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Some Invariants of Flower Graph

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Abstract

Let G be a graph and let $m_{ij}(G), i, j \geq 1$, represents the number of edge of G, where i and j are the degrees of vertices u and v respectively. In this article, we will compute different polynomials of flower graph $f_{(n\times m)}$, namely $M polynomial$ and $Forgotten$ polynomial. These polynomials will help us to find many degree based topological indices, included different Zagreb indices, harmonic indices and forgotten index.

Keywords: Topological index, Flower Graph, Zagreb index

1 Introduction

In discrete mathematics, graph theory in general, not only the study of different properties of objects but it also tells us about objects having same properties as investigating object. These properties of different objects if of main interest. In particular, graph polynomials related to graph are rich in information about subgraph of investigating graph. Mathematical tools like polynomials and topological based numbers play an important rule, these tools gives important information about properties of chemical compounds. We can find out many hidden information about compounds through theses tools. Multifold graph polynomials are present in literature, various of them turned out to be applicable in mathematical chemistry. Actually, topological index is a numeric quantity that tells us about the whole structure of graph. Commonly used topological indices are degree-based and distance-based topological indices. Theses tools helps us to study physical, chemical reactivities and biological properties [1–5]. In 1947, Winner, firstly introduce the concept of topological index while working on boiling point. In particular, Hosoya polynomial [6] palys an important in the area if distance-based topological indices, we can find out Winer index, Hyper winer index and Tratch-stankevich-zefirove index by Hosoya polynomial. Another important degree-based polynomial is M-polynomial [7] that was introduced by Emeric

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Deutsch and Sandi Klavaza in 2015. Mainly, M-polynomial helps us to gain many information related to degree based topological indices. [8–11].

Definition 1. If G=(V,E) is a graph and $v\varepsilon V$, the $d_v(G)$ denoted the degree of v. Let $m_{ij}(G), i, j \geq 1$, be the number of edges uv of G such that $\{d_v(G), d_u(G)\} = \{i, j\}$. The M-polynomial [7] is defined as

$$M(G;x,y) = \sum_{i \leq j} m_{ij}(G) x^i y^j$$

The firstly introduced topological index was Winer index [12], some one named this winer index **path number**. While studying chemical graph structures, commonly studied index in winer index because of having huge number of application in chemical graph theory [13], [14]. Another oldest topological index is Randic index introduced by Milan Randic in 1975. The Randic index [15] is defined as:

$$R_{-\frac{1}{2}}(G) = \sum_{uv\varepsilon E(G)} \frac{1}{\sqrt{d_u d_v}}$$

After some time Ballobas and Erdos [16] in 1998,and Amic et al. [17] gives the concept of generalized Randic index. Mathematicians and chemists gain many results from this concept [18]. Many results related to generalized randic index have been discussed in [19].

Ghorbani and Azimi [21] in 2012, introduced two new variants of zagreb indices

Definition 2. The first multiple Zagreb index of a graph G can be defined as:

$$PM_1(G) = \prod_{uv\varepsilon E(G)} [d_u + d_v]$$

Definition 3. The second multiple Zagreb index of a graph G is:

$$PM_2(G) = \prod_{uv\varepsilon E(G)} [d_u . d_v]$$

Definition 4. The first Zagreb polynomial of a graph G is:

$$M_1(G,x) = \Sigma_{uv\varepsilon E(G)} x^{[d_u + d_v]}$$

Definition 5. The second Zagreb polynomial of a graph G is:

$$M_1(G,x) = \Sigma_{uv\varepsilon E(G)} x^{[d_u . d_v]}$$

Definition 6. [22] The first and second Zagreb Coindices are define as,

$$\overline{M_1} = \overline{M_1}(G) = \sum_{xy \notin E(G)} [d(x) + d(y)]$$

$$\overline{M_2} = \overline{M_2}(G) = \sum_{xy \notin E(G)} [d(x) . d(y)]$$

Definition 7. [**?**] F-Coindex can be define as

$$\overline{F}(G) = \sum_{xy \notin E(G)} [d(x)^2 + d(y)^2]$$

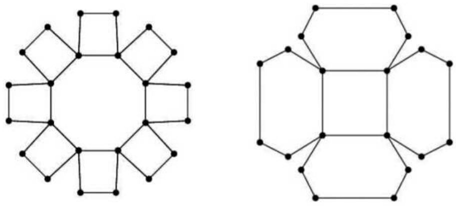

Fig. 1 Flower Graphs $f_{8\times4}$ and $f_{4\times6}$

2 Main Result

Here, we will compute different polynomials of flower graph $f_{(n\times m)}$, namely, M-Polynomial, Forgotten Polynomials and with the help of these polynomials, we will discuss different variants of Flower graph $f_{(n\times m)}$.

Theorem 1. *Let $f_{(n\times m)}$ be Flower graph, where $n=\{0,1,2,...\}$ and $m=\{0,1,2,...\}$. Then the M-polynomial is*

$$M(f_{(n\times m)};x,y)=n(m-3)x^2y^2+2nx^2y^4+nx^4y^4$$

Proof. For Flower graph $f_{(n\times m)}$. The vertex set and edge sets are respectively, $|V(f_{(n\times m)})|=n(m-1)$ and $|E(f_{(n\times m)})|=nm$ The possible vertex degrees are 2 and 4, the contributed edges are $(2,2)$ $(2,4)$ and $(4,4)$. From above condition, we can divided edge set of $f_{(n\times m)}$ into three partitions,

$$E_1f_{(n\times m)}=\{e=uv\varepsilon E(f_{(n\times m)})\quad ;d_u=2,d_v=2\}$$

$$E_2f_{(n\times m)}=\{e=uv\varepsilon E(f_{(n\times m)})\quad ;d_u=2,d_v=4\}$$

$$E_3f_{(n\times m)}=\{e=uv\varepsilon E(f_{(n\times m)})\quad ;d_u=4,d_v=4\}$$

where, $|E_1(f_{(n\times m)})|=n(m-3)$, $|E_2(f_{(n\times m)})|=n$, $|E_3(f_{(n\times m)})|=2n$

From the definition of M-polynomial, we have

$$M(f_{(n\times m)};x,y)=\sum_{i\leq j}m_{ij}(G)x^iy^j$$

$$M(f_{(n\times m)};x,y)=\sum_{2\leq 2}m_{22}(G)x^2y^2+\sum_{2\leq 4}m_{24}(G)x^2y^4+\sum_{4\leq 4}m_{44}(G)x^4y^4$$

$$M(f_{(n\times m)};x,y)=\sum_{uv\varepsilon E_1(f_{n\times m})}m_{22}(G)x^2y^2+\sum_{uv\varepsilon E_2(f_{n\times m})}m_{24}(G)x^2y^4+\sum_{uv\varepsilon E_3(f_{n\times m})}m_{44}(G)x^4y^4$$

$$M(f_{(n\times m)};x,y)=|E_1f_{(n\times m)}|x^2y^2+|E_2f_{(n\times m)}|x^2y^4+|E_3f_{(n\times m)}|x^4y^4$$

$$M(f_{(n\times m)};x,y)=n(m-3)x^2y^2+2nx^2y^4+nx^4y^4$$

Proposition 2. *Let $f_{(n\times m)}$ be Flower graph, where $n=\{0,1,2,...\}$ and $m=\{0,1,2,...\}$. Then the Forgotten Polynomial and Forgotten Index / F-index is given as, respectively,*

$$F(f_{(n\times m)},x)=n(m-3)x^8+nx^{32}+2nx^{20}$$

$$F(f_{(n\times m)})=8mn+48n$$

Proposition 3. *Let $f_{(n\times m)}$ be a flower graph. Then,*

1. $M_1(f_{(n\times m)})=4mn+8n$

2. $M_2(f_{(n\times m)}) = 4mn + 20n$
3. $''M_2(f_{(n\times m)}) = \frac{1}{4}mn - \frac{7}{16}n$
4. $R_\alpha(f_{(n\times m)}) = 2^{2\alpha}mn + (2^{2\alpha} + 2^{(\alpha+1)} - 3)2^{2\alpha}n$
5. $RR_\alpha(f_{(n\times m)}) = 2^{(-2\alpha)}mn + (2^{(-\alpha+1)}.4^\alpha - 2^{(-2\alpha)}.3 + 4^{(-2\alpha)})n$
6. $SSD(f_{(n\times m)}) = 2mn + 2$
7. $H(f_{(n\times m)}) = \frac{1}{4}mn - \frac{7}{24}n$
8. $I(f_{(n\times m)}) = mn + \frac{5}{3}n$
9. $A(f_{(n\times m)}) = 2^3mn + (2^5 - 2^3.3 + 2.3^{(-3)}.4^4)n$

Proof. If

$$M(f_{(n\times m)};x,y) = n(m-3)x^2y^2 + 2nx^2y^4 + nx^4y^4$$

then,

$D_xf(x,y) = 2n(m-3)x^2y^2 + 4nx^2y^4 + 4nx^4y^4$
$D_xD_yf(x,y) = 4n(m-3)x^2y^2 + 16nx^2y^4 + 16x^4y^4$
$S_xS_yf(x,y) = \frac{1}{4}n(m-3)x^2y^2 + \frac{1}{4}nx^2y^4 + \frac{1}{16}nx^4y^4$
$S_x^\alpha S_y^\alpha f(x,y) = 2^{(-2\alpha)}n(m-3)x^2y^2 + 2^{(-\alpha+1)}.4^\alpha nx^2y^4 + 4^{(-2\alpha)}nx^4y^4$
$2S_xJ(x,y) = \frac{1}{4}n(m-3)x^4 + \frac{1}{3}nx^6 + \frac{1}{8}nx^8$
$S_xJD_xD_y(x,y) = n(m-3)x^4 + \frac{8}{3}nx^6 + 2nx^8$
$D_x^3D_y^3f(x,y) = 2^6n(m-3)x^2y^2 + 2^3.4^4nx^2y^4 + 4^6nx^4y^4$
$JD_x^3D_y^3f(x,y) = 2^6n(m-3)x^4 + 2^3.4^4nx^6 + 4^6nx^8$
$Q_{(n-2)}JD_x^3D_y^3f(x,y) = 2^6n(m-3)x^2 + 2^3.4^4nx^4 + 4^6nx^6$
$S_x^3Q_{(n-2)}JD_x^3D_y^3f(x,y) = 2^3n(m-3)x^2 + 2^3.4nx^4 + 4^6.6^{(-3)}nx^6$

2.1 Degree Based Topological Index

In this section, we will discuss different variants of flower graph $f_{n\times m}$ with the help of M-polynomial, Forgotten polynomial and F-index.

- First Zagreb Index $M_1(f_{(n\times m)})$

  $(D_x + D_y)(f_{n\times m})|_{x=y=1} = 4mn + 8n$

- Second Zagreb Index $M_2(f_{(n\times m)})$

  $(D_x.D_y)(f_{n\times m})|_{x=y=1} = 4mn + 20n$

- Modified Second Zagreb Index $''M_2(f_{(n\times m)})$

  $(S_x.S_y)(f_{n\times m})|_{x=y=1} = \frac{1}{4}mn - \frac{7}{16}n$

- Generalized Randic Index $R_\alpha(f_{(n\times m)})$

  $(D_x^\alpha.D_y^\alpha)(f_{n\times m})|_{x=y=1} = 2^{2\alpha}mn + (2^{2\alpha} + 2^{(\alpha+1)} - 3)2^{2\alpha}n$

- Inverse Randic Index $RR_\alpha(f_{(n\times m)})$

  $(S_x^\alpha.S_y^\alpha)(f_{n\times m})|_{x=y=1} = 2^{(-2\alpha)}mn + (2^{(-\alpha+1)}.4^\alpha - 2^{(-2\alpha)}.3 + 4^{(-2\alpha)})n$

- Symmetric Division Index $SSD(f_{(n\times m)})$
  $(S_yD_x+S_xD_y)(f_{n\times m})|_{x=y=1}=2mn+n$
- Harmonic Index $H(f_{(n\times m)})$
  $2S_xJ(f_{n\times m})|_{x1}=\frac{1}{4}mn-\frac{7}{24}n$
- Inverse Index $I(f_{(n\times m)})$
  $S_xJD_xD_y(f_{n\times m})|_{x=1}=mn+\frac{5}{3}n$
- Augmented Zagreb Index $A(f_{(n\times m)})$
  $S_x^3Q_{(n-2)}JD_x^3D_y^3f(x,y)|_{x=1}$
  $=2^3mn+(2^5-2^3.3+2.3^{(-3)}.4^4)n$
- Measure of Irregularity
  $IRM(f(x,y))=8n$
- Reformulated Zagreb Index
  $M_1[L(f_{n\times m})]=8mn+72n+4m$

2.2 Zagreb Coindices

For a graph G and its complement $\overline{G}$. we have,
$F(\overline{G})=n(n-1)^3-6m(n-1)^2+3(n-1)M_1(G)-F(G)$
$F(\overline{f_{n\times m}})=n^4-3n^3+3n^2-73n-8mn+6mn^2-6m$
$\overline{F}(G)=(n-1)M_1(G)-F(G)$
$\overline{F}(f_{n\times m})=8n^2-56n-12mn+4mn^2$
$\overline{F}(\overline{G})=2m(n-1)^2-2(n-1)M_1(G)+F(G)$
$\overline{F}(\overline{f_{n\times m}})=24mn-6mn^2+64n+2m-16n^2$

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