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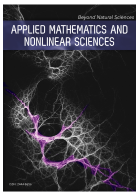

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Reckoning of the Dissimilar Topological indices of Human Liver

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Abstract

The molecular topological indices as validly demonstrated its high performance in the discovery and design of new drugs. The goal of this paper is to study the structurally constructed a graph model of human Liver using graph operator. After the construction, nurtured the model using various topological indices. Also, established a diagnosis defect in the human Liver. Basically, considered structure of Liver can divide into healthy Liver and affected Liver. In this case study the topological indices are used in describe the structure of Liver using graph operator. Constructed model can be useful further in the medical field for any diagnosis with special care.

Keywords: Liver Structure, double graph, first and second Zagreb, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond connectivity (ABC) index, Geometric-Arithmetic (GA) index.
AMS 2010 codes: AMS Subject Classification: 05C05, 05C12, 05C75.

1 Introduction and Preliminaries

A topological index is a numerical value associated with chemical constitution for correlation of chemical structure with various physical properties, chemical reactivity or biological activity. Most of the useful topological indices are distance based or degree based. In order to study the structure and functioning of the human

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Liver is the largest solid organ in the body. This Liver having storing excess nutrients and returning some of the nutrients to the bloodstream, ridding the body of harmful substances in the bloodstream, including drugs and alcohol, helping the body store sugar(glucose) in the form of glycogen also breaking down saturated fat and producing cholesterol. This structure of Liver can divide into normal Liver and cirrhosis of the Liver also it is called as healthy Liver and affected Liver. In this study the topological indices are used in describe the structure of Liver using double graph operator. A healthy Liver is defined as structure in which all the edges and vertices are present in graph Figure 3 while in the case of affected Liver is disconnecting some edges in the healthy Liver graph. Therefore, these topological indices help in the determination of changing in the healthy and affected Liver structure. Motivated from the work [17], we established new model study of the cirrhosis of the Liver using double graph operator.

For our study we recalled degree based topological indices are defined as follows:
The first and second Zagreb indices, have been introduced more than thirty years ago by I. Gutman and Trinajstic [10]. They defined as:

$$M_1(G) = \sum_{u\in V(G)} (d_u)^2$$

$$M_2(G) = \sum_{uv\in E(G)} (d_u.d_v).$$

The Zagreb indices found many applications in QSPR and QSAR studies. For more details on this topological indices we refer to [11–13, 21, 24, 25].

There are many topological indices defined on the basis of the vertex-degree of graph one of the vertex-degree based index namely Harmonic index $H(G)$ is defined as,

$$H(G) = \sum_{uv\in E(G)} \frac{2}{d_u + d_v}$$

for more results on Harmonic index we refer to the articles [18, 26, 28].

Randic index: The connectivity index introduced in 1975 by Milan Randic [22], who has shown this index to reflect molecular branching, Randic index was defined as,

$$R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{(d_u.d_v)}}.$$

results on the Randic index we refer the articles [14, 16].

Symmetric division deg index: Among 148 discrete adriatic indices [1,2] we considered Symmetric division deg index discrete adriatic indices. For collection of recent results on [9, 15], the Symmetric division deg index according to is defined as,

$$SDD(G) = \sum_{uv\in E(G)} \frac{d_u^2 + d_v^2}{d_u.d_v}.$$

The Atom-Bond Connectivity index: (ABC) index according, proposed by Ernesto Estrada [5] is defined as follows,

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u.d_v}}.$$

We refer the articles [4, 8].

The Geometric-Arithmetic index: Another topological index namely, Geometric-Arithmetic index, defined by Vukicevic and Furtula [27] as follows

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u.d_v}}{d_u + d_v}.$$

results on the GA index we refer the articles, [3, 7, 29, 30].

Munarini et al. [20] defined the double graph of a simple graph denoted as $D[G]$. The double graph of a simple graph G can be build up taking two distinct copies of the graph G and joining every vertex V in one copy to every vertex w' in the other copy corresponding to a vertex W adjacent to V in the first copy. In this paper we study some distance based topological indices for double graph also we refer the articles [19].

Example:

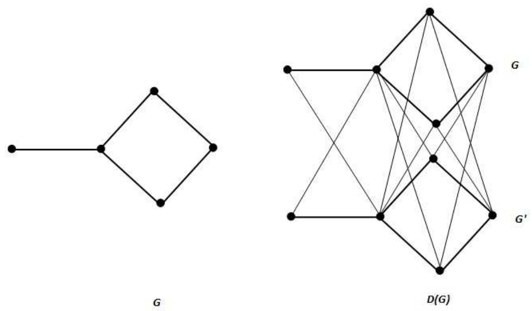

Fig. 1 A graph G and its double graph $D[G]$

V. Lokesha and co-workers and M. Faisal Nadeem and et. al., [6, 17, 23] utilized the following operators.

The subdivision graph $S(G)$ is the graph obtained from G by replacing each edge by a path of length 2 or by inserting a vertex in every edge of the graph G.

$R(G)$ is the graph obtained from G by adding a new vertex corresponding to every edge of G and by joining each new vertex to the end vertices of the edge corresponding to it.

Recently V. Lokesha, A. Usha and et. al., [17] worked on Topological indices on model graph structure of Alveoli in human lungs. Motivated from the above works, we computed the first, second Zagreb indices, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity index and Geometric-Arithmetic index of $D(G)$ of model of healthy Liver.

The construction of paper is organized as follows:
Section 1, consists of introduction and essential definitions which is necessary for the main results. section 2, contains topological indices of $D(G)$ of healthy Liver, section 3, contains topological indices of $D(G)$ of cirrhosis of human Liver and Final section will consists of topological indices of $S(G)$ and $R(G)$ for healthy Liver.

2 Topological indices of Double Graph of Healthy Liver

In this segment, we concentrated basic result on Double graph of healthy Liver, A healthy Liver is defined as structure in which all the edges and vertices are present in graph as shown in the below Figure.

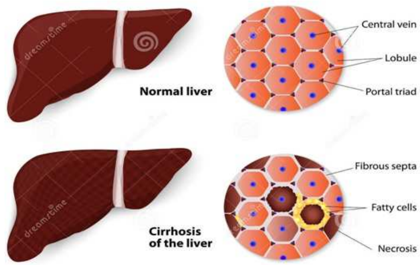

Fig. 2 Structure of Normal Liver and cirrhosis Liver(From Dreamstime.com

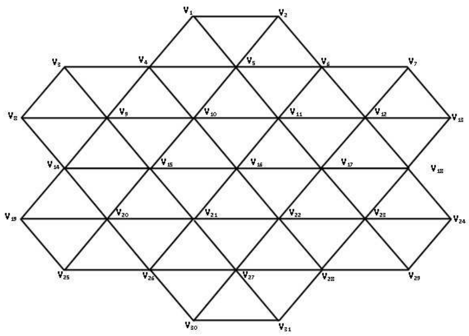

Fig. 3 Model of Healthy Liver

Theorem 1. *The first Zagreb index, second Zagreb index, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity* (ABC) *index, Geometric-Arithmetic* (GA) *index of double graph of model of healthy Liver are*

$$M_1(D(G)) = 5808 \qquad SDD(D(G)) = 615.2$$
$$M_2(D(G)) = 29664 \qquad ABC(D(G)) = 125.38$$
$$H(D(G)) = 29.88 \qquad GA(D(G)) = 283.43$$
$$R(D(G)) = 30.43$$

Proof. Let G be a healthy Liver graph and G' be the copy of healthy Liver graph, then $D(G)$ is a Double graph of Healthy Liver. This healthy Liver graph as 31 vertices ; of which 12 vertices are of degree 3, 6 vertices are of degree 5 and 13 vertices of degree 6.

In all, there are 72 edges of which 6 edges are of the type $(3,3)$, 12 edges are of type $(3,5)$, 12 edges of type $(3,6)$, 18 edges of type $(5,6)$ and 24 edges of type $(6,6)$ Therefore, a double graph of healthy Liver has 62 vertices and 288 edges. Considering the edges and degree of their end vertices, we obtained the indices result.

3 Topological indices of Double graph of Cirrhosis of the Liver

In this segment, This cirrhosis of a Liver is a slowly progressing disease in which healthy Liver tissue is replaced with scar tissue. Hepatitis C, fatty Liver, and alcohol abuse are the most common causes of cirrhosis of the Liver. This cirrhosis of the fatty Liver Diseases is Alcoholic Liver Disease (ALD), Non-alcoholic Fatty Liver Disease ($NAFLD$) and Acute Fatty Liver of Pregnancy.

In such a case, the graph of healthy Liver results in removal of edges of the inner structure of the graph which is containing fatty Liver. This affected fatty Liver can cause the removal of one edge, two edges and more edges, applying different types of topological indices and found the variations in the indices.

Theorem 2. *The first Zagreb index, second Zagreb index, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity* (ABC) *index, Geometric-Arithmetic* (GA) *index double graph of affected Liver on removal of one edge* (V_{28},V_{29}) *are*

$$\begin{aligned} M_1(D(G)) &= 5696 & SDD(D(G)) &= 611 \\ M_2(D(G)) &= 28944 & ABC(D(G)) &= 124.29 \\ H(D(G)) &= 29.75 & GA(D(G)) &= 279.67 \\ R(D(G)) &= 30.36 \end{aligned}$$

Proof. The graph of affected Liver on removal of the edge (V_{28},V_{29}) result in 71 edges. The double graph of the same will have 284 edges. This affected Liver graph as 31 vertices of which 1 vertex is of degree 2, 11 vertices of degree 3, 1 vertex is of degree 4, 5 vertices of degree 5, 13 vertices of degree 6. In all there are 71 edges of which 1 edge of type $(2,3)$, 1 edge of type $(2,6)$, 5 edges of type $(3,3)$, 1 edge of type $(3,4)$, 10 edges of type $(3,5)$, 11 edges of type $(3,6)$, 3 edges of type $(4,6)$, 15 edges of type $(5,6)$ and 24 edges of type $(6,6)$. Also, the number of edges of double graph of healthy Liver on removal of one edge will have 284 edges and 62 vertices. Considering the edges and degrees of their end vertices, using the above topological indices definitions we get the required results.

NOTE: Using the same technique adopted in the above Theorem different removal of one edge sets are as shown in the table.

Removing of One Edge

Indices \ Vertices	(V_{28}, V_{29})	(V_{22}, V_{28})	(V_{17}, V_{22})	(V_{17}, V_{18})	(V_{18}, V_{24})	(V_{24}, V_{29})
$M_1(D(G))$	5696	5648	5632	5648	5696	5728
$M_2(D(G))$	28944	28416	28160	28416	28944	29168
$H(D(G))$	29.75	25.22	29.87	25.22	29.75	29.47
$R(D(G))$	30.36	30.43	30.14	30.43	30.36	30.20
$SDD(D(G))$	611	607.33	608	607.33	611	618.93
$ABC(D(G))$	124.29	124.43	124.51	124.43	124.29	124.33
$GA(D(G))$	279.07	279.41	279.33	279.41	279.07	278.31

Fig. 4 Removing of one edge sets of healthy Liver graph

Theorem 3. *The first Zagreb index, second Zagreb index, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity* (ABC) *index, Geometric-Arithmetic* (GA) *index Of double graph of affected*

Liver on removal of two edges (V_{22},V_{17}) *and* (V_{17},V_{18}) *are*

$$M_1(D(G)) = 5488 \qquad SDD(D(G)) = 601.73$$
$$M_2(D(G)) = 27088 \qquad ABC(D(G)) = 123.53$$
$$H(D(G)) = 29.83 \qquad GA(D(G)) = 275.12$$
$$R(D(G)) = 30.4$$

Proof. The graph of affected Liver on removal of the edges (V_{22},V_{17}) and (V_{17},V_{18}) result in 70 edges. The double graph of the same will have 280 edges. This affected Liver graph as 31 vertices of which 12 vertices is of degree 3, 2 vertices of degree 4, 6 vertices of degree 5, 11 vertices of degree 6. In all there are 70 edges of which 6 edges of type (3, 3), 2 edges of type (3, 4), 10 edges of type (3, 5), 12 edges of type (3, 6), 6 edges of type (4, 6), 1 edge of type (5, 5), 18 edges of type (5, 6), 15 edges of type (6, 6). Also, the number of edges of double graph of healthy Liver on removal of two edges will have 280 edges and 62 vertices. Considering the edges and degrees of their end vertices, using the above topological indices definitions we get the required results.

NOTE: Using the same technique adopted in the above Theorem different removal of two edge sets are as shown in the table.

Removing of Two Edges

Indices \ Vertices	(V_{22},V_{17}) and (V_{17},V_{18})	(V_{18},V_{24}) and (V_{24},V_{29})	(V_{22},V_{28}) and (V_{28},V_{29})	(V_{17},V_{23}) and (V_{18},V_{23})	(V_{24},V_{23}) and (V_{29},V_{23})	(V_{28},V_{23}) and (V_{22},V_{23})
$M_1(D(G))$	5488	5632	5552	5560	5568	5560
$M_2(D(G))$	27088	28576	27840	27600	28144	27600
$H(D(G))$	29.83	29.15	29.68	29.75	29.76	29.75
$R(D(G))$	30.4	30	30.32	30.25	30.36	30.25
$SDD(D(G))$	601.73	619.53	605.33	596.87	601.73	596.87
$ABC(D(G))$	123.53	123.19	123.32	123.08	123.12	123.08
$GA(D(G))$	275.12	273.92	274.81	275.68	275.2	275.68

Fig. 5 Removal of two edge sets of healthy Liver graph

Theorem 4. *The first Zagreb index, second Zagreb index, harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity* (ABC) *index, Geometric-Arithmetic* (GA) *index Of double graph of affected Liver on removal of three edges* (V_{17},V_{18}), (V_{18},V_{24}) *and* (V_{24},V_{29}) *are*

$$M_1(D(G)) = 5488 \qquad SDD(D(G)) = 613.87$$
$$M_2(D(G)) = 27472 \qquad ABC(D(G)) = 122.21$$
$$H(D(G)) = 29.08 \qquad GA(D(G)) = 269.65$$
$$R(D(G)) = 32.64$$

Proof. The graph of affected Liver on removal of the edges (V_{17},V_{18}), (V_{18},V_{24}) and (V_{24},V_{29}) result in 69 edges. The double graph of the same will have 276 edges. This affected Liver graph as 31 vertices of which 1 vertex is of degree 1, 1 vertex of degree 2, 11 vertices of degree 3, 6 vertices of degree 5, 12 vertices of degree 6. In all there are 69 edges of which 1 edge of type (1, 6), 1 edge of type (2, 6), 6 edges of type (3, 3), 9 edges of type (3, 5), 12 edges of type (3, 6), 20 edges of type (5, 6), 19 edges of type (6, 6), 1 edge of type (2, 5). Also, the number of edges of double graph of healthy Liver on removal of three edges will have 276 edges and 62 vertices. Considering the edges and degrees of their end vertices, using the above topological indices definitions we get the required results.

NOTE: Using the same technique adopted in the above Theorem different removal of three edge sets are as shown in the table.

Removing of Three Edges

Vertices / Indices	(V_{17}, V_{18}), (V_{18}, V_{24}) and (V_{24}, V_{29})	(V_{28}, V_{29}), (V_{28}, V_{22}) and (V_{22}, V_{17})	(V_{23}, V_{18}), (V_{24}, V_{23}) and (V_{23}, V_{29})	(V_{22}, V_{23}), (V_{23}, V_{28}) and (V_{17}, V_{23})
$M_1(D(G))$	5488	4588	5440	5344
$M_2(D(G))$	27472	26512	27280	26320
$H(D(G))$	29.08	29.63	29.71	29.97
$R(D(G))$	32.64	30.29	30.33	30.48
$SDD(D(G))$	613.87	599.73	595.8	587.8
$ABC(D(G))$	122.21	122.43	122.09	122.37
$GA(D(G))$	269.65	270.52	270.93	271.81

Fig. 6 Removing of three edge sets of healthy Liver graph

Theorem 5. *The first Zagreb index, second Zagreb index, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity (ABC) index, Geometric-Arithmetic (GA) index Of double graph of affected Liver on removal of six edges (V_{17}, V_{18}), (V_{18}, V_{24}), (V_{24}, V_{29}), (V_{28}, V_{29}), (V_{22}, V_{28}) and (V_{17}, V_{22}) are*

$$M_1(D(G)) = 5104 \qquad SDD(D(G)) = 604.8$$
$$M_2(D(G)) = 24624 \qquad ABC(D(G)) = 119.17$$
$$H(D(G)) = 28.6 \qquad GA(D(G)) = 256.53$$
$$R(D(G)) = 29.68$$

Proof. The graph of affected Liver on removal of the edges (V_{17}, V_{18}), (V_{18}, V_{24}), (V_{24}, V_{29}), (V_{28}, V_{29}), (V_{22}, V_{28}) and, (V_{17}, V_{22}) result in 66 edges. The double graph of the same will have 264 edges. This affected Liver graph as 31 vertices of which 2 vertices of degree 1, 12 vertices of degree 3, 2 vertices of degree 4, 4 vertices of degree 5, 11 vertices of degree 6. In all there are 66 edges of which 2 edges of type (1, 6), 7 edges of type (3, 3), 8 edges of type (3, 5), 14 edges of type (3, 6), 8 edges of type (4, 6), 12 edges of type (5, 6), 15 edges of type (6, 6). Also, the number of edges of double graph of healthy Liver on removal of six edges will have 264 edges and 62 vertices. Considering the edges and degrees of their end vertices, using the above topological indices definitions we get the required results.

NOTE: Using the same technique adopted in the above Theorem different removal of six edge sets are as shown in the table.

Removing of Six Edges

Vertices / Indices	(V_{17}, V_{18}), (V_{18}, V_{24}), (V_{24}, V_{29}), (V_{28}, V_{29}), (V_{22}, V_{28}) and (V_{17}, V_{22})	(V_{17}, V_{23}), (V_{18}, V_{23}), (V_{24}, V_{23}), (V_{29}, V_{23}), (V_{28}, V_{23}) and (V_{22}, V_{23})
$M_1(D(G))$	5104	5120
$M_2(D(G))$	24624	25312
$H(D(G))$	28.6	28.89
$R(D(G))$	29.68	29.43
$SDD(D(G))$	604.8	565.33
$ABC(D(G))$	119.17	117.36
$GA(D(G))$	256.53	259.65

Fig. 7 Removing of six edge sets of healthy Liver graph

The following graphs shows a comparison between $M_1(G)$, $M_2(G)$, $H(G)$, $R(G)$, $SDD(G)$, $ABC(G)$, $GA(G)$ indices using $D(G)$ of healthy and cirrhosis of the Liver as shown in the following Figure-8.

Fig. 8 Comparison between healthy and cirrhosis of Liver graph.

4 Effect of Subdivision and $R(G)$ operators on human Liver

Here we demonstrate the $S(G)$ and $R(G)$ operators also computed the subdivision and $R(G)$ of healthy Liver.

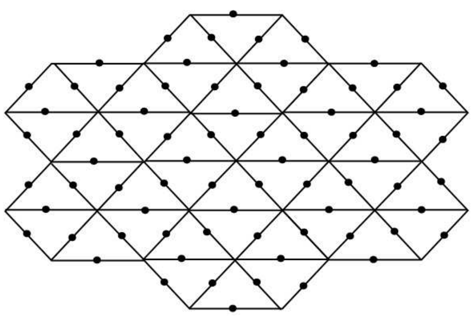

Fig. 9 Subdivision graph of Model of healthy Liver.

Theorem 6. *The first Zagreb index, second Zagreb index, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity* (ABC) *index, Geometric-Arithmetic* (GA) *index of subdivision graph of healthy Liver are*

$$M_1(S(G)) = 1014 \qquad SDD(S(G)) = 425$$
$$M_2(S(G)) = 1452 \qquad ABC(S(G)) = 101.82$$
$$H(S(G)) = 42.47 \qquad GA(S(G)) = 129.93$$
$$R(S(G)) = 46.70$$

Proof. Let G be a healthy Liver graph and $S(G)$ be the subdivision graph of Healthy Liver. This healthy Liver graph as 103 vertices ; of which 72 vertices are of degree 2, 12 vertices are of degree 3 and 6 vertices of degree 5 and 13 vertices of degree 6.

In all, there are 144 edges of which 36 edges are of the type $(2,3)$, 30 edges are of type $(2,5)$, 78 edges are of the type $(2,6)$. Considering the edges and degree of their end vertices, and we obtain the required results.

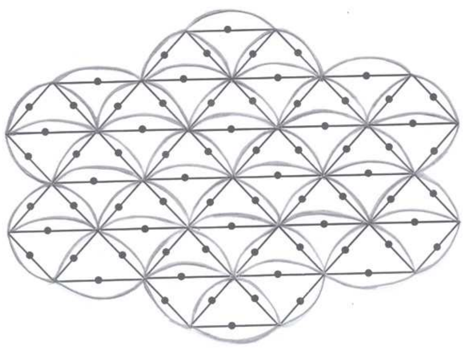

Fig. 10 $R(G)$ of Model of healthy Liver.

Theorem 7. *The first Zagreb index, second Zagreb index, Harmonic index, Randic index, Symmetric division deg index, Atom-Bond Connectivity* (ABC) *index, Geometric-Arithmetic* (GA) *index of* $K(G)$ *be the* $R(G)$ *of healthy Liver are*

$$M_1(K(G)) = 3192 \qquad SDD(K(G)) = 910.8$$
$$M_2(K(G)) = 10320 \qquad ABC(K(G)) = 133.17$$
$$H(K(G)) = 32.61 \qquad GA(K(G)) = 178.98$$
$$R(K(G)) = 40.63$$

Proof. Let G be a healthy Liver graph and $K(G)$ be the $R(G)$ of healthy Liver. This healthy Liver of $K(G)$ graph as 103 vertices ; of which 72 vertices are of degree 2, 12 vertices are of degree 6 and 6 vertices of degree 10 and 13 vertices of degree 12.

In all, there are 216 edges of which 36 edges are of the type $(2,6)$, 30 edges are of type $(2,10)$, 78 edges are of the type $(2,12)$, 6 edges are of the type $(6,6)$, 12 edges are of the type $(6,10)$, 12 edges are of the type $(6,12)$, 18 edges are of the type $(10,12)$, 24 edges are of the type $(12,12)$. Considering the edges and degree of their end vertices, and we obtain the required results.

The following graphs shows a comparison between $M_1(G)$, $M_2(G)$, $H(G)$, $R(G)$, $SDD(G)$, $ABC(G)$, $GA(G)$ indices using $S(G)$ and $R(G)$ operators shown in Figure 8.

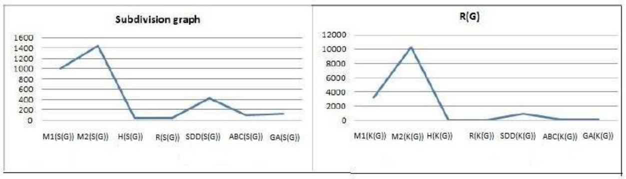

Fig. 11 The comparison between $M_1(G)$, $M_2(G)$, $H(G)$, $R(G)$, $SDD(G)$, $ABC(G)$, $GA(G)$ indices using $S(G)$ and $R(G)$ operators.

5 Conclusions

In this article, we compare the results of the computed topological indices for the normal Liver and the cirrhosis of the Liver by applying three graph operators viz., $D(G)$, $S(G)$ and $R(G)$ respectively and the comparison graphs are shown in Figure 8 and Figure 11.

From this we observed that the normal Liver of $D(G)$ having 288 edges which is maximum and in the case of cirrhosis of the Liver its ranging from 264 to 284 edges. So, the computation of topological indices of $D(G)$ are shrinking when extracting the edges and it is analogous for other cases also. Such that in all this cases, it is avail to take suggestion and treatment of medical personnel. Thus it can be deduce that the work proposes a model which is effective in diagnosing the defects in the cirrhosis of the Liver.

References

[1] V. Alexander, (2014), *Upper and lower bounds of Symmetric division deg index*, Iranian Journal of Mathemtical Chemistry, 5(2), 91 - 98.
[2] Damir Vukicevic and Mariji Gasperov, (2010), *Bond additive modeling1. Adriatic indices*, Croat. Chem. Acta, 83(3), 243 - 260.
[3] K. C. Das, (2010), *On Geometric-Arithmetic index of graphs*, MATCH Commun. Math. Comput. Chem., 64, 619 - 630.
[4] K. C. Das, (2010), *Atom-Bond connectivity index of graphs*, Discrete Appl. Math., 158, 1181 - 1188. 10.1016/j.dam.2010.03.006
[5] E. Estrada, L. Torres, L. Rodriguez, I. Gutman, (1998), *An Atom-Bond connectivity index modeling the enthalpy of formation of alkane*, India J. Chem., 37A, 849 - 855.
[6] M. Faisal Nadeem, Sohail Zafar, Zohaib Zahid, (2017), *Some Topological indices of* $L(S(CNC_k[n]))$, Punjab University Journal of Mathematics, 49(1), 13 - 17.
[7] G. H. Fath Tabar and I. Gutman, (2010), *A new Geometric-Arithmetic index*, J. Math.Chem., 47, 477 - 486. 10.1007/s10910-009-9584-7
[8] B. Furtula, A. graovac, D.Vukievi, (2009), *Atom-Bond connectivity index of trees*, Discrete Appl. Math., 157, 2828 - 2835. 10.1016/j.dam.2009.03.004
[9] C. K. Gupta, V. Lokesha, B. Shwetha Shetty and P. S. Ranjini, (2016), *On the Symmetric division deg index of graph*, Southeast Asian Bulletin of Mathematics, 41(1), 1 - 23.
[10] I. Gutman and K. C. Das, (2004), *The first Zagreb indices 30 years after*, MATCH Commun. Math. Comput. Chem, 50, 83-92.
[11] I. Gutman, N. Trinajstic, (1972), *Graph theory and molecular orbitals. Total* π*-electron energy of alternate hydrocarbons*, Chem. phy. Lett., 17, 535 - 538.
[12] A. Ilic and D. Stevanovic, (2009), *On comparing Zagreb indices*, MATCH Commun. Math. Comput. Chem., 62, 681 - 687.
[13] M. H. Khalifeh, H. Yousefi-Azari and A. R. Ashrafi, (2009), *The first and second Zagreb indices of some graph operations*, Discrete Appl. Math., Vol. 157, 804 - 811. 10.1016/j.dam.2008.06.015
[14] Li, X., Shi, Y., (2008), *A survey on the randic index*, MATCH Commun. Math. Comput. Chem., 59(1), 127 - 156.
[15] V. Lokesha, T. Deepika, P. S. Ranjini, I. N. Cangul, (2017), *Operations of nanostructures via SDD,* ABC_4*,* GA_5 *indices*, Applied Math. Non-linear Science, 2(1), 173 - 180.
[16] V. Lokesha, B. Shwetha Shetty, P. S. Ranjini, Ismail Naci Cangul and Ahmet Sinan Cevik, (2013), *New bounded for*

Randic and GA, Journal of Inequalities and Applications, 180, 1 - 7.

[17] V. Lokesha, A. Usha, P. S. Ranjini and K. M. Devendraiah, (2015), *Topological indices on model graph structure of Alveoli in human lungs*, proceedings of the Jangjeon Mathematical Society, 18(4), 435 - 453.

[18] V. Lokesha, A. Usha, P. S. Ranjini and T. Deepika, (2015), *Harmonic index of cubic polyhedral graphs using bridge graphs*, App. Math. Sci., 9, 4245 - 4253.

[19] Muhammad Kamran Jamil, (2017), *Distance based topological indices and double graph*. Iranian J. Math, Chem., 8(1), 83 - 91.

[20] E. Munarini, C. Perelli Cippo, A. Scagliola, N. Zagagalia Salvi, (2008), *Double graphs*, Discrete Math., 308, 242 - 254. 10.1016/j.disc.2006.11.038

[21] S. Nikolic, G. Kovacevic, A. Milicevic and N. Trinajstic, (2003), *The Zagreb indices 30 years after*, Croat. Chem. Acta, 76, 113 - 124.

[22] M. Randic, *Characterization of molecular branching* J. Amer. chem. Soc., (1975), 97, 6609 - 6615. 10.1021/ja00856a001

[23] P. S. Ranjini and V. Lokesha, (2010), *Smarandache-Zagreb index on three graph operators*, International J. Math. Combin., 3, 01 - 10.

[24] P. S. Ranjini, V. Lokesha and M. A. Rajan, (2010), *On Zagreb indices of the subdivision graphs*, Int. J. Math. Sc. Eng. Appl., 4, 221 - 228.

[25] P. S. Ranjini, V. Lokesha, (2011),*On the Zagreb indices of the line graphs of the subdivision graphs*, Appl. Math. Comput., 218, 699 - 702. 10.1016/j.amc.2011.03.125

[26] B. Shwetha Shetty, V. Lokesha and P. S. Ranjini, (2015), *On the Harmonic index of graph operations*. Transactions on Combinatorics, 4(4), 5 - 14.

[27] D. Vukicevic and B. Furtula, (2009), *Topological index based on the ratios of geometrical and arithmetical means of end-vertex degree of edges*, J Math. chem., 4, 1369 - 1376. 10.1007/s10910-009-9520-x

[28] Xinli Xu, (2012), *Relationships between Harmonic index and other topological indices*, Applied Mathematical Sciences, 6(41), 2013 - 2018.

[29] Y. Yuan, B. Zhou and N. Trinajstic, (2010), *On Geometric-Arithmetic index*, J.Math.Chem., 47, 833 - 841. 10.1007/s10910-009-9603-8

[30] B. Zhou, I. Gutman, B. Furtula and Z.Du, (2009), *On two types of Geometric-Arithmetic index*, Chem. Phys. Lett., 482, 153 - 155.