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Revan and hyper-Revan indices of Octahedral and icosahedral networks

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Abstract

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. Recently, the Revan vertex degree concept is defined in Chemical Graph Theory. The first and second Revan indices of G are defined as $R_1(G) = \sum_{uv\in E}[r_G(u)+r_G(v)]$ and $R_2(G) = \sum_{uv\in E}[r_G(u)r_G(v)]$, where uv means that the vertex u and edge v are adjacent in G. The first and second hyper-Revan indices of G are defined as $HR_1(G) = \sum_{uv\in E}[r_G(u)+r_G(v)]^2$ and $HR_2(G) = \sum_{uv\in E}[r_G(u)r_G(v)]^2$. In this paper, we compute the first and second kind of Revan and hyper-Revan indices for the octahedral and icosahedral networks.

Keywords: Revan index, hyper-Revan, Octahedral, Icosahedral, Networks.
AMS 2010 codes: 05C05, 05C07, 05C35.

1 Introduction

Topological indices of large chemical structures such as metal organic frameworks can be extremely useful in both characterization of structures and computing their physico-chemical properties that are otherwise difficult to compute for such large networks of importance in reticular chemistry. Synthesis of novel reticular metal organic frameworks and networks in which covalent fibers weaved into crystals are becoming increasingly important in recent years [1], [2]. A topological index is a numerical parameter mathematically derived from the graph structure. Numerous topological indices have been considered in Theoretical Chemistry and many topological indices are defined by using vertex degree concept. Among them, topological descriptors are of great importance, as they deal with topological characterizations of the molecules. In general, it acts like advanced models in the chemical and control field of applications.

The concept of topological index originated from the pioneering work of Wiener while he was attempting to find structural relationships to boiling points of paraffins. There are a large number of topological indices which

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are classified based on the structural properties of the graphs used for their calculations. In general, they are classified into distance-based topological indices, degree-based topological indices and counting related indices of graphs. The degree-based topological indices, which have reliable predicting power and thus they have been employed in deriving multi-linear regression models for statistical correlation of properties. The topological indices such as atom-bond connectivity and geometric-arithmetic are also used to predict the bio-activity of the chemical compounds. These classes of topological indices are of great importance and play a vital role in chemical characterization. More related contexts can refer to Zhao et al. [3], Basavanagoud et al. [4], Dobrynin et al. [5], Imran et al. [6] and Sardar et al. [7].

Very recently, Kulli defined a novel degree concept in graph theory: the Revan vertex degree and determined exact formulae for oxide and honeycomb networks. We consider only finite, simple and connected graph G. Let $\delta(G)$ denote the minimum degree and $\Delta(G)$ denote the maximum degree of graph G. The Revan vertex degree of a vertex v in G is defined as $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$. The first and second Revan indices of G are defined as:

$$R_1(G) = \sum_{uv\in E} [r_G(u) + r_G(v)],$$

and

$$R_2(G) = \sum_{uv\in E} [r_G(u) r_G(v)],$$

where uv means that the vertex u and edge v are adjacent in G. The first and second hyper Revan indices of G are defined as

$$HR_1(G) = \sum_{uv\in E} [r_G(u) + r_G(v)]^2$$

and

$$HR_2(G) = \sum_{uv\in E} [r_G(u) r_G(v)]^2$$

where $r_G(v) = \Delta(G) + \delta(G) - d_G(v)$. We refer to [8] for details about these indices.
An octahedral sheet-like structure is a ring of octahedral structures which are linked to other rings by sharing corner vertices in a two dimensional plane. An octahedral network of dimension n is denoted by OT_n, where n is the order of circumscribing. The numbers of vertices and edges in OT_n with $n \geq 1$ are $27n^2 + 3n$ and $72n^2$ respectively. We study the Revan indices for the octahedral network as follows.

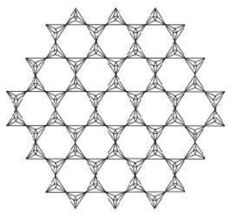

Fig. 1 Octahedral network OT_3

Theorem 1. *Let $G = OT_n$ be the octahedral network. Then the first and the second Revan indices of OT_n are*

$$R_1[OT_n] = 864n^2 + 96n,$$
$$R_2[OT_n] = 2592n^2 + 576n.$$

Table 1 Edge partition of octahedral network

(d_u, d_v)	Number of edges	$r_G(u)$	$r_G(v)$
$(4,4)$	$18n^2+12n$	8	8
$(4,8)$	$36n^2$	8	4
$(8,8)$	$18n^2-12n$	4	4

Proof. Let $G = OT_n$ be the octahedral network. From figure 1, the edge partition of octahedral network OT_n based on degrees of end vertices of each edge is given in table 1. First Revan index of OT_n is calculated as

$$\begin{aligned}
R_1[OT_n] &= \sum_{uv\in E}[r_G(u)+r_G(v)] \\
&= \sum_{uv\in E_{4,4}}[r_G(u)+r_G(v)] + \sum_{uv\in E_{4,8}}[r_G(u)+r_G(v)] \\
&\quad + \sum_{uv\in E_{8,8}}[r_G(u)+r_G(v)] \\
&= (18n^2+12n)[8+8] + 36n^2[8+4] + (18n^2-12n)[4+4] \\
&= 864n^2+96n
\end{aligned}$$

Second Revan index of OT_n is calculated as

$$\begin{aligned}
R_2[OT_n] &= \sum_{ue}[r_G(u)r_G(v)] \\
&= \sum_{uv\in E_{4,4}}[r_G(u)r_G(v)] + \sum_{uv\in E_{4,8}}[r_G(u)r_G(v)] \\
&\quad + \sum_{uv\in E_{8,8}}[r_G(u)r_G(v)] \\
&= (18n^2+12n)[8\times 8] + 36n^2[8\times 4] + (18n^2-12n)[4\times 4] \\
&= 2592n^2+576n.
\end{aligned}$$

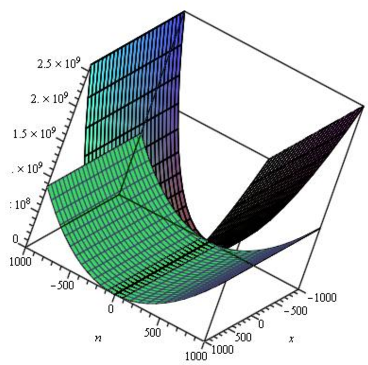

Fig. 2 The green and blue sheets show the comparison result for Revan index of first and second kind Octahedral network OT_n respectively.

Theorem 2. *Let $G = OT_n$ be the octahedral network. Then the first and the second hyper Revan indices of OT_n are*

$$\begin{aligned}
HR_1[OT_n] &= 10944n^2+2304n, \\
HR_2[OT_n] &= 115200n^2+46080n.
\end{aligned}$$

Proof. Let $G = OT_n$ be the octahedral network. Then first hyper Revan index of OT_n is calculated as

$$\begin{aligned}
HR_1[OT_n] &= \sum_{uv\in E}[r_G(u)+r_G(v)]^2 \\
&= \sum_{uv\in E_{4,4}}[r_G(u)+r_G(v)]^2 + \sum_{uv\in E_{4,8}}[r_G(u)+r_G(v)]^2 \\
&\quad + \sum_{uv\in E_{8,8}}[r_G(u)+r_G(v)]^2 \\
&= (18n^2+12n)[8+8]^2 + 36n^2[8+4]^2 + (18n^2-12n)[4+4]^2 \\
&= 10944n^2 + 2304n.
\end{aligned}$$

Second hyper Revan index of OT_n is calculated as

$$\begin{aligned}
HR_2[OT_n] &= \sum_{uv}[r_G(u)r_G(v)]^2 \\
&= \sum_{uv\in E_{4,4}}[r_G(u)r_G(v)]^2 + \sum_{uv\in E_{4,8}}[r_G(u)r_G(v)]^2 \\
&\quad + \sum_{uv\in E_{8,8}}[r_G(u)r_G(v)]^2 \\
&= (18n^2+12n)[8\times 8]^2 + 36n^2[8\times 4]^2 + (18n^2-12n)[4\times 4]^2 \\
&= 115200n^2 + 46080n.
\end{aligned}$$

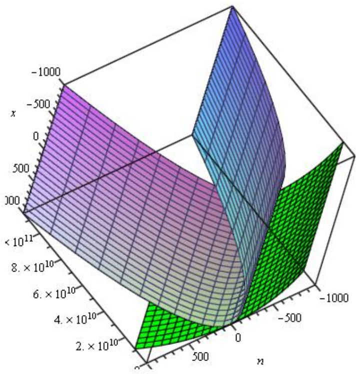

Fig. 3 The green and blue sheets show the comparison result for hyper Revan index of first and second kind Octahedral network OT_n respectively.

Now we introduce another new network based on icosahedron. Icosahedral network is obtained from the octahedral network by replacing all the octahedra with the icosahedra. An n-dimensional icosahedral network is denoted by IS_n. It has $63n^2+3n$ number of vertices and $180n^2$ number of edges. We study the Revan and hyper Revan indices for the icosahedral network as follows.

Theorem 3. *Let $G = IS_n$ be the Icosahedral network. Then the first and the second Revan indices of IS_n are*

$$\begin{aligned}
R_1[IS_n] &= 3150n^2 + 150n, \\
R_2[IS_n] &= 13950n^2 + 1200n.
\end{aligned}$$

Proof. Let $G = IS_n$ be the Icosahedral network. Table 2 shows the edge partition of Icosahedral network of IS_n based on degrees of end vertices of each edge.

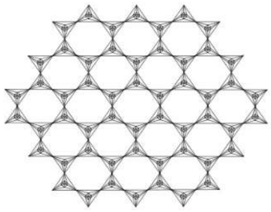

Fig. 4 Icosahedral network IS_3

Table 2 Edge partition of Icosahedral Network

(d_u, d_v)	Number of edges	$r_G(u)$	$r_G(v)$
$(5,5)$	$108n^2 + 18n$	10	10
$(5,10)$	$54n^2 - 6n$	10	5
$(10,10)$	$18n^2 - 12n$	5	5

First Revan index of IS_n is calculated as

$$\begin{aligned}
R_1[IS_n] &= \sum_{uv\in E}[r_G(u)+r_G(v)] \\
&= \sum_{ue\in E_{5,5}}[r_G(u)+r_G(v)] + \sum_{ue\in E_{5,10}}[r_G(u)+r_G(v)] \\
&\quad + \sum_{ue\in E_{10,10}}[r_G(u)+r_G(v)] \\
&= (108n^2+18n)[10+10]+(54n^2-6n)[10+5] \\
&\quad +(18n^2-12n)[5+5] \\
&= 3150n^2+150n.
\end{aligned}$$

Second Revan index of IS_n is calculated as

$$\begin{aligned}
R_2[IS_n] &= \sum_{uv\in E}[r_G(u)r_G(v)] \\
&= \sum_{ue\in E_{5,5}}[r_G(u)r_G(v)] + \sum_{ue\in E_{5,10}}[r_G(u)r_G(v)] \\
&\quad + \sum_{ue\in E_{10,10}}[r_G(u)r_G(v)] \\
&= (108n^2+18n)[10\times 10]+(54n^2-6n)[10\times 5] \\
&\quad +(18n^2-12n)[5\times 5] \\
&= 13950n^2+1200n.
\end{aligned}$$

Theorem 4. *Let $G = IS_n$ be the icosahedral network. Then the first and the second hyper Revan indices of IS_n are*

$$\begin{aligned}
HR_1[IS_n] &= 57150n^2+4650n, \\
HR_2[IS_n] &= 1226250n^2+157500n.
\end{aligned}$$

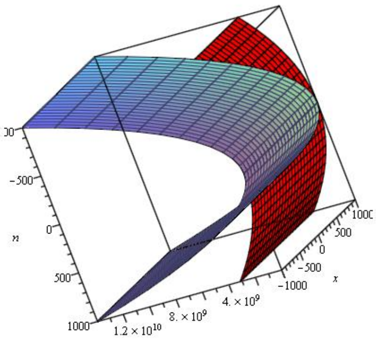

Fig. 5 The red and blue sheets show the comparison result for Revan index of first and second kind Isocahedral network IS_n respectively.

Proof. Let $G = IS_n$ be the icosahedral network. First hyper Revan index of IS_n is calculated as

$$\begin{aligned} HR_1[IS_n] &= \sum_{uv\in E}[r_G(u)+r_G(v)]^2 \\ &= \sum_{ue\in E_{5,5}}[r_G(u)+r_G(v)]^2 + \sum_{ue\in E_{5,10}}[r_G(u)+r_G(v)]^2 \\ &\quad + \sum_{ue\in E_{10,10}}[r_G(u)+r_G(v)]^2 \\ &= (108n^2+18n)[10+10]^2 + (54n^2-6n)[10+5]^2 \\ &\quad +(18n^2-12n)[5+5]^2 \\ &= 57150n^2+4650n. \end{aligned}$$

Second hyper Revan index of IS_n is calculated as

$$\begin{aligned} HR_2[IS_n] &= \sum_{uv\in E}[r_G(u)r_G(v)]^2 \\ &= \sum_{ue\in E_{5,5}}[r_G(u)r_G(v)]^2 + \sum_{ue\in E_{5,10}}[r_G(u)r_G(v)]^2 \\ &\quad + \sum_{ue\in E_{10,10}}[r_G(u)r_G(v)]^2 \\ &= (108n^2+18n)[10\times 10]^2 + (54n^2-6n)[10\times 5]^2 \\ &\quad +(18n^2-12n)[5\times 5]^2 \\ &= 1226250n^2+157500n. \end{aligned}$$

2 Conclusion

In the present report, we have computed First and second Revan and hyper Revan indices of Octahedral and icosahedral networks.

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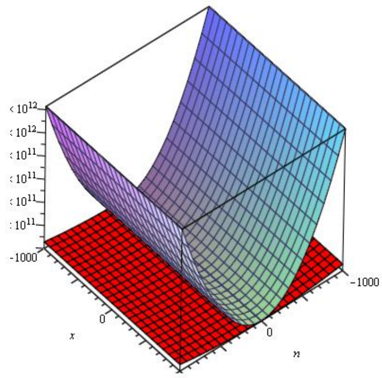

Fig. 6 The red and blue sheets show the comparison result for hyper Revan index of first and second kind Isocahedral network IS_n respectively.

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