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# Solution of the Maximum of Difference Equation $x_{n+1} = max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_n}\right\}; \quad y_{n+1} = max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_n}\right\}$

Dagistan Simsek[1,2], Burak Ogul[1] †, Fahreddin Abdullayev[1,3].

[1]Kyrgyz Turkish Manas University, Bishkek, Kyrgyzstan.
[2]Konya Technical University, Konya, Turley.
[3]Mersin University, Mersin, Turkey.

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**Abstract**

In the recent years, there has been a lot of interest in studying the global behavior of, the socalled, max-type difference equations; see, for example, [1–17]. The study of max type difference equations has also attracted some attention recently. We study the behaviour of the solutions of the following system of difference equation with the max operator:paper deals with the behaviour of the solutions of the max type system of difference equations,

$$x_{n+1} = max\left\{\frac{A}{x_{n-1}}, \frac{y_n}{x_n}\right\}; \quad y_{n+1} = max\left\{\frac{A}{y_{n-1}}, \frac{x_n}{y_n}\right\}, \tag{1}$$

where the parametr $A$ and initial conditions $x_{-1}, x_0, y_{-1}, y_0$ are positive reel numbers.

**Keywords:** Difference equations, Periodicity, Max type difference equations
**AMS 2010 codes:** 39A10.

## 1 Introduction

Recently, there has been a great concern in studying nonlinear difference equations since many models describing real life situations in population biology, economics, probability theory, genetics, psychology, sociology etc. are represented by these equations. See for example [1–28].

**Definition 1.** Let $I$ be an interval of reel numbers and let $f : I^{s+1} \to I$ be a continuously differentiable function

†Corresponding author.
Email address: burak_1745@hotmail.com

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where s is a non-negative integer. Consider the difference equation

$$x_{n+1} = f(x_n, x_{n-1}, ..., x_{n-s}) \text{ for } \quad n = 0, 1, ..., \tag{2}$$

with the initial values $x_{-s}, ..., x_0 \in I$. A point $\bar{x}$ called an equilibrium point of equation 2. if $\bar{x} = f(\bar{x}, ..., \bar{x})$.

**Definition 2.** *A* positive semi sycle of a solution $\{x_n\}_n^\infty = -s$ of 2 consist of a string of terms $\{x_l, x_{l+1}, ..., x_m\}$ all greater than or equal to equilibrium $\bar{x}$ with $l \geq -s$ and $m \leq \infty$ such that either $l = -s$ or $l > s$ and $x_{l-1} < \bar{x}$ and either $m = \infty$ or $m < \infty$ and $x_{m+1} < \bar{x}$.

**Definition 3.** *A* negative semisycle of a solution $\{x_n\}_n^\infty = -s$ of 2 consist of a string of terms $\{x_l, x_{l+1}, ..., x_m\}$ all less than or equal to equilibrium $\bar{x}$ with $l \geq -s$ and $m \leq \infty$ such that either $l = -s$ or $l > -s$ and $x_{l-1} \geq \bar{x}$ and either $m = \infty$ or $m \leq \infty$ and $x_{m+1} \geq \bar{x}$.

## 2 Main Results

In some cases of parameter $A$ and initial conditions, the solution of the system of max type difference equation has been studied. Let $\bar{x}$ and $\bar{y}$ be the unique positive equilibrium of 1, then clearly,

$$\bar{x} = max\left\{\frac{A}{\bar{x}}, \frac{\bar{y}}{\bar{x}}\right\}; \bar{y} = max\left\{\frac{A}{\bar{y}}, \frac{\bar{x}}{\bar{y}}\right\}.$$

The parameter $A$ is the greatest value in all initial conditions that we select, so

$$\bar{x} = \frac{A}{\bar{x}} \Rightarrow \bar{x}^2 = A \Rightarrow \bar{x} = \pm\sqrt{A}; \quad \bar{y} = \frac{A}{\bar{y}} \Rightarrow \bar{y}^2 = A \Rightarrow \bar{y} = \pm\sqrt{A},$$

we can obtain $\bar{x} = \sqrt{A}$ and $\bar{y} = \sqrt{A}$.

**Lemma 1.** *Assume that, A and $x_0, x_{-1}, y_0, y_{-1}$ are positive integer sequence for 1*
$A > x_0 > x_{-1} > y_0 > y_{-1}, A > x_0 > y_0 > x_{-1} > y_{-1}, A > y_0 > x_0 > x_{-1} > y_{-1}$,
*Then the following statements are true:*
$n \geq 0$ *for* $x_n$ *and* $n \geq 1$ *for* $y_n$

a) *Every positive semi-cycle consist two term.*

b) *Every negative semi-cycle consist two term.*

c) *Every positive semi-cycle of length two is followed by a negative semi-cycle of length two.*

d) *Every negative semi-cycle of length two is followed by a positive semi-cycle of length two.*

*Proof.* $A > x_0 > x_{-1} > y_0 > y_{-1}, A > x_0 > y_0 > x_{-1} > y_{-1}, A > y_0 > x_0 > x_{-1} > y_{-1}$ The solution $x_n$ and $y_n$ can be obtained as follows:

$$x_1 = max\left\{\frac{A}{x_{-1}}, \frac{y_0}{x_0}\right\} = \frac{A}{x_{-1}} < \bar{x}; \quad y_1 = max\left\{\frac{A}{y_{-1}}, \frac{x_0}{y_0}\right\} = \frac{A}{y_{-1}} > \bar{y},$$

$$x_2 = max\left\{\frac{A}{x_0}, \frac{y_1}{x_1}\right\} = max\left\{\frac{A}{x_0}, \frac{x_{-1}}{y_{-1}}\right\} = \frac{x_{-1}}{y_{-1}} < \bar{x}; \quad y_2 = max\left\{\frac{A}{y_0}, \frac{x_1}{y_1}\right\} = max\left\{\frac{A}{y_0}, \frac{y_{-1}}{x_{-1}}\right\} = \frac{A}{y_0} < \bar{y},$$

$$x_3 = max\left\{\frac{A}{x_1}, \frac{y_2}{x_2}\right\} = max\left\{x_{-1}, \frac{Ay_{-1}}{x_{-1}y_0}\right\} = x_{-1} > \bar{x}; \quad y_3 = max\left\{\frac{A}{y_1}, \frac{x_2}{y_2}\right\} = max\left\{y_{-1}, \frac{x_{-1}y_0}{Ay_{-1}}\right\} = y_{-1} < \bar{y},$$

$$x_4 = max\left\{\frac{A}{x_2},\frac{y_3}{x_3}\right\} = max\left\{\frac{Ay_{-1}}{x_{-1}},\frac{y_{-1}}{x_{-1}}\right\} = \frac{Ay_{-1}}{x_{-1}} > \bar{x}; \quad y_4 = max\left\{\frac{A}{y_2},\frac{x_3}{y_3}\right\} = max\left\{y_0,\frac{x_{-1}}{y_{-1}}\right\} = y_0 > \bar{y},$$

$$x_5 = max\left\{\frac{A}{x_3},\frac{y_4}{x_4}\right\} = max\left\{\frac{A}{x_{-1}},\frac{x_{-1}y_0}{Ay_{-1}}\right\} = \frac{A}{x_{-1}} < \bar{x}; \quad y_5 = max\left\{\frac{A}{y_3},\frac{x_4}{y_4}\right\} = max\left\{\frac{A}{y_{-1}},\frac{Ay_{-1}}{x_{-1}y_0}\right\} = \frac{A}{y_{-1}} > \bar{y},$$

$$x_6 = max\left\{\frac{A}{x_4},\frac{y_5}{x_5}\right\} = max\left\{\frac{x_{-1}}{y_{-1}},\frac{x_{-1}}{y_{-1}}\right\} = \frac{x_{-1}}{y_{-1}} < \bar{x}; \quad y_6 = max\left\{\frac{A}{y_4},\frac{x_5}{y_5}\right\} = max\left\{\frac{A}{y_0},\frac{y_{-1}}{x_{-1}}\right\} = \frac{A}{y_0} < \bar{y},$$

$$x_7 = max\left\{\frac{A}{x_5},\frac{y_6}{x_6}\right\} = max\left\{x_{-1},\frac{Ay_{-1}}{y_0x_{-1}}\right\} = x_{-1} > \bar{x}; \quad y_7 = max\left\{\frac{A}{y_5},\frac{x_6}{y_6}\right\} = max\left\{y_{-1},\frac{y_0x_{-1}}{Ay_{-1}}\right\} = y_{-1} < \bar{y},$$

$$x_8 = max\left\{\frac{A}{x_6},\frac{y_7}{x_7}\right\} = max\left\{\frac{Ay_{-1}}{x_{-1}},\frac{y_{-1}}{x_{-1}}\right\} = \frac{Ay_{-1}}{x_{-1}} > \bar{x}; \quad y_8 = max\left\{\frac{A}{y_6},\frac{x_7}{y_7}\right\} = max\left\{y_{-1},\frac{x_{-1}}{y_{-1}}\right\} = y_0 > \bar{y},$$

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Hence we obtained. $x_1 < \bar{x}, x_2 < \bar{x}, x_3 > \bar{x}, x_4 > \bar{x}, x_5 < \bar{x}, x_6 < \bar{x}, x_7 > \bar{x}, x_8 > \bar{x}, \ldots$
$y_1 > \bar{y}, y_2 < \bar{y}, y_3 < \bar{y}, y_4 > \bar{y}, y_5 > \bar{y}, y_6 < \bar{y}, y_7 < \bar{y}, y_8 > \bar{y}, \ldots$
Hence, the solution $n \geq 0$ for $x_n$ and $n \geq 1$ for $y_n$, every positive semi-cycle consists of two terms, every negative semi-cycle consists of two terms.

**Lemma 2.** *Assume that, A and* $x_0, x_{-1}, y_0, y_{-1}$ *are positive integer sequence for* 1
$A > x_0 > y_0 > y_{-1} > x_{-1}, A > y_0 > x_0 > y_{-1} > x_{-1}, A > y_0 > y_{-1} > x_0 > x_{-1}$,
*Then the following statements are true:*
$n \geq 1$ *for* $x_n$ *and* $n \geq 0$ *for* $y_n$

a) *Every positive semi-cycle consist two term.*

b) *Every negative semi-cycle consist two term.*

c) *Every positive semi-cycle of length two is followed by a negative semi-cycle of length two.*

d) *Every negative semi-cycle of length two is followed by a positive semi-cycle of length two.*

*Proof.* Lemma 2 proof's can be obtained similarly Lemma 1.

**Lemma 3.** *Assume that, A and* $x_0, x_{-1}, y_0, y_{-1}$ *are positive integer sequence for* 1
$A > x_{-1} > y_{-1} > x_0 > y_0, A > y_{-1} > x_{-1} > x_0 > y_0, A > y_{-1} > x_0 > x_{-1} > y_0$,
*Then the following statements are true:*
$n \geq 0$ *for* $x_n$ *and* $n \geq 1$ *for* $y_n$

a) *Every positive semi-cycle consist two term.*

b) *Every negative semi-cycle consist two term.*

*c) Every positive semi-cycle of length two is followed by a negative semi-cycle of length two.*

*d) Every negative semi-cycle of length two is followed by a positive semi-cycle of length two.*

*Proof.* Lemma 3 proof's can be obtained similarly Lemma 1.

**Theorem 4.** *Let $(x_n, y_n)$ be a solution of 1 for*
$A > x_0 > x_{-1} > y_0 > y_{-1}, A > x_0 > y_0 > x_{-1} > y_{-1}, A > y_0 > x_0 > x_{-1} > y_{-1}$.
*Then for $n = 0, 1, \ldots$ we have,*

$$x_n = \left\{ \frac{A}{x_{-1}}, \frac{x_{-1}}{y_{-1}}, x_{-1}, \frac{Ay_{-1}}{x_{-1}}, \ldots \right\},$$
$$y_n = \left\{ \frac{A}{y_{-1}}, \frac{A}{y_0}, y_{-1}, y_0, \ldots \right\}.$$

*Proof.* We obtain,

$$x_1 = max\left\{ \frac{A}{x_{-1}}, \frac{y_0}{x_0} \right\} = \frac{A}{x_{-1}}; \quad y_1 = max\left\{ \frac{A}{y_{-1}}, \frac{x_0}{y_0} \right\} = \frac{A}{y_{-1}},$$

$$x_2 = max\left\{ \frac{A}{x_0}, \frac{y_1}{x_1} \right\} = max\left\{ \frac{A}{x_0}, \frac{x_{-1}}{y_{-1}} \right\} = \frac{x_{-1}}{y_{-1}}; \quad y_2 = max\left\{ \frac{A}{y_0}, \frac{x_1}{y_1} \right\} = max\left\{ \frac{A}{y_0}, \frac{y_{-1}}{x_{-1}} \right\} = \frac{A}{y_0},$$

$$x_3 = max\left\{ \frac{A}{x_1}, \frac{y_2}{x_2} \right\} = max\left\{ x_{-1}, \frac{Ay_{-1}}{x_{-1}y_0} \right\} = x_{-1}; \quad y_3 = max\left\{ \frac{A}{y_1}, \frac{x_2}{y_2} \right\} = max\left\{ y_{-1}, \frac{x_{-1}y_0}{Ay_{-1}} \right\} = y_{-1},$$

$$x_4 = max\left\{ \frac{A}{x_2}, \frac{y_3}{x_3} \right\} = max\left\{ \frac{Ay_{-1}}{x_{-1}}, \frac{y_{-1}}{x_{-1}} \right\} = \frac{Ay_{-1}}{x_{-1}}; \quad y_4 = max\left\{ \frac{A}{y_2}, \frac{x_3}{y_3} \right\} = max\left\{ y_0, \frac{x_{-1}}{y_{-1}} \right\} = y_0,$$

$$x_5 = max\left\{ \frac{A}{x_3}, \frac{y_4}{x_4} \right\} = max\left\{ \frac{A}{x_{-1}}, \frac{x_{-1}y_0}{Ay_{-1}} \right\} = \frac{A}{x_{-1}}; \quad y_5 = max\left\{ \frac{A}{y_3}, \frac{x_4}{y_4} \right\} = max\left\{ \frac{A}{y_{-1}}, \frac{Ay_{-1}}{x_{-1}y_0} \right\} = \frac{A}{y_{-1}},$$

$$x_6 = max\left\{ \frac{A}{x_4}, \frac{y_5}{x_5} \right\} = max\left\{ \frac{x_{-1}}{y_{-1}}, \frac{x_{-1}}{y_{-1}} \right\} = \frac{x_{-1}}{y_{-1}}; \quad y_6 = max\left\{ \frac{A}{y_4}, \frac{x_5}{y_5} \right\} = max\left\{ \frac{A}{y_0}, \frac{y_{-1}}{x_{-1}} \right\} = \frac{A}{y_0},$$

$$x_7 = max\left\{ \frac{A}{x_5}, \frac{y_6}{x_6} \right\} = max\left\{ x_{-1}, \frac{Ay_{-1}}{y_0x_{-1}} \right\} = x_{-1}; \quad y_7 = max\left\{ \frac{A}{y_5}, \frac{x_6}{y_6} \right\} = max\left\{ y_{-1}, \frac{y_0x_{-1}}{Ay_{-1}} \right\} = y_{-1},$$

$$x_8 = max\left\{ \frac{A}{x_6}, \frac{y_7}{x_7} \right\} = max\left\{ \frac{Ay_{-1}}{x_{-1}}, \frac{y_{-1}}{x_{-1}} \right\} = \frac{Ay_{-1}}{x_{-1}}; \quad y_8 = max\left\{ \frac{A}{y_6}, \frac{x_7}{y_7} \right\} = max\left\{ y_{-1}, \frac{x_{-1}}{y_{-1}} \right\} = y_0,$$

$$\vdots$$

Thus,

$$x_n = \left\{ \frac{A}{x_{-1}}, \frac{x_{-1}}{y_{-1}}, x_{-1}, \frac{Ay_{-1}}{x_{-1}}, \ldots \right\},$$
$$y_n = \left\{ \frac{A}{y_{-1}}, \frac{A}{y_0}, y_{-1}, y_0, \ldots \right\},$$

the solutions are shown to be 4-peirod.

**Theorem 5.** *Let* $(x_n, y_n)$ *be a solution of 1 for*
$A > x_0 > y_0 > y_{-1} > x_{-1}, A > y_0 > x_0 > y_{-1} > x_{-1}, A > y_0 > y_{-1} > x_0 > x_{-1}$
*Then for* $n = 0, 1, \ldots$ *we have,*

$$x_n = \left\{ \frac{A}{x_{-1}}, \frac{A}{x_0}, x_{-1}, x_0, \ldots \right\},$$
$$y_n = \left\{ \frac{A}{y_{-1}}, \frac{y_{-1}}{x_{-1}}, y_{-1}, \frac{Ax_{-1}}{y_{-1}}, \ldots \right\}.$$

*Proof.* Proof of the Theorem 5 can be obtain similar way to the Theorem 4.

**Theorem 6.** *Let* $(x_n, y_n)$ *be a solution of 1 for*
$A > x_{-1} > y_{-1} > x_0 > y_0, A > y_{-1} > x_{-1} > x_0 > y_0, A > y_{-1} > x_0 > x_{-1} > y_0$
*Then for* $n = 0, 1, \ldots$ *we have,*

$$x_n = \left\{ \frac{A}{x_{-1}}, \frac{A}{x_0}, x_{-1}, x_0, \ldots \right\},$$
$$y_n = \left\{ \frac{x_0}{y_0}, \frac{A}{y_0}, \frac{Ay_0}{x_0}, y_0, \ldots \right\}.$$

*Proof.* Proof of the Theorem 6 can be obtain similar way to the Theorem 4.

**Example 7.** *If the initial conditions are selected follows for Lemma 1* $A > x_0 > x_{-1} > y_0 > y_{-1}$*:*
$A = 36; x[-1] = 25; x[0] = 30; y[-1] = 15; y[0] = 20;$
*The graph of the solution is given below:*

$x_n = \{1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6,$
$1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667,$
$25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44,$
$1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6,$
$1.44, 1.66667, 25., 21.6, 1.44, \ldots\}.$

$y_n = \{2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8,$
$15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8,$
$25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44, 1.66667, 25., 21.6, 1.44,$
$15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8, 15., 20., 2.4, 1.8,$
$15., 20., 2.4, \ldots\}.$

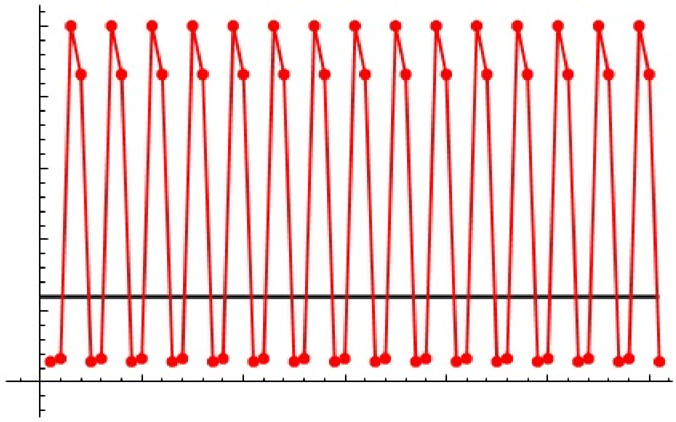

**Fig. 1** $x_n$ graph solution.

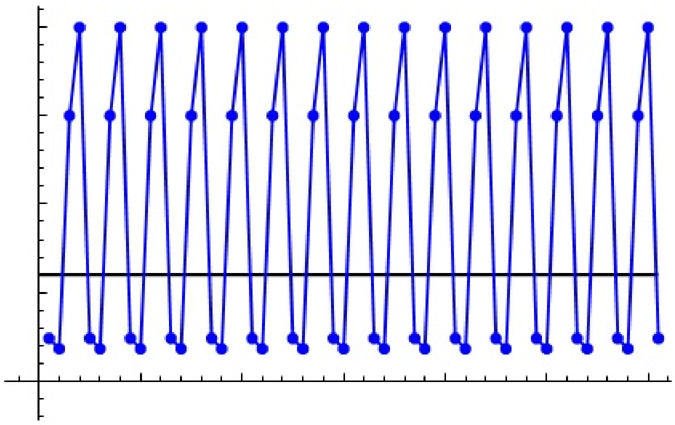

**Fig. 2** $y_n$ graph solution.

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