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Acousto-optic modulation in ion implanted semiconductor plasmas having SDDC

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Abstract

The present paper is aimed to the exploration of acousto-optic (AO) modulational amplification in ion implanted semiconductors. The AO modulational process has been treated as a four wave parametric mixing process and the effective third-order acousto-optic susceptibility characterizing the instability process has been deduced. By considering that the origin of modulational interaction lies in the third order AO susceptibility arising from the nonlinear induced current density and using the coupled mode theory, an analytical investigation of an intense laser beam in a strain dependent dielectric constant (SDDC) semiconductor crystal is presented. We found a significant change in threshold and gain characteristics with changes in charge imbalance parameter. The presence of colloidal grains (CGs) plays an effective role in changing the threshold intensity and effective gain constant.

1 Introduction

The acousto-optic modulation is the convenient and widely used means of controlling intensity and/or phase of propagating radiation [1, 2]. The conventional way of controlling phonons takes advantage of size confinement. These are used to tailor the propagation properties of acoustic and optical phonons, as well as their interactions with optical fields. The concept of transverse modulational instability originates from a space-time analogy that exists when the dispersion is replaced by diffraction and instability of a plane wave in self focusing Kerr medium.

A large number of attempts have been made on the investigation of modulational interactions. It has found that growth rate of instability in strain dependent dielectric (SDDC) semiconductor crystals such as $BaTiO_3$ is large enough [3–8]. These modulations have number of applications including the impression of information onto optical pulses, mode-locking, and optical beam deflection [9, 11]. An important field of study in nonlinear

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acoustics is the amplification/attenuation and frequency mixing of waves in III-V semiconductors because of its immediate relevance to problems of optical communication systems. In most of the cases of nonlinear optical interactions, the SDDC materials are normally ignored. Looking at the potential of the semiconductors in modern optoelectronic technology, the analytical investigations of some basic nonlinear processes in such crystals are of considerable significance due to its vast technological potentialities.

Ion implantation a process in which use of ions is made to dope and modify semiconductor materials. The colloids that act as third species or foreign particles are the result of the implantation of any metal ion inside the medium. Colloidal plasmas are a new and fascinating field of plasma physics. These colloids acquire a negative charge through the sticking of high mobility free electrons on them. The negatively charged colloidal grains (CGs) are assumed to be of uniform size and smaller than both the wavelength under study and the carrier Debye length [12, 13]. The high mobility charge carrier makes diffusion effects even more relevant in semiconductor technology as they (charge carriers) travel significant distances before recombining. Therefore inclusion of carrier diffusion in theoretical studies of nonlinear wave-wave interactions seems to be very important from the fundamental as well as application view points and thus attracted many researchers in the last decades [14].

Present analysis is based on coupled mode theory for investigating the effects of negatively charged CGs on the acousto-optic modulation due to parametric four wave mixing process in ion implanted semiconductors having SDDC. The intense laser beam electrostrively generates an acoustic wave within the semiconductor medium that induces an interaction between the free carriers (through electron plasma wave) and the acoustic phonons (through material vibration). This interaction induces a strong space charge field that modulates the pump beam. Thus the optical and acoustic waves present in an acousto-optic modulator can be strongly amplified through nonlinear optical pumping. The presence of charged CGs in semiconductor plasma medium add new dimensions to the analysis presented in n-doped semiconductors with strain dependent dielectric constants (SDDC). It is found that the presence of colloidal grains (CGs) plays an effective role in changing the threshold intensity and effective gain constant.

2 Theoretical Formulation

The well-known hydrodynamic description of semiconductor plasmas has been considered to study the acousto-optic (AO) modulation in ion-implanted n-type semiconductor plasma having SDDC (for which $k_a l \ll 1$, k_a and l being the acoustic wave number and mean free path of an electron, respectively). In order to study the crystal, a uniform $(|k_0| \approx 0)$ and homogeneous optical pump field $\overrightarrow{E_0} = \hat{x}E_0 \exp(i\omega_0 t)$ that propagates with parametrically generated acoustic wave within the medium is considered. Since these acoustic grating results in a proportional refractive index variation through the medium photo- elastic response, the incident optical field will be diffracted by this grating to generate an additional field within the medium. The diffracted beam is either frequency up-shifted (anti –stokes mode) or down-shifted (stokes mode) depending on the orientation of the incident wave. In presence of strain, stokes and anti stokes mode can be coupled over a long interaction path. This coupled wave propagates as a solitary wave form in the non dispersive regime of acoustic wave and can be amplified under appropriate phase matching condition.

We proceed with the following basic equations describing acousto-optic modulation under one dimensional configuration (along x-axis):

$$\frac{\partial v_0}{\partial t} + \upsilon v_0 = -\frac{eE_{0.}}{m} \tag{1}$$

$$\frac{\partial v_1}{\partial t} + \upsilon v_1 + \left(v_0 \frac{\partial}{\partial x} \right) v_1 = -\frac{eE_1}{m} - \frac{k_B T}{m n_0} . \nabla n_1 \tag{2}$$

$$\frac{\partial n_1}{\partial t} + \delta_d n_0 \frac{\partial v_1}{\partial x} + v_0 \frac{\partial n_1}{\partial x} + D \frac{\partial^2 n_1}{\partial x^2} = 0 \tag{3}$$

in which diffusion coefficient

$$D = \frac{k_B T}{e}\mu \tag{4}$$

Equations (1) and (2) represent the zeroth and first order oscillatory fluid velocities under influence of the respective fields. m and υ represents the effective mass and momentum transfer collision frequency of electrons. Equation (3) is the continuity equation, where n_0 and n_1 are the equilibrium and perturbed carrier densities. In III-V semiconductor plasmas (i.e. $n_{0e} \approx n_{0h} \approx n_0$) embedded with CGs, the electrons and holes from all directions colloidal with CGs and get stick onto them. However, greater number of electrons stick onto the CGs as compared to holes in given time interval. The charge imbalance parameter is define as $\delta_d = \frac{Z_d e n_{0d}}{n_{0h}}$. The plasma is quasi neutral and the conservation of particle number density must always holds. Thus charged should be hold:$n_{0e}e = n_{0h}e - Z_d e n_{0d}$
where n_{0d} is concentration of CGs and Z_d is charge state of colloids [15–17]. In equation (4), $\mu \left(= {}^{e}/_{m\nu}\right)$ is the electron mobility, k_B is the Boltzmann's constant and T the electron temperature in ^0K.

$$\frac{\partial^2 u}{\partial t^2} = \frac{c}{\rho}\frac{\partial^2 u}{\partial x^2} - \frac{(\varepsilon g E_0)}{\rho}\cdot\frac{\partial E_1^*}{\partial x} \tag{5}$$

$$\frac{\partial E_1}{\partial x} = -\frac{n_1 e}{\varepsilon} + gE_{0.}\frac{\partial^2 u^*}{\partial x^2} \tag{6}$$

$$P_{ao} = -\varepsilon g E_{0.}\Delta u^* \tag{7}$$

The equation (5) describes the lattice displacement in an ion implanted semiconductor plasma in which ρ, u and c being mass density of the crystal, lattice displacement and crystal elastic stiffness, respectively. The migration of charge that leads to strong space charge field $\overrightarrow{E_1}$ is determined by the Poisson's equation (6) in which $\overrightarrow{D} = \varepsilon \overrightarrow{E}\,(1+gS)$ and $\varepsilon = \varepsilon_0.\varepsilon_s$ where $\overrightarrow{D}$ is the electric displacement, ε_s is dielectric constant in absence of any strain $S(=du/dx)$ and $g\,(=\varepsilon_s/3)$ is coupling constant due to SDDC. The electrostatic force thus produced is the origin of acousto-optic strain within the medium given by equation (7), the nonlinear polarization P_{ao} .

Physically in acousto-optic modulation process a carrier density perturbation is created in the medium under the influence of a strong pump beam, which is associated with phonon mode and varies as the acoustic frequency. The equation for density fluctuation of the coupled electron plasma wave in ion implanted semiconductor is obtained from equations (1) to (7) using linearized perturbation theory as

$$\frac{\partial^2 n_1}{\partial t^2} + \upsilon\frac{\partial n_1}{\partial t} + \upsilon D\frac{\partial^2 n_1}{\partial x^2} + \bar{\omega}_p^2 n_1 + \frac{ikP_{ao}}{e}\delta_d\omega_p^2 = ik_{\pm}n_1\bar{E}_b \tag{8}$$

In which $\bar{\omega}_p^2 = \delta_d\left(\omega_p^2 + k_{\pm}^2\left(\frac{k_B T}{m}\right)\right)$ being modified plasma frequency, $\omega_p^2 = \left(\frac{n_0 e^2}{m\varepsilon}\right)$ is electron plasma frequency and $\bar{E}_b = \frac{eE_0}{m}$.

The pump beam is thus phase modulated by the density perturbations to produce enforced disturbances at the upper $(\omega_a + \omega_0)$ and lower $(\omega_a - \omega_0)$ sideband frequencies. The higher order components are filtered out by assuming a long interaction path (by considering the crystal to be of infinite extent). The modulation process under consideration must also fulfill the phase matching conditions $k_{\pm} = k_a \pm k_a$ and $\omega_{\pm} = \omega_a \pm \omega_a$ known as momentum and energy conservation relation, respectively under spatially uniform laser irradiations $|k_0| \approx 0$ such that $|k_{\pm}| = |k_a \pm k_0| \approx |k_a| = k$ (say). The density modulation oscillating at the upper $(\omega_a + \omega_0)$ and lower $(\omega_a - \omega_0)$ sideband frequencies can be represented after simplification as

$$n_1(\omega_{\pm}, k_{\pm}) = -\frac{ikP_{ao}}{e}\delta_d\omega_p^2\left[\bar{\omega}_p^2 - \omega_{\pm}^2 - \upsilon k_{\pm}^2 D - i\upsilon\omega_{\pm} - ik_{\pm}\bar{E}_b\right]^{-1} \tag{9}$$

The density perturbations oscillating at the forced frequency in equation (9) are obtained under quasi-static approximation and by neglecting the Doppler shift under the assumption that $\omega_0 \gg \upsilon \gg k_0 v_0$. We also neglected the contribution of transition dipole momentum in the analysis of modulational instability to study the effect of nonlinear current density.

The induced nonlinear current densities for upper and lower sidebands may be expressed as

$$J_+(\omega_+, k_+) = -eD\frac{\partial n_1}{\partial x}(\omega_+, k_+) \tag{10a}$$

and

$$J_-(\omega_-, k_-) = -eD\frac{\partial n_1}{\partial x}(\omega_-, k_-) \tag{10b}$$

In centrosymmetric system, the four-wave parametric interaction involving the incident pump, the upper and lower side band signals and induced acousto-optical idler wave characterized by the cubic nonlinear susceptibility tensor effectively results in the modulational instability of the pump. Hence the induced cubic nonlinear optical polarization at the modulated frequencies

$P_{NL}(\omega_\pm, k_\pm)$ as the time integral of the nonlinear current density $J_{NL}(\omega_\pm, k_\pm)$ as we have

$$P_{NL}(\omega_\pm, k_\pm) = \int J_{NL}(\omega_\pm, k_\pm)\, dt \tag{11}$$

The effective nonlinear third order polarization has contribution from both individual side band and can be represented as

$$P_{eff}(\omega_\pm, k_\pm) = P_{NL}(\omega_+, k_+) + P_{NL}(\omega_-, k_-) \tag{12}$$

The effective nonlinear third order polarization after algebraic simplification as

$$P_{eff} = -\frac{2\nu\varepsilon^2 g^2 k^4 D\delta_d \omega_p^2 |E_0|^2 E_1}{\rho(\omega_a^2 - k_a^2 v_a^2)}\left[\frac{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2) + (2i\bar{E}_b\Delta)}{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2)^2 + 4k^2\bar{E}_b^2\Delta^2}\right] \tag{13}$$

Thus the effective acousto-optic nonlinear susceptibility of medium including the contribution due to the drift and diffusion of charge carriers can be obtained as

$$\xi_{eff} = -\frac{2\nu\varepsilon^2 g^2 k^4 D\delta_d \omega_p^2}{\rho(\omega_a^2 - k_a^2 v_a^2)}\left[\frac{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2) + (2i\bar{E}_b\Delta)}{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2)^2 + 4k^2\bar{E}_b^2\Delta^2}\right] \tag{14}$$

Equation (14) may be separated into real and imaginary parts $(\xi_{eff.}) = (\xi_{eff.} + i(\xi_{eff.}))$ as,

$$[\xi_{eff}]_{real} = -\frac{2\nu\varepsilon^2 g^2 k^4 D\delta_d \omega_p^2}{\rho(\omega_a^2 - k_a^2 v_a^2)}\left[\frac{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2)}{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2)^2 + 4k^2\bar{E}_b^2\Delta^2}\right] \tag{15}$$

and

$$[\xi_{eff}]_{imag.} = -\frac{2\nu\varepsilon^2 g^2 k^4 D\delta_d \omega_p^2}{\rho(\omega_a^2 - k_a^2 v_a^2)}\left[\frac{2\bar{E}_b\Delta}{(\Delta^2 - k^2\bar{E}_b^2 + \omega_0^2\nu^2)^2 + 4k^2\bar{E}_b^2\Delta^2}\right] \tag{16}$$

In order to explore the possibility of the modulational amplification, the nonlinear steady state growth rate of the modulated waveform of the pump exceeding a threshold value is obtained through the relation

$$[g_{eff}]_{ao} = -\frac{k}{2\varepsilon}[\xi_{eff.}]_{imga.}|E_0|^2 \tag{17}$$

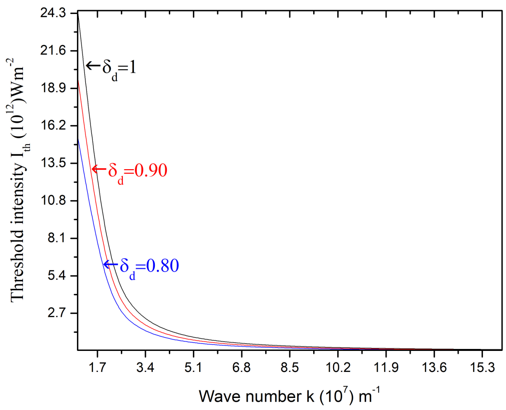

Fig. 1 Variation of threshold intensity I_{th} Vs wave number k.

Thus from equations (16) and (17), in order to attain a growth of the modulated signal, it may be infer that $[\xi_{eff}]_{imag.}$ should be negative. Thus condition for achieving a positive growth rate is as follows:

$$k^2\bar{E}_b^2 > (\Delta^2 + \omega_0^2 \nu^2), \tag{18}$$

It is evident from above discussions that not only the presence of particle diffusion is an necessity to induce instability but also the value of applied pump intensity must be well above the threshold defined by equation (18). The threshold value of the pump amplitude require for the onset of the modulational amplification is obtained as

$$E_{th} = \frac{m}{ek} \cdot \left(\Delta^2 + \omega_0^2 \nu^2\right)^{1/2} \tag{19}$$

The corresponding pump intensity can be obtained by using the relation

$$I_{th} = \frac{1}{2}\eta \varepsilon_0 c |E_{0th}|^2. \tag{20}$$

3 Results and Discussion

The numerical calculations are performed for the n-type semiconductor sample ($BaTiO_3$) at 300 K duly irradiated by 10.6 µm CO_2 laser.

The following material parameters have been considered as follows: $m = 0.0145m_0$ (m_0 being the free electron rest mass), $m_d = 1.67 \times 10^{-27} kg \cdot \varepsilon_s = 2000$,$\rho = 4 \times 10^3 kg \cdot m^{-3}$, $\eta = 2.4 n_0 = 10^{25} m^{-3}$, $\nu_e = 5 \times 10^{11} s^{-1}$, $\omega_0 = 1.78 \times 10^{13} s^{-1}$, $\omega_a = 1.6 \times 10^{13} s^{-1}$, and $v_a = 3 \times 10^3 m \cdot s^{-1}$.

The threshold characteristics are illustrated in Figures 1. Figure 1 shows the variation of threshold intensity I_{th} with wave number (equation 20). The I_{th} decreases abruptly as the k increases in ion implanted semiconductors with chosen values of charge imbalance parameter δ_d. It can be seen that at $k \approx 1 \times 10^6 m^{-1}$ the intensity is $I_{th} \approx 2.451 \times 10^{13} W \cdot m^{-2}$. The I_{th} decreases abruptly with wave number k up to $k \gg 4 \times 10^7 m^{-1}$ and then afterwards decreases slowly. It is evident from the figure that as the fraction of negative charge stick on to the CGs δ_d, decreases (i.e. $\delta_d \approx 1 > 0.90 > 0.80$), the I_{th} gets lowered.

The nature of dispersion arising due to third order real effective susceptibility $[\chi_{eff.}]_{real}$ for ion implanted semiconductor plasmas has been displayed in Figure 2 with respect carrier concentration[in terms of ω_p/ω_0] when$E_0 = 5 \times 10^7 V m^{-1}$ (equation 15). It can be observed that there exists a distinct anomalous parametric dispersion regime that varies with n_0 and δ_d both. The $[\chi_{eff.}]_{real}$, can be both positive and negative under

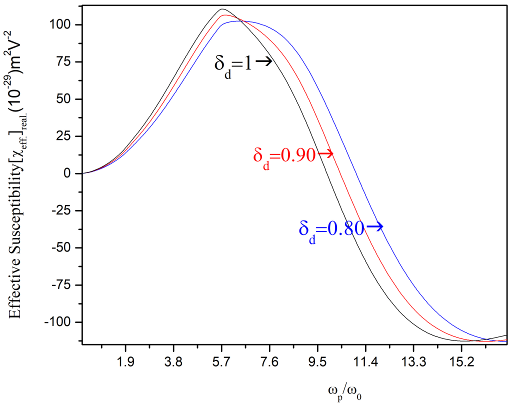

Fig. 2 Effective susceptibility $[\chi_{eff.}]_{real}$ with carrier concentration [in terms of ω_p/ω_0] when $E_0 = 5\times10^7 Vm^{-1}$.

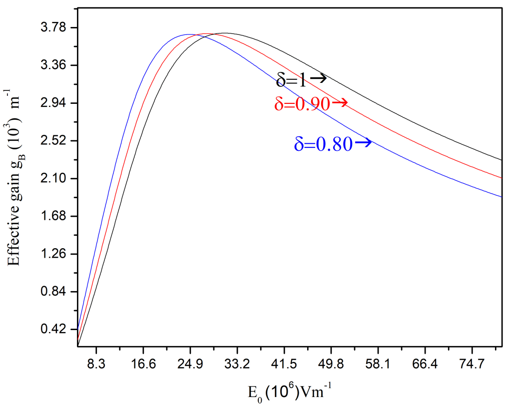

Fig. 3 Effective gain $[g_{eff.}]_{ao}$ with pump electric fieldE_0 at $k_0 = 5\times10^7 m^{-1}$ and $n_0 = 10^{26} m^{-1}$.

the anomalous regime. For ${}^{\omega_p}/_{\omega_0} < 8.136$, $[\chi_{eff.}]_{real}$ is a positive quantity which initially increases and then achieves a maximum value at about ${}^{\omega_p}/_{\omega_0} \approx 5.88$. A slight turning in ${}^{\omega_p}/_{\omega_0}$ beyond this point causes a sharp fall in $[\chi_{eff.}]_{real}$, which first vanishes at near ${}^{\omega_p}/_{\omega_0} \approx 10$. After this resonance condition$[\chi_{eff.}]_{real}$, decreases very sharply and achieves a minimum value.

The variation of effective gain $[g_{eff.}]_{ao}$ with pump electric field E_0 is depicted in Figure 3 when $k_0 = 5\times10^7 m^{-1}$, $D = 0.896$, $n_0 = 10^{26} m^{-3}$ (equation 17). It may be inferred for this set of data that at initially the $[g_{eff.}]_{ao}$ increases abruptly with increase in E_0 and attains maximum value at $E_0 < 2.5\times10^7 Vm^{-1}$. Beyond this point $E_0 > 4\times10^7 Vm^{-1}$ the $[g_{eff.}]_{ao}$ decreases slowly and almost constant in further increase. On changing the fraction δ_d, the $[g_{eff.}]_{ao}$ shows anomalous behavior as shown in figure. Figure 4 shows the effective gain $[g_{eff.}]_{ao}$ curve with wave number k_0. The $[g_{eff.}]_{ao}$ increase gradually as increase in k_0. On decrease in value of δ_d, the $[g_{eff.}]_{ao}$ increases slightly

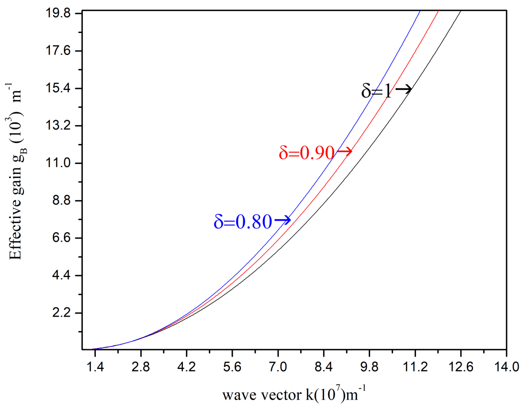

Fig. 4 Effective gain $\left[g_{eff.}\right]_{ao}$ with wave number k_0 at $E_0 = 5 \times 10^7 V m^{-1}$ and $n_0 = 10^{26} m^{-1}$.

4 Conclusion

In present analysis, we have analytically studied the influence of CGs on the threshold intensity, effective susceptibility and effective gain constant. It is observed from the study that significant change in threshold and gain characteristics when the charge imbalance parameter is slightly changed. The presence of CGs plays an effective role in changing the threshold intensity and effective gain constant.

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