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Wall Properties and Slip Consequences on Peristaltic Transport of a Casson Liquid in a Flexible Channel with Heat Transfer

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Abstract

In this paper, the peristaltic wave propagation of a Non-Newtonian Casson liquid in a non-uniform (flexible)channel with wall properties and heat transfer is analyzed. Long wavelength and low Reynolds number approximations are considered. Analytical solution for velocity, stream function and temperature in terms of various physical parameters is obtained. The impact of yield stress, elasticity, slip and non-uniformity parameters on the peristaltic flow of Casson liquidare observed through graphs and discussed. The important outcome is that an increase in rigidity, stiffness and viscous damping force of the wall results in the enhancement of the size and number of bolus formed in the flow pattern.

Keywords: Peristaltic wave; wall effects; rigidity; casson liquid; heat transfer.
AMS 2010 codes: 7605.

1 Introduction

Peristaltic wave proliferation is a system of transporting liquid from lower pressure to higher pressure. This peristaltic wave spread discovers its applications in biology, medical and engineering field. Further, in view of this guideline, modern peristaltic pumps are likewise outlined. Numerous examinations on peristaltic

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stream wonder have been performed in tube and channels. A large portion of the examinations in industry and science demonstrate that the liquid conduct is non-Newtonian. Subsequently a few specialists are focusing on the stream of non-Newtonian liquids through peristalsis in tubes and channels (Scott Blair [1], Vajravelu et al. [2]- [3]. A large portion of the examinations in the literature has influenced a way to deal with concentrate to and comprehend the urine move through ureter and flow of blood in large arteries where shear rate is high, yet it neglects to clarify the complex rheological conduct of blood in thin veins where the shear rates are low. The examination on non-Newtonian nature of bloodstream has been of most significance to scientists lately because of their application in exploring the conduct of blood in narrow arteries. Casson liquid is one such non-Newtonian liquid which shows yield pressure and fits the streaming blood when the shear rates are low (See Casson [4]). Ref. [1] watched that at low shear rates, Casson model was more precise in anticipating the physiological practices of blood. Srivastava and Srivastava [5] assumed blood as immiscible fluids and studied on the peristaltic pumping of blood by considering Casson fluid. Recently, Vajravelu et al. [6] investigated the flow of Casson liquid under peristalsis and elasticity. Numerous researchers did the investigation of Casson model under various geometric conditions as of late (Nagarani [7]; Prasad et al. [8]- [9]; Vajravelu et al. [10]; Prasad et al. [11]). In the event that the liquid is moving with the impact of peristalsis, it is extremely intriguing reality to think about the elastic nature of the channel. Numerous examinations have been conveyed with a stream of non-Newtonian liquids in elastic tubes (Vajravelu et al. [12], Nadeem and Ijaz [13], Siva et. al. [14], Badari et. al. [15]- [16], Rajashekhar et al. [17]).

Heat transfer in a biological framework is a natural phenomenon. Further, bio-heat transfer show is considered keeping in mind the end goal to outline the impacts of blood perfusion and metabolic heat age in living tissues. Heat is a type of energy that is transferred across a boundary because of the temperature distinction. The standards of variety in temperature in designing structures can be associated with the human body to choose how the body trades heat. Heat is created in the body by the constant absorption of nutrient supplements which offers vitality to the systems of the body. Blood moving through the vessels goes about as a convective liquid and keeps in charge any advancement of heat inside the tissues of the body. The heat conveyed by the blood is administered by the temperature of the neighbouring tissues, the measurement of the veins, the thickness of the liquid, liquid speed and the heat trade coefficient of the blood. The investigation of heat transfer impacts alongside slip conditions on peristalsis has procured a tremendous measure of enthusiasm among the scientists in the previous decades. The examination of heat transfer has discovered its application in the field of biofluid mechanics, substance designing and prescription(See Prasad et al. [18]- [20]). Numerous analysts analysed the communication amongst peristalsis and heat transfer in various geometries along with/without slip conditions.Hayat et al [**?**] employed analytical method to examine the Influence of slip and heat transferon the peristaltic transport in a channel.Radhakrishnamacharya et al. [23] observed the flow of Newtonian liquid in a channel with wall effects and heat transfer.Hayat et al. [24] continued the work of Ref. [23] by considering Power law fluid. Further,Lakshminarayana et al. [25] examined the heat transfer analysis on the MHD peristaltic flow of a Bingham fluid in a channel with wall properties.Recently,Nabil et al. [26] concentrated on the peristaltic motion of couple stress liquid in a porous channel with heat transfer. In a flexible channel the peristaltic motion of dusty fluid with MHD and heat transfer was investigated by Hayat andJaved [27].

In perspective of this, the analysis concentrates on the impact of elasticity,boundary slip and heat transfer on the flow of Casson liquid in a channel. An expression for velocity, stream function,and temperature has been calculated analytically under the assumptions of long wavelength and small Reynolds number approximations. The liquid flow depends on many physical expressions such as wall properties, slip parameter, non-uniformity parameter and yield stress. These effects of parameters are discussed in detail through graphs by using MATLAB. Since Casson model closely describes blood flow in physiological systems, the results obtained have important applications in the cardiovascular system.

2 Mathematical Formulation

Consider a peristaltic flow of a Casson liquid in a channel with heat transfer and wall effects (see Fig. 1) on which sinusoidal waves of moderate amplitude are imposed. The walls are taken like stretched membranes. The geometry of the channel wall is given by

$$y = \eta(x,t) = D(x) + aSin\frac{2\pi}{\lambda}(X - ct) \tag{1}$$

where $D(x) = d + \omega' X, \omega' \ll 1$, a is the amplitude, λ is the wave length, d is the mean half width of the channel, $\omega'$is the dimensional non-uniformity of the channel.

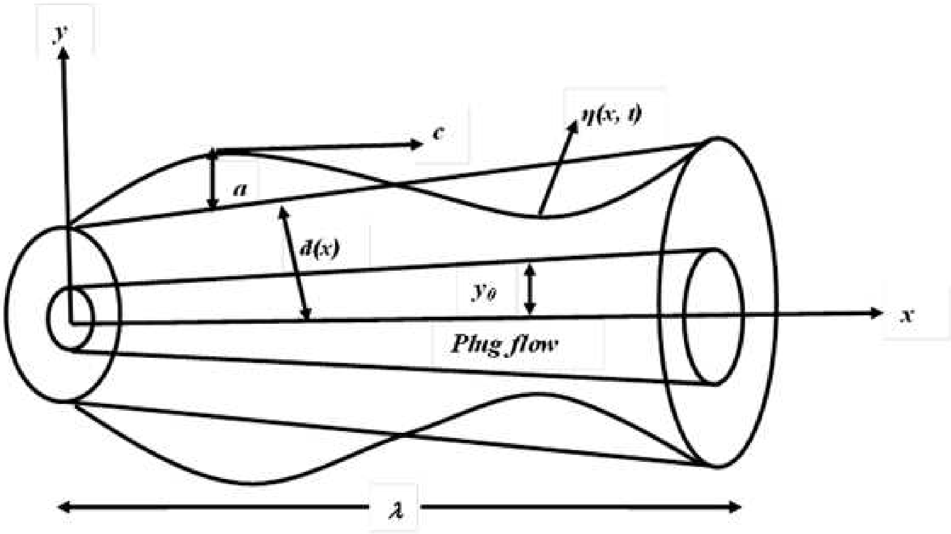


Fig.1: Physical model of the problem

The equations governing the motion for the present problem are

$$\frac{\partial \Omega}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{2}$$

$$\rho\left(\frac{\partial \Omega}{\partial t} + \Omega\frac{\partial \Omega}{\partial x} + V\frac{\partial \Omega}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 \Omega}{\partial x^2}\right) + \frac{\partial}{\partial y}\left(\tau_0^{\frac{1}{2}} + \left(-\mu\frac{\partial \Omega}{\partial y}\right)^{\frac{1}{2}}\right)^2 \tag{3}$$

$$\rho\left(\frac{\partial V}{\partial t} + \Omega\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) \tag{4}$$

$$\xi\left(\frac{\partial \Theta}{\partial t} + \Omega\left(\frac{\partial \Theta}{\partial x}\right) + V\frac{\partial \Theta}{\partial y}\right) = \frac{k}{\rho}\left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) + \upsilon\left\{2\left[\left(\frac{\partial \Omega}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2\right] + \left(\frac{\partial \Omega}{\partial x} + \frac{\partial V}{\partial y}\right)^2\right\} \tag{5}$$

Where Ω and V are the components of velocity along x and y directions respectively, ρ is the density, $\mu$is the coefficient of viscosity of the liquid, p is the pressure,d is the mean half width of the channel, a is the amplitude, λ is the wave length, c is the phase speed of the wave, m' is the dimensional non-uniformity of the channel, ξ is the specific heat at constant volume, υ is the kinematic viscosity of the liquid, k is the thermal conductivity of the liquid, Θ is the temperature of the liquid.

The governing equations of motion of the flexible wall may be expressed as

$$\Gamma^*(\eta) = p - p_0 \tag{6}$$

where Γ^* is an operator, which is used to represent the motion of stretched membrane with viscosity damping forces such that

$$\Gamma^* = -\alpha\frac{\partial^2}{\partial x^2} + \beta\frac{\partial^2}{\partial t^2} + \gamma\frac{\partial}{\partial t} \tag{7}$$

Here α is the elastic tension in the membrane,β is the mass per unit area, γ is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to the tension in the muscles. Continuity of stress at $y = \eta$ and using x – momentum equation yields

$$\frac{\partial}{\partial x}\Gamma^*(\eta) = \frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2\Omega}{\partial x^2}\right) + \frac{\partial}{\partial y}\left(\tau_0^{\frac{1}{2}} + \left(-\mu\frac{\partial\Omega}{\partial y}\right)^{\frac{1}{2}}\right)^2 - \rho\left(\frac{\partial\Omega}{\partial t} + \Omega\frac{\partial\Omega}{\partial x} + V\frac{\partial\Omega}{\partial y}\right) \tag{8}$$

$$\Omega = -h_1\tfrac{\partial\Omega}{\partial y}\ at\ y = \eta = d + \omega' x + aSin\tfrac{2\pi}{\lambda}(x - ct) \tag{9}$$

$$\frac{\partial\Theta}{\partial y} = 0\ on\ y = y_0,\ \ \Theta = \Theta_1\ on\ y = \eta \tag{10}$$

where $\Gamma^* = -\alpha\frac{\partial^2}{\partial x^2} + \beta\frac{\partial^2}{\partial t^2} + \gamma\frac{\partial}{\partial t}$. For simplicity, we assume $p_0 = 0$ and introduce the non-dimensional stream function and non-dimensional quantities as

$$\Psi(\Omega, V) = \Psi\left(\frac{\partial\Psi}{\partial y}, -\frac{\partial\Psi}{\partial x}\right),$$

$$x' = \frac{x}{\lambda}, y' = \frac{y}{d}, \Psi' = \frac{\Psi}{cd}, t' = \frac{ct}{\lambda}, \eta' = \frac{\eta}{d}, p' = \frac{d^2}{c\lambda\mu}, k' = \frac{k}{d^2}, \tau_0' = \frac{d}{\mu c}, T = \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0} \tag{11}$$

After dropping primes, we obtain non-dimensional governing equations

$$R\delta\left(\frac{\partial^2\Psi}{\partial t\partial y} + \frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial x\partial y} - \frac{\partial\Psi}{\partial x}\frac{\partial^2\Psi}{\partial y^2}\right) = -\frac{\partial p}{\partial x} + \delta^2\left(\frac{\partial^3\Psi}{\partial x^2\partial y}\right) + \frac{\partial}{\partial y}\left(\tau_0{}^{\frac{1}{2}} + \left(-\frac{\partial^2\Psi}{\partial y^2}\right)^{\frac{1}{2}}\right)^2, \tag{12}$$

$$R\delta\left(\frac{\partial^2\Psi}{\partial t\partial x} + \frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial x^2} - \frac{\partial\Psi}{\partial x}\frac{\partial^2\Psi}{\partial x\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^4\left(\frac{\partial^3\Psi}{\partial x^3}\right) + \delta^2\left(\frac{\partial^3\Psi}{\partial x\partial y^2}\right), \tag{13}$$

$$R\delta\left(\tfrac{\partial T}{\partial t} + \tfrac{\partial\Psi}{\partial y}\tfrac{\partial T}{\partial x} - \tfrac{\partial\Psi}{\partial x}\tfrac{\partial T}{\partial y}\right) = \tfrac{1}{\mathrm{Pr}}\left(\delta^2\tfrac{\partial^2}{\partial x^2} + \tfrac{\partial^2}{\partial y^2}\right)T + Ec\left(4\delta^2\left(\tfrac{\partial^2\Psi}{\partial x\partial y}\right)^2 + \left(\tfrac{\partial^2\Psi}{\partial y^2} - \delta^2\tfrac{\partial^2\Psi}{\partial x^2}\right)^2\right), \tag{14}$$

$$\frac{\partial\Psi}{\partial y} = -\zeta\frac{\partial^2\Psi}{\partial y^2}\ at\ y = \eta = 1 + \omega x + \varepsilon Sin2\pi(x - t) \tag{15}$$

$$\delta^2\left(\tfrac{\partial^3\Psi}{\partial x^2\partial y}\right) + \tfrac{\partial}{\partial y}\left(\tau_0^{\frac{1}{2}} + \left(-\tfrac{\partial^2\Psi}{\partial y^2}\right)^{\frac{1}{2}}\right)^2 - R\delta\left(\tfrac{\partial^2\Psi}{\partial t\partial y} + \tfrac{\partial\Psi}{\partial y}\tfrac{\partial^2\Psi}{\partial x\partial y} - \tfrac{\partial\Psi}{\partial x}\tfrac{\partial^2\Psi}{\partial y^2}\right) = \left(A_1\tfrac{\partial^3}{\partial x^3} + A_2\tfrac{\partial^3}{\partial x\partial t^2} + A_3\tfrac{\partial^2}{\partial x\partial t}\right)\eta \tag{16}$$

Non-dimensional boundary conditions are

$$\begin{aligned} &\Psi_p = 0, \Psi_{yy} = \tau_0 \text{ at } y = 0, \\ &\Psi = \Psi_p, \tfrac{\partial T}{\partial y} = 0 \text{ at } y = y_0, \\ &\theta = 1 \text{ at } y = \eta. \end{aligned} \tag{17}$$

where $\varepsilon = \frac{a}{d}, \delta = \frac{d}{\lambda}$ are geometric parameters, $R = \frac{cd\rho}{\mu}$ is the Reynolds number, $A_1 = -\frac{\alpha d^3}{\lambda^3 \mu c}, A_2 = \frac{\beta c d^3}{\lambda^3 \mu}$, $A_3 = \frac{\gamma d^3}{\lambda^2 \mu}$ are the non-dimensional elasticity parameters, $\omega = \frac{\lambda \omega'}{d}$ is the non-uniform parameter, $\Pr = \frac{\rho \upsilon \xi}{k}$ is the Prandtl number, $Ec = \frac{c^2}{\xi(\Theta_1 - \Theta_0)}$ is the Eckert number, ζ is the Knudsen number (slip parameter).

3 Solution of the Problem

Using the long wavelength and low Reynolds number approximations, one can find from equations (12) to (16) that

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\tau_0^{\frac{1}{2}} + \left(-\frac{\partial^2 \Psi}{\partial y^2}\right)^{\frac{1}{2}}\right)^2 \tag{18}$$

$$0 = -\frac{\partial p}{\partial y} \tag{19}$$

Equation (19) shows that p is not a function of y

$$0 = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + Ec\left(\frac{\partial^2 \Psi}{\partial y^2}\right)^2 \tag{20}$$

On differentiating Eq. (18) with respect to y, we get

$$\frac{\partial^2}{\partial y^2}\left(\tau_0^{\frac{1}{2}} + \left(-\frac{\partial^2 \Psi}{\partial y^2}\right)^{\frac{1}{2}}\right)^2 = 0 \tag{21}$$

From Eq. (16) we get

$$\frac{\partial}{\partial y}\left(\tau_0^{\frac{1}{2}} + \left(-\frac{\partial^2 \Psi}{\partial y^2}\right)^{\frac{1}{2}}\right)^2 = \left(A_1\frac{\partial^3}{\partial x^3} + A_2\frac{\partial^3}{\partial x \partial t^2} + A_3\frac{\partial^2}{\partial x \partial t}\right)\eta \tag{22}$$

The closed form solution for equation (21) using the boundary conditions (15), (17) and (22) can be obtained as

$$\begin{aligned}\Omega &= \tfrac{A}{2}(\eta^2 - y^2) + (B^2 + \tau_0)(\eta - y + \zeta) - \tfrac{4}{3}\tfrac{\tau_0^{\frac{1}{2}}}{A}\left(\left(A\eta + B^2\right)^{\frac{3}{2}} - \left(Ay + B^2\right)^{\frac{3}{2}}\right) \\ &+ A\zeta\eta - 2\zeta\,\tau_0^{\frac{1}{2}}\left(A\eta + B^2\right)^{\frac{1}{2}}, y_0 \le y \le \eta\end{aligned} \tag{23}$$

We find the upper limit of plug flow region using the boundary condition that $\Psi_{yy} = 0$ at $y = y_0$. It is given by

$$y_0 = \frac{\tau_0 - B^2}{A} \tag{24}$$

Taking $y = y_0$ in equation (23) and using the relation (24), we get the velocity in the plug flow region as

$$\begin{aligned}\Omega_p &= \tfrac{A}{2}\eta^2 + \eta(A\zeta + 2B^2) + B^2(2\zeta + \tfrac{4B^2}{3A}) - \tfrac{1}{6}Ay_0^2 \\ &+ (A\eta + \tfrac{2}{3}B^2 + \zeta A)y_0 - (Ay_0 + B^2)^{\frac{1}{2}}\left(2\zeta + \tfrac{4}{3A}(A\eta + B^2)^{\frac{3}{2}}\right), \quad 0 \le y \le y_0\end{aligned} \tag{25}$$

By using Equations (23) and (25), we get

$$\Psi = \frac{A\eta^2 y}{2} - \frac{Ay^3}{3} + (B^2+\tau_0)\eta y - (B^2+\tau_0)\left(\frac{y^2}{2} - \frac{y_0^2}{2}\right) + A\eta\zeta y + \zeta(B^2+\tau_0)y$$
$$-\frac{4\tau_0^{\frac{3}{2}}}{3A}\left(y\left(A\eta+B^2\right)^{\frac{3}{2}} - \frac{2}{5A}\left(Ay+B^2\right)^{\frac{5}{2}}\right) - 2\zeta\tau_0^{\frac{1}{2}}\left(A\eta+B^2\right)^{\frac{1}{2}}y - \frac{A}{6}{y_0}^3 \tag{26}$$
$$+\frac{4}{3A}{\tau_0}^2 y_0 - \frac{8}{15A^2}{\tau_0}^3$$

$$\Psi_p = y\begin{pmatrix} \frac{A}{2}\eta^2 + \eta(A\zeta+2B^2) + B^2(2\eta+\frac{4B^2}{3A}) - \frac{1}{6}Ay_0^2 \\ +(A\eta+\frac{2}{3}B^2+\zeta A)y_0 - (Ay_0+B^2)^{\frac{1}{2}}\left(2\zeta+\frac{4}{3A}(A\eta+B^2)^{\frac{3}{2}}\right)\end{pmatrix} \tag{27}$$

where

$$A = -8\varepsilon\pi\left[(A_1+A_2)\cos 2\pi(x-t) - \frac{A_3}{2\pi}\sin 2\pi(x-t)\right], \quad B = \tau_0^{\frac{1}{2}} + (-\tau_0)^{\frac{1}{2}} \tag{28}$$

By solving equation (20) with the help of equation (26) and (17), an expression for temperature field is obtained

$$T = 1 + Br\left\{\frac{A^2}{3}\left({y_0}^3 y - \frac{y^4}{4} - {y_0}^3\eta + \frac{\eta^4}{4}\right) + \left((B^2+\tau_0)^2 + 4B^2\tau_0\right)\left(y_0 y - \frac{y^2}{2} - y_0\eta + \frac{\eta^2}{2}\right)\right.$$
$$+\left(B^2+3\tau_0\right)A\left({y_0}^2 y - \frac{y^3}{3} - {y_0}^2\eta + \frac{\eta^3}{3}\right) - \frac{4\tau_0^{\frac{1}{2}}}{105A^2}\left(\left(Ay+B^2\right)^{\frac{7}{2}} - \left(A\eta+B^2\right)^{\frac{7}{2}}\right)$$
$$-\frac{8\tau_0^{\frac{1}{2}}}{3A}(y-\eta)\left(Ay_0+B^2\right)^{\frac{3}{2}}\left(Ay_0+B^2+\tau_0\right) + \frac{16\tau_0^{\frac{1}{2}}}{15A}\left[y\left(Ay+B^2\right)^{\frac{5}{2}} - \eta\left(A\eta+B^2\right)^{\frac{5}{2}}\right. \tag{29}$$
$$\left.\left.+(y-\eta)\left(Ay_0+B^2\right)^{\frac{5}{2}}\right] + \frac{16\tau_0^{\frac{1}{2}}}{15A^2}(B^2+\tau_0)\left[\left(Ay+B^2\right)^{\frac{5}{2}} - \left(A\eta+B^2\right)^{\frac{5}{2}}\right]\right\}$$

4 Results and Discussions

The velocity, stream function and temperature are calculated analytically via MATLAB under the assumptions of long wave length and low Reynolds number. The effect of physical parameters like elastic parameters A_1, A_2 and A_3, non-uniform parameter ω, yield stress τ_0 and Brinkman number Br on velocity, temperature and stream function are discussed graphically from Fig 2 to 16.

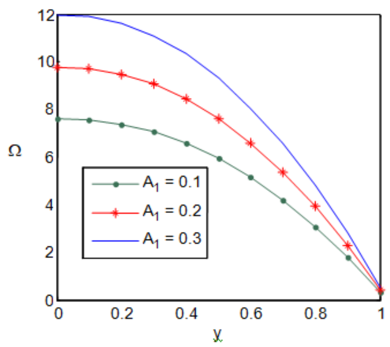


Fig 2. Effect of A_1 **on the velocity distribution for fixed values of** $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1$ $\tau_0 = 0.1, A_2 = 0.3, A_3 = 0.5, \zeta = 0.1$

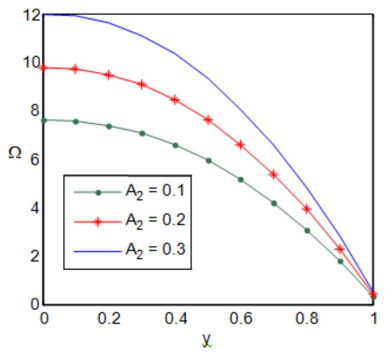


Fig 3. Effect of A_2 **on the velocity distribution for fixed values of** $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1$
$\tau_0 = 0.1, A_1 = 0.1, A_3 = 0.5, \zeta = 0.1$

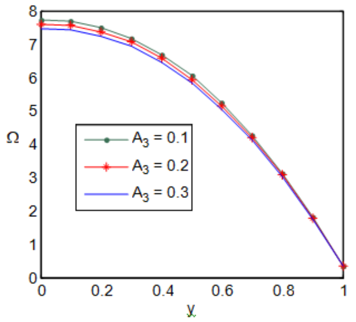


Fig 4. Effect of A_3 **on the velocity distribution for fixed values of** $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1$
$\tau_0 = 0.1, A_1 = 0.1, A_2 = 0.3, \zeta = 0.1$

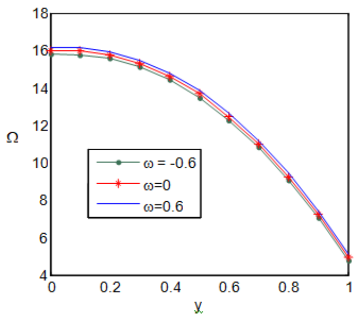


Fig 5. Effect of ω **on the velocity distribution for fixed values of** $x = 0.01, t = 0.4, \varepsilon = 0.2, \zeta = 0.1$
$\tau_0 = 0.1, A_1 = 0.1, A_2 = 0.3, A_3 = 0.5$

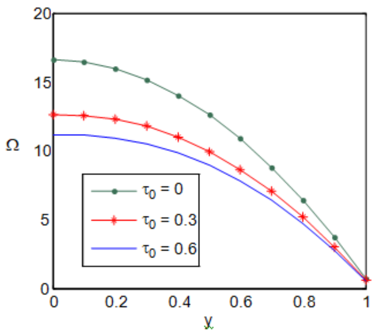


Fig 6. Velocity Distribution for different τ_0 **for fixed values of** $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1$
$\zeta = 0.1, A_1 = 0.1, A_2 = 0.3, A_3 = 0.5$

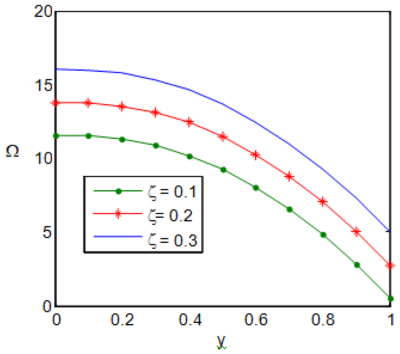


Fig 7. Velocity Distribution for different ζ **for fixed values of** $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1$
$\tau_0 = 0.1, A_1 = 0.1, A_2 = 0.3, A_3 = 0.5$

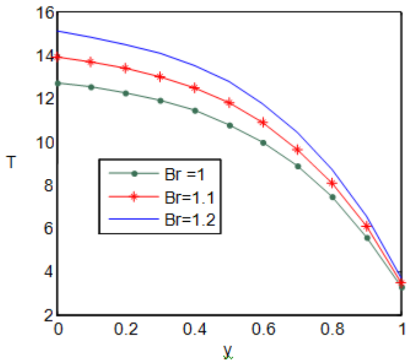


Fig 8. Effect of Br **on the Temperature for fixed values of** $x = 0.2, t = 0.1, \varepsilon = 0.1, \omega = 0.1,$
$\tau_0 = 0.1, A_1 = 0.1, A_2 = 0.3, A_3 = 0.5$

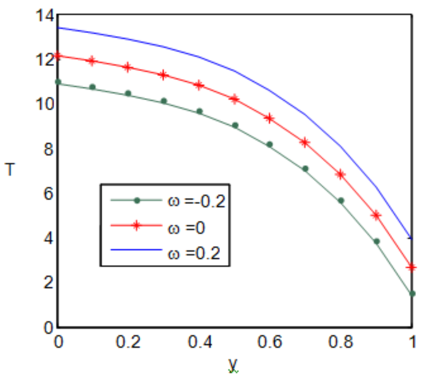


Fig 9. Effect of ω **on the temperature for fixed values of** $x = 0.2, t = 0.1, \varepsilon = 0.1, Br = 1,$
$\tau_0 = 0.1, A_1 = 0.1, A_2 = 0.3, A_3 = 0.5$

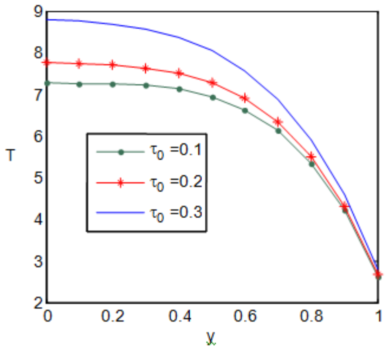


Fig 10. Temperature profiles for different τ_0 for fixed values of
$x = 0.2, t = 0.1, \varepsilon = 0.1, \omega = 0.1, Br = 1, A_1 = 0.1, A_2 = 0.3, A_3 = 0.5$

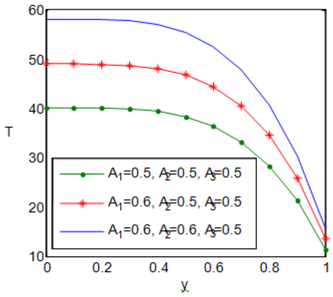
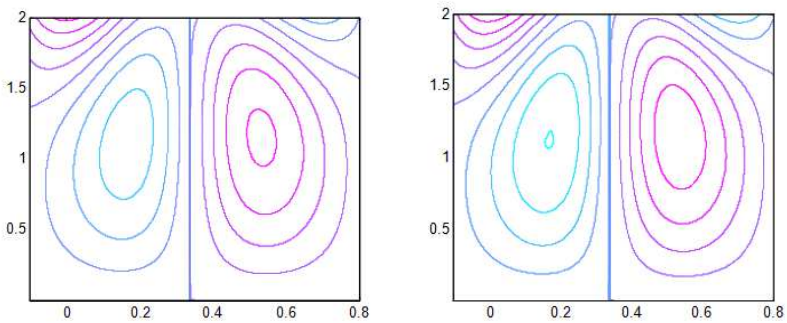
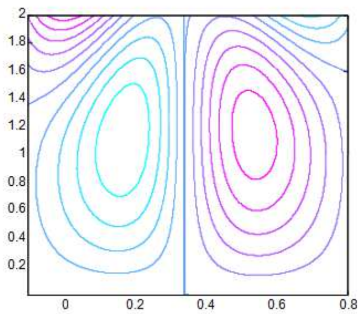


Fig11. Effect of elastic parameters on Temperature profiles for fixed values of
$x = 0.2, t = 0.1, \varepsilon = 0.1, \omega = 0.1, Br = 1, \tau_0 = 0.1$

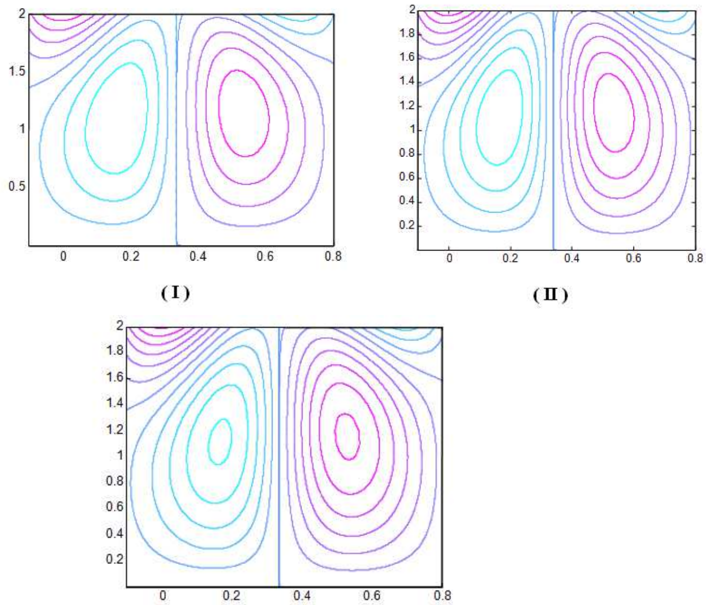


(I) **(II)**



(III)

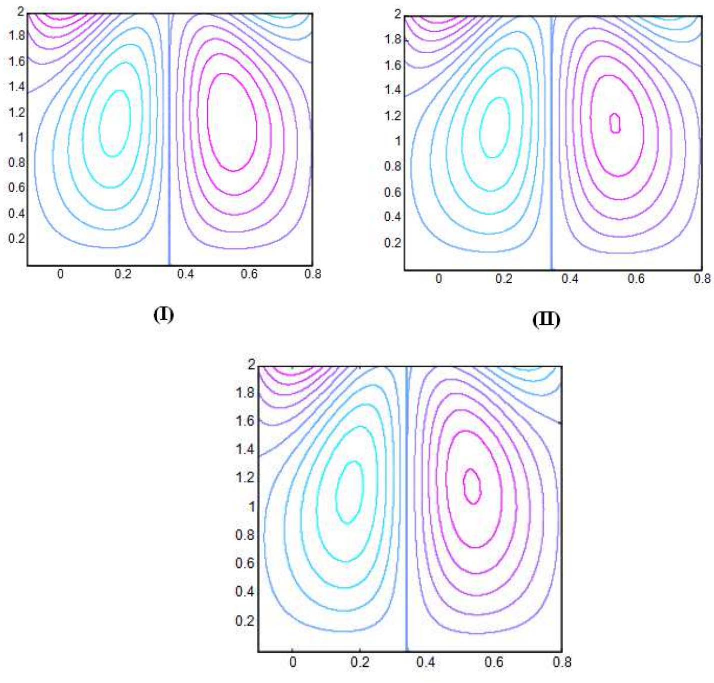
Fig. 12: Effect of A_1 on Trapping (I) $A_1 = 0.5$ (II) $A_1 = 0.6$ (III) $A_1 = 0.8$
for fixed values of $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1, \tau_0 = 0.1, A_2 = 0.3, A_3 = 0.5, \zeta = 0.1$



(III)

Fig. 13: Effect of A_2 on Trapping (I) $A_2 = 0.2$ **(II)** $A_2 = 0.4$ **(III)** $A_2 = 0.5$

for fixed values of $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1, \tau_0 = 0.1, A_1 = 0.1, A_3 = 0.5, \zeta = 0.1$



(III)

Fig. 14: Effect of A_3 on Trapping (I) $A_3 = 0.1$ **(II)** $A_3 = 0.3$ **(III)** $A_3 = 0.5$

for fixed values of $x = 0.01, t = 0.4, \varepsilon = 0.2, \omega = 0.1, \tau_0 = 0.1, A_1 = 0.1, A_2 = 0.3, \zeta = 0.1$

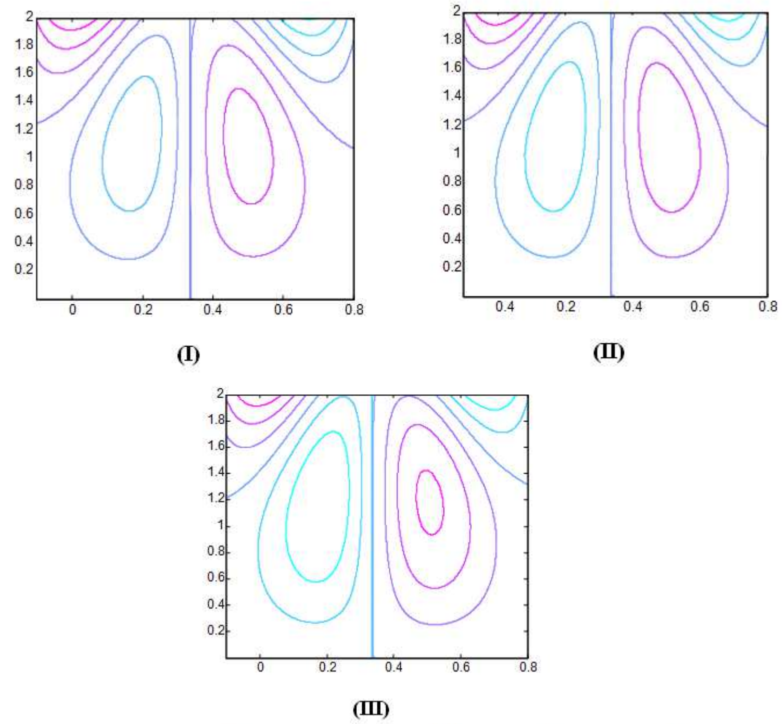


Fig. 15: Effect of ω **on Trapping (I)** $\omega=-0.1$ **(II)** $\omega=0$ **(III)** $\omega=0.1$
for fixed values of $x=0.01, t=0.4, \varepsilon=0.2, \tau_0=0.1, A_1=0.1, A_2=0.3, A_3=0.5, \zeta=0.1$

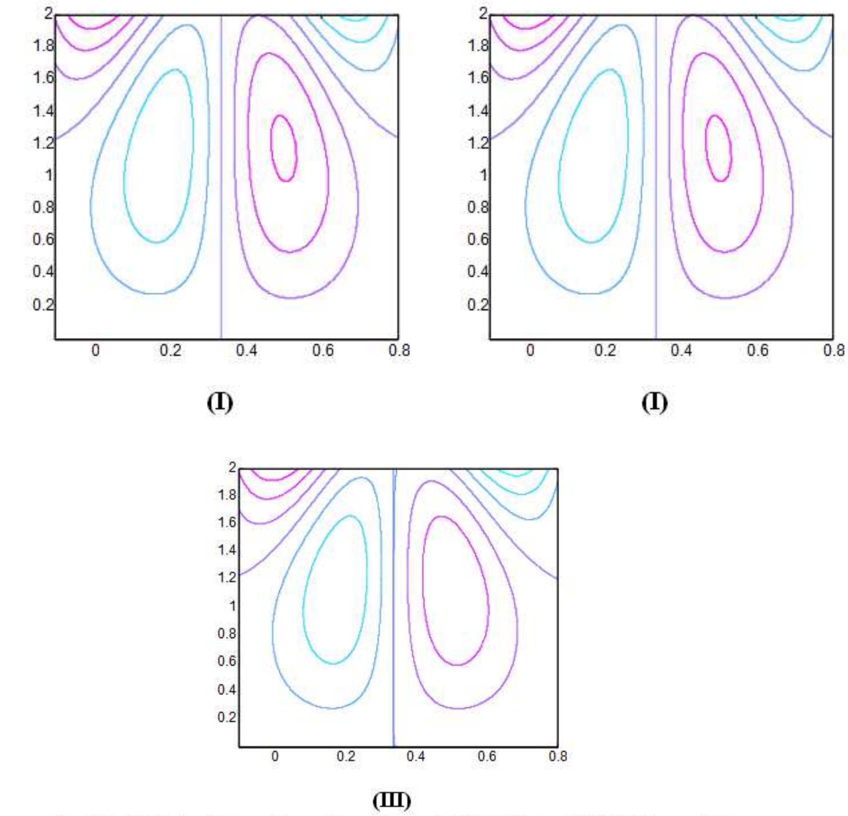


Fig. 16: Effect of τ_0 **on Trapping (I)** $\tau_0=0.001$ **(II)** $\tau_0=0.01$ **(III)** $\tau_0=0.1$
for fixed values of $x=0.01, t=0.4, \varepsilon=0.2, \omega=0.1, A_1=0.1, A_2=0.3, A_3=0.5, \zeta=0.1$

4.1 Velocity Field and Temperature distribution

Figures 2 to 4 shows the profiles for velocity Ω versus y with different elastic wall parameters namely A_1, A_2 and A_3. An increase in the rigidity parameter A_1 gives rise to an increment in velocity Ω. A similar trend may be observed in the case of elastic parameter A_2 whereas the parameter A_3 exhibits opposite behavior.From this it is clear that the fluid flow is more if the tension in the membrane and the mass per unit area of the elastic wall is more and fluid flow will be less if the viscous damping forces in the fluid are more. The change in the velocity Ω for different values of non-uniformity parameter ω is elucidated in the Fig. 5. Physically, for the convergent channel $(\omega < 0)$, uniform channel $(\omega = 0)$ and divergent channel $(\omega > 0)$. It is noted from these profiles that the velocity Ω decreases with increase in non-uniformity parameter and in order to obtain a better flow of liquid the channel must be convergent. Fig. 6 demonstrates that the velocity is a decreasing function of yield stress τ_0.The effect of slip parameter on velocity is observed in Fig. 7. An increase in slip parameter results in the enhancement of the velocity of the liquid.

The effect of various parameters on temperature is illustrated in Figures 8 to 11. From Fig. 8 it can be noticed that an increase in Br increments in the temperature field. Further, it is noted from the Fig. 9 that temperature increase for large values of ω. Fig. 10 depicts that an increase in the value of the yield stress decreases the magnitude of temperature. This behaviour is expected due to the presence of τ (minimum amount of energy required to begin the flow) in the Casson model. We see from Fig.11 that as increasing values of $A_1$and A_2 increases the temperature. Physically, the tension in the membrane and mass per unit area is more, then we have high temperature.

4.2 Trapping Phenomenon

The most important concept to be noted in peristalsis is trapping, a closed bolus of liquid that moves along with the peristaltic wave. Fig. 12 shows that the size of the bolus increases with increase in rigidity A_1. From Fig. 13 we observe that as stiffness parameter (A_2) increases the number of bolus increases. Further increment in A_3 increases the size of the trapped bolus which is noticed in Fig. 14. From Fig. 15 we conclude that the pattern and size of bolus based on the change of non-uniformity parameter. From these figures it is noticed that there is a symmetric behaviour in uniform channel and non-symmetric behaviour in convergent and divergent channel. Also, the pattern of the formation of bolus is opposite in nature for convergent and divergent channels. The effect of yield stress parameter on the trapping is illustrated in Fig. 16. It can be concluded that the size of the trapped bolus decreases with increase in $\tau_{0.}$ Thus, we have seen the effect of associated parameters A_1, A_2, A_3, ω *and* τ_0 on the progress of the trapping phenomena.These qualitative results may have some significance in understanding the transport of blood in the small blood vessels.

5 Concluding remarks

The main findings are listed below.

- As the elastic effect of the channel increases the velocity of the liquid increases.
- The increase in yield stress results out in the decrease in velocity of the liquid flow.
- Increase in rigidity A_1, stiffness A_2, viscous damping force A_3 of the wall increases the size and number of bolus formed in the flow pattern.
- The size of the tapered bolus decreases as the yield stress increases.
- If the channel is uniform, then the bolus is symmetric and opposite behavior is observed if the channel is either convergent or divergent.

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