



# Applied Mathematics and Nonlinear Sciences

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## New Complex and Hyperbolic Forms for Ablowitz–Kaup–Newell–Segur Wave Equation with Fourth Order

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### Abstract

Researching different solutions of nonlinear models has been interesting in different fields of science and application. In this study, we investigated different solutions of fourth-order nonlinear Ablowitz–Kaup–Newell–Segur wave equation. We have used the sine-Gordon expansion method (SGEM) during this research. We have given the 2D, 3D, and contour graphs acquired from the values of the solutions obtained using strong SGEM.

**Keywords:** The sine-Gordon expansion method, Ablowitz–Kaup–Newell–Segur wave equation, complex hyperbolic function solutions.

**AMS 2010 codes:** Put here the AMS 2010 codes of the paper.

## 1 Introduction

The study of nonlinear differential equation (NLDE) solutions attracts the attention of scientists. NLDE is used in many areas such as physics and chemistry. We need to explore the solutions of NLDE that have an important place in applied mathematics. Some scientists have explored these solutions.

In recent years, several effective methods, including extended tanh method [1, 2], first integral method [3, 4], He's semi-inverse method [5, 6], sine-cosine method [7, 8], dynamical system method [9], modified simple equation method [10, 11], Bell-polynomial method [12], simplified Hirota's method [13], Cole–Hopf transformation method [14], sine-cosine method [15], tanh method [16], generalized tanh function method [17], improved F-expansion method with Riccati equation [18, 19], modified  $\exp(-\Omega(\xi))$ -expansion function method [20, 21],

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and improved Bernoulli subequation function method [22], have been successfully considered to find the exact solutions of a wide variety of NLDEs and many others [27–88].

In this study, we obtain new complex hyperbolic function solutions to the nonlinear Ablowitz-Kaup–Newell–Segur wave equation (AKNSWE) with fourth order [23], which is defined as

$$4u_{xt} + u_{xxx} + 8u_x u_{xy} + 4u_{xx} u_y - \gamma u_{xx} = 0, \quad (1)$$

where  $\gamma$  is a real constant with a non-zero value, by using the sine-Gordon expansion method (SGEM).

## 2 Fundamental Properties of the SGEM

Let us consider the following sine-Gordon equation [24–26]:

$$u_{xx} - u_{tt} = m^2 \sin(u), \quad (2)$$

where  $u = u(x, t)$  and  $m$  is a real constant. When we apply the wave transform  $\xi = \mu(x - ct)$  to Eq. (2), we obtain the nonlinear ordinary differential equation (NODE) as follows:

$$U'' = \frac{m^2}{\mu^2(1-c^2)} \sin(U), \quad (3)$$

where  $U = U(\xi)$ ,  $\xi$  is the amplitude of the travelling wave, and  $c$  is the velocity of the travelling wave. If we reconsider Eq. (3), we can write it in the full simplified version as follows:

$$\left[ \left( \frac{U}{2} \right)' \right]^2 = \frac{m^2}{\mu^2(1-c^2)} \sin^2 \left( \frac{U}{2} \right) + K, \quad (4)$$

where  $K$  is the integration constant. When we resubmit as  $K = 0$ ,  $w(\xi) = \frac{U}{2}$ , and  $a^2 = \frac{m^2}{\mu^2(1-c^2)}$  in Eq. (4), we can obtain the following equation:

$$w' = a \sin(w). \quad (5)$$

If we put  $a = 1$  in Eq. (5), we can obtain the following equation:

$$w' = \sin(w). \quad (6)$$

If we solve Eq. (6) by using separation of variables, we find the following two significant equations:

$$\sin(w) = \sin(w(\xi)) = \frac{2pe^\xi}{p^2e^{2\xi} + 1} \Bigg|_{p=1} = \operatorname{sech}(\xi), \quad (7)$$

$$\cos(w) = \cos(w(\xi)) = \frac{p^2e^{2\xi} - 1}{p^2e^{2\xi} + 1} \Bigg|_{p=1} = \tanh(\xi), \quad (8)$$

where  $p$  is the integral constant having a non-zero value. For the solution of following nonlinear partial differential equation

$$P(u, u_x, u_t, \dots) = 0, \quad (9)$$

let us consider

$$U(\xi) = \sum_{i=1}^{\delta} \tanh^{i-1}(\xi) [B_i \operatorname{sech}(\xi) + A_i \tanh(\xi)] + A_0. \quad (10)$$

We can rewrite Eq. (10) according to Eqs. (7 and 8) as follows:

$$U(w) = \sum_{i=1}^{\delta} \cos^{i-1}(w) [B_i \sin(w) + A_i \cos(w)] + A_0. \tag{11}$$

Under the terms of homogenous balance technique, we can determine the values of  $n$  under the terms of *NODE*. Let the coefficients of  $\sin^i(w) \cos^j(w)$  all be zero; it yields a system of equations. Solving this system by using Wolfram Mathematica 9 gives the values of  $A_i, B_i, \mu$  and  $c$ . Finally, substituting the values of  $A_i, B_i, \mu$  and  $c$  in Eq.(10), we can find new analytical solutions to Eq. (9).

### 3 Application of the SGEM

SGEM has been successfully used to obtain analytical solutions to the AKNSWE. Using  $u(x, y, t) = U(\xi)$ ,  $\xi = x + y + \omega t$  in Eq. (1), we get

$$(4\omega - \gamma)U' + \omega U''' + 6(U')^2 = 0.$$

If we consider  $V = U'$ , we can obtain the following nonlinear differential equation:

$$(4\omega - \gamma)V + 6V^2 + \omega V'' = 0. \tag{12}$$

After balancing, we find  $\delta = 2$ . For this value, Eq. (11) can be written as

$$V(w) = B_1 \sin(w) + A_1 \cos(w) + B_2 \cos(w) \sin(w) + A_2 \cos^2(w) + A_0. \tag{13}$$

If we put Eq. (13) with second derivation into Eq. (12), we can find a trigonometric equation. Solving this, we can choose the following coefficients:

*Case 1*

$$A_0 = \frac{\omega}{3}; A_1 = 0; A_2 = -\frac{\omega}{2}; B_1 = 0; B_2 = -\frac{i\omega}{2}; \gamma = 3\omega.$$

Inserting these values into Eq. (10) with  $V = U'$  yields

$$U_1(x, y, t) = -\frac{1}{6}\omega(x + y + t\omega - 3\text{isech}[x + y + t\omega] - 3\tanh[x + y + t\omega]). \tag{14}$$

*Case 2* Considering another coefficient as

$$A_0 = \frac{\omega}{2}; A_1 = 0; A_2 = -\frac{\omega}{2}; B_1 = 0; B_2 = -\frac{i\omega}{2}; \gamma = 5\omega$$

into Eq. (10) with  $V = U'$  produces following complex hyperbolic function solutions:

$$U_2(x, y, t) = \frac{1}{2}\omega(\text{isech}[\omega t + x + y] + \tanh[\omega t + x + y]) \tag{15}$$

*Case 3* Taking another coefficient as follows

$$A_0 = \frac{\gamma}{9}; A_1 = 0; A_2 = -\frac{\gamma}{6}; B_1 = 0; B_2 = -\frac{i\gamma}{6}; \omega = \frac{\gamma}{3};$$

into Eq. (10) with  $V = U'$ , we find the following result:

$$U_3(x, y, t) = -\frac{1}{18}\gamma\left(\frac{\gamma t}{3} + x + y - 3\text{isech}\left[\frac{\gamma t}{3} + x + y\right] - 3\tanh\left[\frac{\gamma t}{3} + x + y\right]\right). \tag{16}$$

Case 4 Getting the following items into Eq. (10) with  $V = U'$ ;

$$A_0 = \frac{\gamma}{8}; A_1 = 0; A_2 = -\frac{\gamma}{8}; B_1 = 0; B_2 = 0; \omega = \frac{\gamma}{8};$$

we can find

$$U_4(x, y, t) = \frac{1}{8} \gamma \tanh \left[ \frac{\gamma t}{8} + x + y \right] \tag{17}$$

Case 5 Inserting

$$A_0 = \frac{\gamma}{10}; A_1 = 0; A_2 = -\frac{\gamma}{10}; B_1 = 0; B_2 = -\frac{i\gamma}{10}; \omega = \frac{\gamma}{5}$$

into Eq. (10) with  $V = U'$  gives rise to

$$U_5(x, y, t) = \frac{\gamma}{5i + 5 \coth \left[ \frac{1}{2} \left( \frac{\gamma t}{5} + x + y \right) \right]}. \tag{18}$$

Case 6 When we take one another coefficient as follows into Eq. (10) with  $V = U'$

$$A_0 = \frac{\gamma}{9}; A_1 = 0; A_2 = -\frac{\gamma}{6}; B_1 = 0; B_2 = \frac{i\gamma}{6}; \omega = \frac{\gamma}{3},$$

we get the following solution:

$$U_6(x, y, t) = -\frac{1}{18} \gamma \left( \frac{\gamma t}{3} + x + y + 3 \operatorname{sech} \left[ \frac{\gamma t}{3} + x + y \right] - 3 \tanh \left[ \frac{\gamma t}{3} + x + y \right] \right). \tag{19}$$

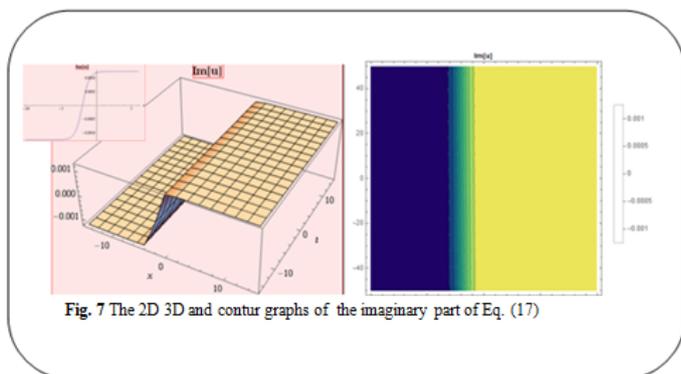
Case 7 Finally, if we take into account the following coefficient into Eq. (10) with  $V = U'$ ,

$$A_0 = \frac{\gamma}{10}; A_1 = 0; A_2 = -\frac{\gamma}{10}; B_1 = 0; B_2 = \frac{i\gamma}{10}; \omega = \frac{\gamma}{5},$$

we obtain another complex hyperbolic function solutions as follows:

$$U_7(x, y, t) = \frac{\gamma}{-5i + 5 \coth \left( \frac{1}{2} \left( \frac{\gamma t}{5} + x + y \right) \right)}. \tag{20}$$

$$U_7(x, y, t) = \frac{\gamma}{-5i + 5 \coth \left[ \frac{1}{2} \left( \frac{\gamma t}{5} + x + y \right) \right]}. \tag{21}$$



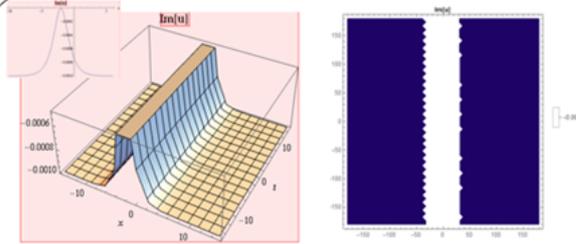


Fig. 8 The 2D 3D and contour graphs of the imaginary part of Eq. (18)

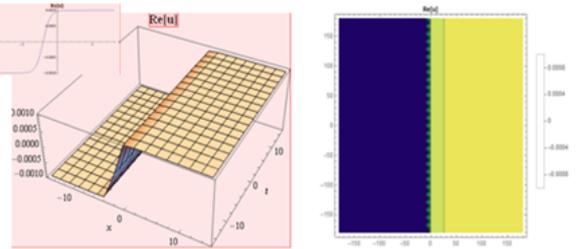


Fig. 9 The 2D 3D and contour graphs of the real part of Eq. (18)

#### 4 Conclusions

In summary, we have successfully applied the SGEM to Eq. (1) to find new complex hyperbolic function solutions. We have plotted 2D and 3D surfaces of the solutions along with contour surfaces under the suitable values of parameters by using computational program along with contour surfaces of them.

It has been observed that the travelling wave solutions obtained in this paper are entirely new complex hyperbolic function solutions compared with [23]. To the best of our knowledge, the application of SGEM to the nonlinear Ablowitz–Kaup–Newell–Segur wave equation with fourth order has not been submitted to literature before.

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