant*

Note 7. *In [18], U. K. Kim studied generalized Sasakian space forms and proved that if a generalized Sasakian space forms $M^{2n+1}(f_1,f_2,f_3)$ of dimension greater than three is conformally flat and ξ is Killing, then it is locally symmetric. Moreover, if $M^{2n+1}(f_1,f_2,f_3)$ is locally symmetric, then $f_1 - f_3$ is constant. In the above theorem it is shown that a conformally flat generalized Sasakian space form of dimension greater than 3 is locally ϕ-symmetric if and only if f_1 and scalar curvature is constant. Thus, we observe the difference between locally symmetric generalized Sasakian space forms and concircularly locally ϕ-symmetric generalized Sasakian space forms.*

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