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Design of Gravity Assist trajectory from Earth to Jupiter

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Abstract

The goal of this paper is to find a combination of conical trajectories, using gravitational assisted maneuvers (swing-by), which perform the transfer from a nearby of the departure planet (Earth) to the vicinity of the arrival planet (Jupiter), making a closest approaches with Mars (flyby) to reduce the fuel consumption for the journey. A detailed description of the mission from Earth− Mars−Jupiter, that used this technique is presented. The table of flyby dates, altitudes of closest approaches is also included. A methodology known as the Patched Conics was used, where the trajectory is divided into three parts:
1) Departure phase, inside of the sphere of influence of the departure planet,
2) Heliocentric phase, during the journey between the planets,
3) Arrival phase, inside the sphere of influence of the arrival planet.

Keywords: Interplanetary trajectories − Gravity Assist −Flyby − Departure and arrival trajectories.
AMS 2010 codes: Primary: 70F15; Secondary: 70M20.

Nomenclature

S = Index for Sun
E =Index for Jupiter
M =Index for Mars
μ = Gravitational parameter
R = Planet radius
r_p = Periapsis radius
$\overrightarrow{V_\infty}^{(1)}$ = Input hyperbolic excess velocity
$\overrightarrow{V_\infty}^{(2)}$ = Output hyperbolic excess velocity
Δt_1 = The time of flight between Earth and Mars

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Δt_2 = The time of flight between Mars and Jupiter
T_0 = The spacecraft departure date from the Earth (22/4/2018 $0^h\ 0^m\ 0^s$)
T_1 = The spacecraft departure and arrival date to Mars (22/8/2018 $0^h\ 0^m\ 8^s$)
T_2 = The spacecraft departure and arrival date to Jupiter (3/4/2021 $10^h\ 40^m\ 44^s$)
r_{0k} = the component of position vector of Earth and the S/C at T_0 relative to the sun in k - direction (k = x, y, z)
r_{1k} = the component of position vector of Mars and the S/C at T_1 relative to the sun in k - direction (k = x, y, z)
r_{3k} = the component of position vector of Jupiter and the S/C at T_2 relative to the sun in k - direction (k = x, y, z)

1 Introduction

In deep space missions, a gravity assist trajectory is often used, which uses the gravity of a planet (or other celestial body) to alter the path and speed of a spacecraft. This technique allows to reach destinations which would not be accessible with current technology or to reach targets with significantly reduced propulsion requirements. Many spacecrafts such as Voyager, Galileo, and Cassini use the gravity assist technique to achieve their targets. The two Voyager spacecrafts provide a classic example. Voyager 2 launched in August 1977 took one G. A. from Jupiter, one from Saturn, later from Uranus, and then move up to Neptune and beyond. Galileo passed by Venus then twice by Earth, and finally go up to its path Jupiter. Cassini passed by Venus twice, then Earth, and finally Jupiter on the way to Saturn [1–3].

In a gravity assist trajectory, angular momentum is transferred from the orbiting planet to a spacecraft, while the value of it's speed relative to planet is not changed during a gravity assist flyby, but it's direction is changed. However, both value and direction of spacecraft's speed relative to the sun are changed during a gravity assist flyby, due to the planet relative orbital velocity is added to the spacecraft's velocity on its way out.

The application of a"multi-conic method" with differential correction was explored by Wilson and Howell [4] with applications to the Sun-Earth-Moon environment. Their work is based on the original multi-conic method, which approximates trajectory legs by considering separate perturbing influences. This method is somewhat of a compromise between patched conics and fully integrated trajectories. In another work, Marchand, Howell, and Wilson [6] utilized a multi-step correction process for obtaining trajectories in an n-body ephemeris model. This procedure begins with a"seed" trajectory, divides the trajectory into nodes, and performs differential correction on the states at the nodes to satisfy specified constraints in the n-body model. The design of a transfer trajectory combining Solar Electric Propulsion (SEP) and gravity assist (GA) can be regarded as a general trajectory optimization problem [7]. The dynamics of the spacecraft is governed mainly by the gravity attraction of the Sun, when the spacecraft is outside the sphere of influence of a planet, and by the gravity attraction of the planet during a gravity assist maneuver. Low-thrust propulsion is then used to shape trajectory arcs between two subsequent encounters and to meet the best incoming conditions for a swing-by.

An interesting approach is to choice to direct collocation as demonstrated by Betts [8], who efficiently optimized a transfer trajectory to Mars combining low-thrust with two swing-bys of Venus. In this paper an original direct optimization approach has been used to design an optimal interplanetary trajectory. The proposed approach is characterized by a transcription of both states and controls by Finite Elements in Time (DFET) [9]. A set of additional parameters, not included among states and controls, are allowed and can be used for a combined optimization of both the trajectory and other quantities peculiar to the original optimal control problem (parametric optimization). In particular, in this paper, the orbital elements of each hyperbola are treated as additional parameters and opposite to the work of Betts, swing-by trajectories are not transcribed with collocation but using multiple shooting.

In this work we study the interplanetary trajectory of a spacecraft leaving Earth and making fly by with Mars in it's destination to Jupiter. We introduced a simple and accessible algorithms for interplanetary trajectory planning that do not require gross simplifications and are able to find the required solution. The algorithms are implemented in Mathematica program, which allows for their straightforward use in an academic setting.

2 Description of the Mission

The complete trajectory has been divided into five different segments. Three of them are planetary segments around Earth, Mars and Jupiter, respectively, the other two are heliocentric elliptic orbits Fig. (1). The classical

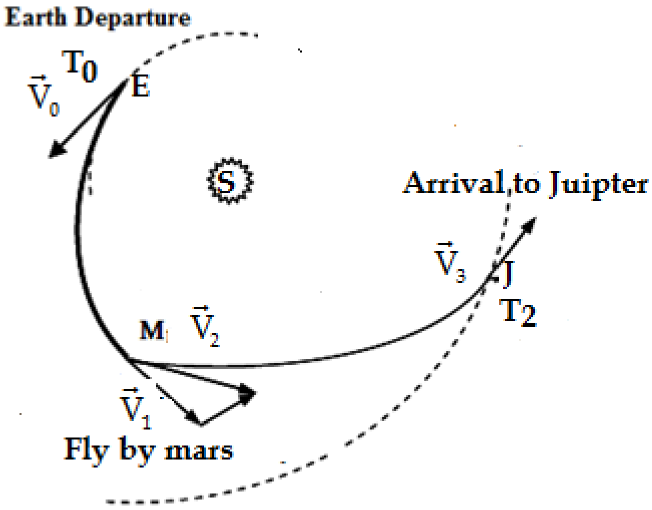

Fig. 1 Description of the mission from Earth to fly by with Mars

analysis of scales for interplanetary missions is adopted [5]. That is, since planetary radii are significantly smaller than planetary Spheres of influence (SoI), the limit of the (SoI) is considered (from the point of view of the planetary segments) to be located at infinity. On the other hand, from the perspective of the heliocentric trajectories Earth - Mars and Mars - Jupiter, the (SoI) are reduced to a point. Finally, using the method of patched conics, the five segments are joined to compose the complete trajectory.

3 Gravity Assists

A gravity assist maneuver is applied in an interplanetary trajectory to use of planet's gravitational field and momentum in order to increase or decrease the spacecraft's heliocentric orbital energy. In the planet centered reference frame of the patched conic method, the trajectory (unpowered gravity assist) does not change in orbital energy, but is simply redirected from entering $\overrightarrow{V_\infty}^{(1)}$ to exiting $\overrightarrow{V_\infty}^{(2)}$. When converting the planetocentric entering and departing V_∞ to a heliocentric spacecraft velocity, the change in heliocentric orbital velocity and energy is apparent, particularly when visualized in Figs. (1) and (3).

$\overrightarrow{V_1}^{(v)}$ is the spacecraft heliocentric velocity pre-encounter,

$\overrightarrow{V_2}^{(v)}$ is the spacecraft heliocentric velocity post-encounter,

$\overrightarrow{V}$ is the planet heliocentric velocity,

δ is the gravitational bend angle (turn angle).
The heliocentric velocity of the spacecraft $V^{(v)}$ resulting from the gravity assist is increased or decreased depending on how the maneuver is performed. If the hyperbolic periapsis occurs on the trailing side of the planet

with respect to the planet's heliocentric velocity, then the spacecraft heliocentric velocity will be increased by the gravity assist. If the hyperbolic periapsis occurs on the leading side of the planet with respect to the planet's heliocentric velocity, then the spacecraft the heliocentric velocity will be decreased by the gravity assist.

4 lambert's problem

Lambert's problem is characterized by taking two position vectors r_1, r_2 and the time of flight between them Δt and solving for the incoming and outgoing velocity vectors of the transfer trajectory $\overrightarrow{V}_1, \overrightarrow{V}_2$. In theory, Lambert's problem will need to be solved twice. The first one for the departure from Earth and the arrival at the flyby planet (Mars), and the second for the departure from the flyby planet and the arrival at the target planet (Jupiter), see Table (1). From these departure and arrival velocities, we can then calculate the ΔV requirements that the spacecraft will need to be able to perform those maneuvers. There are many methods that can be used to solve Lambert's problem. For this design we will be using the Universal Variable method.

Universal Variable Method

The algorithm that is used is taken from Fundamentals of Astrodynamics and Applications [10]. This algorithm utilizes the bisection method which provide a strong solution for a wide variety of transfer orbits. The formulation of this method begins with the f and g universal variable, defined by the following formulas :

$$\begin{aligned} f &= 1 - \frac{\chi^2}{r_1} C(z) \\ g &= \Delta t - \frac{\chi^3}{\sqrt{\mu}} C(z) \\ \dot{g} &= 1 - \frac{\chi^3}{r_2} C(z) \\ \dot{f} &= \frac{\sqrt{\mu}}{r_1 r_2} \chi (z S(z) - 1) \end{aligned} \tag{1}$$

Where r_1 and r_2 are the magnitudes of the initial and final position vectors,
χ is a universal variable, z is square of the difference in eccentric anomalies, E, at two position ($z = (\Delta E)^2$)), $C(z)$ and $S(z)$ are define as:

$$C(z) = \frac{\sqrt{z} - \sin\sqrt{z}}{\sqrt{z^3}}, \quad S(z) = \frac{1 - \cos\sqrt{z}}{z}$$

The f and g expressions in terms of the orbital elements

$$\begin{aligned} g &= \frac{r_1 r_2}{\sqrt{\mu P}} \sin\Delta\nu \\ f &= 1 - \frac{r_2}{P}(1 - \cos\Delta\nu) \\ \dot{g} &= 1 - \frac{r_1}{P}(1 - \cos\Delta\nu) \\ \dot{f} &= \sqrt{\frac{\mu}{P}} \tan\frac{\Delta\nu}{2} \left(\frac{1 - \cos\Delta\nu}{P} - \frac{1}{r_1} - \frac{1}{r_2}\right) \end{aligned} \tag{2}$$

Equating the corresponding equations in the two groups Eqs. (1) and Eqs. (2), we obtain

$$f = 1 - \frac{r_2}{P}(1 - \cos\Delta\nu) = 1 - \frac{\chi^2}{r_1} C(z) \tag{3}$$

$$g = \frac{r_1 r_2}{\sqrt{\mu P}} \sin\Delta\nu = \Delta t - \frac{\chi^3}{\sqrt{\mu}} C(z) \tag{4}$$

$$\dot{g} = 1 - \frac{r_1}{P}(1 - \cos\Delta\nu) = 1 - \frac{\chi^3}{r_2} C(z) \tag{5}$$

$$\dot{f} = \sqrt{\frac{\mu}{P}} \tan\frac{\Delta\nu}{2} \left(\frac{1-\cos\Delta\nu}{P} - \frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{\sqrt{\mu}}{r_1 r_2} \chi (zS(z) - 1) \tag{6}$$

We get from Eq. (3)

$$\chi = \sqrt{\frac{r_1 r_2 (1 - \cos\Delta\nu)}{PC(z)}} \tag{7}$$

Substituting with χ in Eq. (6) and cancelling $h = \sqrt{\frac{\mu}{p}}$ from both sides, we obtain after simplification ;

$$\frac{r_1 r_2 (1 - \cos\Delta\nu)}{P} = r_1 + r_2 - \frac{\sqrt{r_1 r_2} \sin\Delta\nu}{\sqrt{1 - \cos\Delta\nu}} \frac{(1 - zS(z))}{\sqrt{C(z)}} \tag{8}$$

We can write this equation more compactly by defining two auxiliary symbols, A and y as :

$$A = \frac{\sqrt{r_1 r_2} \sin\Delta\nu}{\sqrt{1 - \cos\Delta\nu}}, \quad y = r_1 r_2 \frac{(1 - \cos\Delta\nu)}{P}$$

Using these definitions of A and y, Eqs. (7) and (8) may be written more compactly as

$$\chi = \sqrt{\frac{y}{C(z)}}$$

$$y = r_1 + r_2 - A \frac{(1 - zS(z))}{\sqrt{C(z)}}$$

If we now solve for Δt from Eq. (4), we get

$$\sqrt{\mu}\Delta t = \chi^3 S(z) + A\sqrt{y}$$

Using the auxiliary symbols A and y to write Eqs. (3 - 6) in the following simplified expressions :

$$f = 1 - \frac{y}{r_1}, \quad g = A\sqrt{\frac{y}{\mu}}, \quad \dot{g} = 1 - \frac{y}{r_2}$$

Then the solution of Lambert problems yields the following relations :

$$\overrightarrow{V_1} = \frac{1}{g}(\overrightarrow{r_2} - f\overrightarrow{r_1}), \quad \overrightarrow{V_2} = \frac{1}{g}(\dot{g}\overrightarrow{r_2} - \overrightarrow{r_1})$$

Table 1 Calculations of Lambert problems

	lambert earth - Mars		Lambert Mars -Jupiter
r_{0x} (km)	$-1.280952970127814 \times 10^8$	$r_{2x} = r_{1x}$(km)	$1.588109522284044 \times 10^8$
r_{0y}(km)	$-7.873040871488884 \times 10^7$	$r_{2y} = r_{1y}$(km)	$-1.331196049556935 \times 10^8$
r_{0z}(km)	4241.23706297353500	$r_{2z} = r_{1z}$(km)	-6690470.129802731
r_{1x}(km)	$1.588109522284044 \times 10^8$	r_{3x}(km)	$5.331461279416993 \times 10^8$
r_{1y}(km)	$-1.3311960495569350 \times 10^8$	r_{3y}(km)	$5.392020271071861 \times 10^8$
r_z(km)	-6690470.12980273100	r_{3z}(km)	-9671957.471152349
$\Delta t_1(s)$	$1.054080811623402 \times 10^7$	$\Delta t_2(s)$	$8.25504364820473 \times 10^7$
$\mu_s(km^3/s^2)$	$1.32712428000 \times 10^{11}$	$\mu_s(km^3/s^2)$	$1.32712428 \times 10^{11}$
$V_{0x}(km/s)$	18.61716546638244600	$V_{2x}(km/s)$	30.823073404118684
$V_{0y}(km/s)$	-28.29950136444239000	$V_{2y}(km/s)$	4.934176592416298
$V_{0z}(km/s)$	-1.152116764359739900	$V_{2z}(km/s)$	-0.8994633203622276
$V_{1x}(km/s)$	21.7254264303175400	$V_{3x}(km/s)$	-5.888427200446674
$V_{1y}(km/s)$	13.84469939965363100	$V_{3y}(km/s)$	3.2105733597573787
$V_{1z}(km/s)$	0.013528429195021072	$V_{3z}(km/s)$	0.22569580208145187

5 Mission Analysis

5.1 Initial impulse (Escape from the Earth at T_0)

In order to escape the gravitational pull of a planet, the spacecraft must travel a hyperbolic trajectory relative to the planet, arriving at it's sphere of influence with a relative velocity $V_{\infty 1}$ (hyperbolic excess velocity) greater than zero Fig. (2).

The heliocentric velocity of S/C $\overrightarrow{V_0}$ at departure from the Earth is the sum of the Earth velocity $\overrightarrow{V_E}$ and the hyperbolic excess velocity $\overrightarrow{V}_{\infty 1}$.

$$\overrightarrow{V_0} = \overrightarrow{V_E} + \overrightarrow{V}_{\infty 1}$$

The latter is assumed to be equal to the spacecraft velocity relative to the Earth. In general it is $\overrightarrow{V_E} \gg \overrightarrow{V_{\infty 1}}$, due

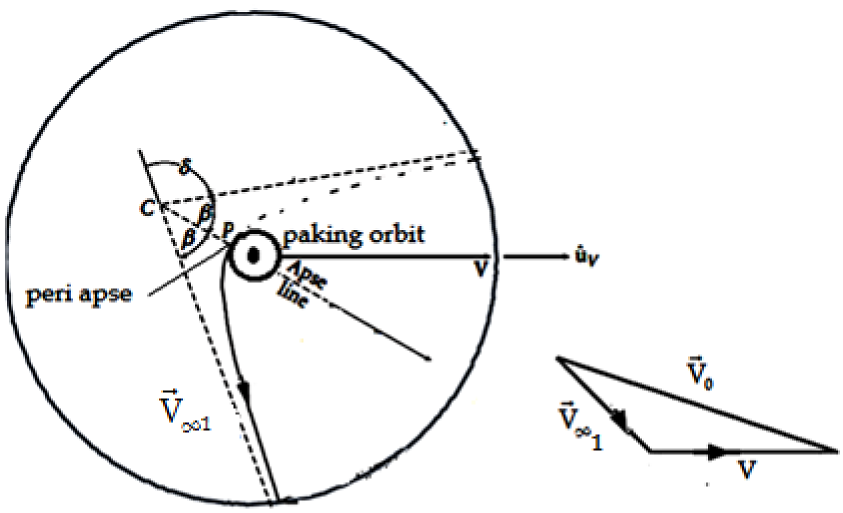

Fig. 2 Escape from Earth

to the modest capabilities of present space propulsion, so that the maximum angle between $\overrightarrow{V_E}$ and $\overrightarrow{V_0}$ is quite small. In particular the heliocentric leg will lie in a plane that can assume only a modest inclination away from the ecliptic plane.

The impulse required to be given at the perigee of the hyperbolic orbit to transfer the spacecraft from the parking orbit to the escape hyperbolic orbit is given by

$$\Delta\overrightarrow{V_1} = \overrightarrow{V_p} - \overrightarrow{V_c}$$

$\overrightarrow{V_p}$ is the velocity of the spacecraft at perigee of hyperbolic orbit,
$\overrightarrow{V_c}$ is the velocity of the spacecraft in the parking orbit.
Clearly the direction of $\overrightarrow{V_p}$ and $\overrightarrow{V_c}$ are the same, then $\Delta\overrightarrow{V_1}$ is also in the same direction.
We can obtain $\overrightarrow{V_0}$ from solving Lambert's problem earth - Mars , see Table (1), we calculate $\overrightarrow{V_E}$ at T_0 to find $\overrightarrow{V}_{\infty 1}$ and it's magnitude $(V_{\infty 1})$ which by it and a given perigee $(r_{p1} = R_E + 300)$ can calculate the hyperbolic trajectory elements. the angular momentum and eccentricity of the hyperbolic orbit can be obtained from the following relations [5] :

$$h_1 = r_{p1}\sqrt{V_{\infty 1}^2 + \frac{2\mu}{r_{p1}}} \quad , e_1 = 1 + \frac{r_{p1}V_{\infty 1}^2}{\mu_1}$$

The velocity at the perigee of the hyperbolic orbit is :

$$V_{P1} = \sqrt{V_{\infty 1}^2 + \frac{2\mu_E}{r_{p1}}}$$

The speed of S/c in its circular parking orbit is given by $V_c = \sqrt{\mu/r_{p1}}$ Then the ΔV_1 required to put the S/C onto the hyperbolic departure trajectory is :

$$\Delta\overrightarrow{V_1} = \overrightarrow{V_{p1}} - \overrightarrow{V_c} = \sqrt{V_{\infty 1}^2 + \frac{2\mu_E}{r_{p1}}} - \sqrt{\frac{\mu_E}{r_{p1}}}$$

$$\Delta\overrightarrow{V_1} = V_c\left(\sqrt{2 + \left(\frac{V_{\infty 1}}{V_c}\right)^2} - 1\right)$$

The orientation of the apse line of the hyperbola to the asymptotes of the hyperbolic trajectory measured by the angle , which can be obtained from the relation [5]

$$\beta = cos^{-1}\left(\frac{1}{e_1}\right) = \cos^{-1}\left(\frac{1}{1 + \frac{r_{p1}V_{\infty 1}^2}{\mu_E}}\right)$$

The results are summarized in the Table (2).

5.2 Gravity assist maneuver (fly by Mars at $T_1 = T_0 + \Delta t_1$)

Now after solving Lambert's problem for earth - Mars trajectory and Mars, Jupiter trajectory we have $\overrightarrow{V}_1$ and $\overrightarrow{V}_2$, see Table (1). Such that :
$\overrightarrow{V}_1$ is the heliocentric S/C velocity at final position for Earth-Mars Lambert's algorithm,
$\overrightarrow{V}_2$ is the heliocentric S/C velocity at initial position for Mars- Jupiter Lambert's algorithm,

Table 2 Escape from Earth at T_0

$\overrightarrow{V}_{E0} = \{15.113446216468128, -25.49053048920682, 0.0008860019561318039\}$

$\overrightarrow{V}_0 = \{18.617165466382446, -28.29950136444239, -1.1521167643597399\}$

$\overrightarrow{V}_{\infty 1} = \{3.50372, -2.80897, -1.153\}$

JD (Julian day No) of $T_0 = 2458230.5$

$V_{(parkingorbit)} = 11.8689$(km/s)

$V_{\infty 1}$ (km/s)	μ_E (km³/s²)	r_{p1} (km)	e_1	h_1	β (red)	V_{p1} (km/s)	ΔV_1 (km/s)
4.63635	3.986004415×10^5	6678.1363	1.36014	79262.1	.744807	7.72576	4.14313

$\overrightarrow{V}_M$ is the velocity of Mars at time T_1.
Then the heliocentric velocity of spacecraft at the SOI of Mars is $\overrightarrow{V}_1$ and it need to out from the SOI by $\overrightarrow{V}_2$, to complete the trajectory and finally reach to the SOI of Jupiter, on the other hand the spacecraft enter the SOI velocity relative to mars is $\overrightarrow{V}_\infty^{(1)}$ and it need to out with velocity relative to Mars is $\overrightarrow{V}_{out}$ Fig. (3).

$$\overrightarrow{V}_\infty^{(1)} = \overrightarrow{V}_1 - \overrightarrow{V}_M$$

$$\overrightarrow{V}_{out} = \overrightarrow{V}_0 - \overrightarrow{V}_M$$

We take the direction of $\overrightarrow{V}_\infty^{(2)}$ (the out velocity of spacecraft in hyperbolic orbit fly by) in the same direction of $\overrightarrow{V}_{out}$ then the turn angle of the hyperbolic orbit flyby can be calculate from this relation [5],

$$cos\delta = \frac{\overrightarrow{V}_\infty^{(1)}.\overrightarrow{V}_\infty^{(2)}}{V_\infty^2} = \frac{\overrightarrow{V}_\infty^{(1)}.\overrightarrow{V}_{out}}{V_\infty \left|\overrightarrow{V}_{out}\right|}$$

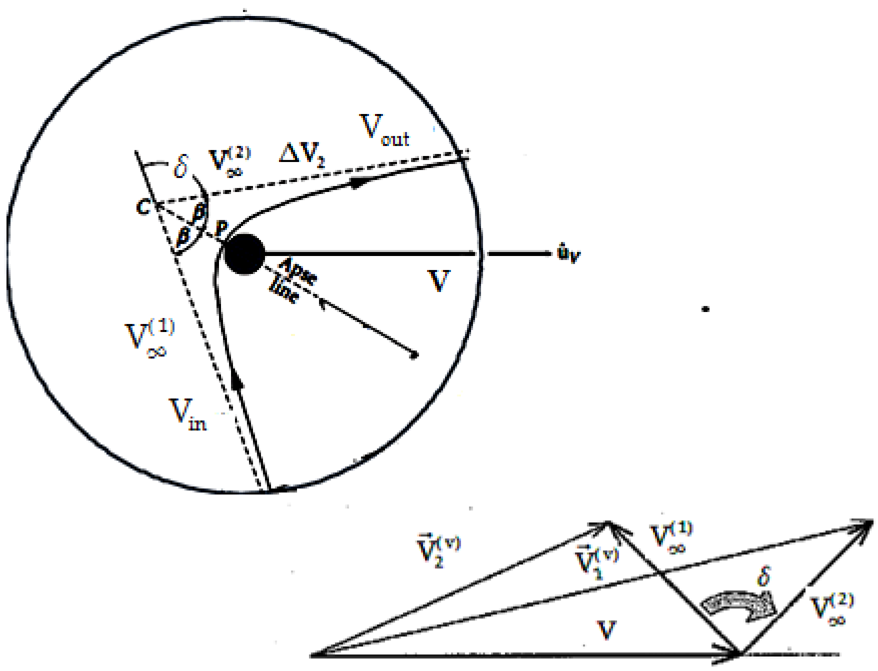

Fig. 3 fly by with Mars

We know that

$$\left|\overrightarrow{V}_\infty^{(1)}\right| = \left|\overrightarrow{V}_\infty^{(2)}\right|$$

Now we can calculate the hyperbolic orbital elements using the relation [5],

$$e_2 = \frac{1}{\sin(\frac{\delta}{2})}, \quad r_{p2} = \frac{\mu_M}{V_{\infty 2}^2}(e_2 - 1), \quad \Delta = r_p\sqrt{1 + \frac{2\mu_M}{r_p V_{\infty 2}^2}}$$

Then we can calculate the perigee of the hyperbolic fly by Mars, now the spacecraft out the SOI of Mars with the velocity $\overrightarrow{V}_{\infty}^{(2)}$ relative to Mars with magnitude is $V_{\infty 2}$ and in the same direction of $\overrightarrow{V}_{out}$, to make the velocity of spacecraft relative to Mars is $\overrightarrow{V}_{out}$ we give it ΔV_2 in the same direction of $\overrightarrow{V_{\infty}}^{(2)}$,

$$\Delta V_2 = \left|\overrightarrow{V}_{out}\right| - V_{\infty 2}$$

After that the spacecraft out from the SOI of Mars with heliocentric velocity is
$\overrightarrow{V}_2 = \overrightarrow{V}_M + \overrightarrow{V}_{out}$
which by it can complete it's trajectory to reach the SOI of Jupiter. The results are summarized in Table (3).

Table 3 Fly by Mars at T_1

$$\overrightarrow{V}_{in} = \overrightarrow{V}_1 = \{21.72542643031754, 13.844699399653631, 0.013528429195021072\}$$
$$\overrightarrow{V}_{out} = \overrightarrow{V}_2 = \{30.823073404118684, 4.934176592416298, -0.8994633203622276\}$$
$$\overrightarrow{V}_M = \{16.48592538172737, 20.64382791494634, 0.02774276866812586\}$$

JD (Julian day No) of $T_1 = 2458230.5$

$V_{\infty 2 (km/s)}$	$\mu_{M (km^3/s^2)}$	$r_{p1 (km)}$	e_2	Δ(km)	$V_{out/M (km/s)}$	δ (red)	$\Delta V_{2 (km/s)}$
8.58375	4.305×10^4	11968.6	21.4844	12539.3	21.2887	0.0931244	12.7049

5.3 Capture by Jupiter at ($T_2 = T_1 + \Delta t_2$)

A spacecraft arrives at the sphere of influence of the Jupiter with a hyperbolic excess velocity $\overrightarrow{V}_{\infty 3}$ relative to Jupiter, where

$$\overrightarrow{V}_{\infty 3} = \overrightarrow{V}_3 - \overrightarrow{V}_J$$

Such that :
$\overrightarrow{V}_3$ is the heliocentric S/C velocity at final position for Mars- Jupiter lambert algorithm. see Table (1), or it is the heliocentric S/C velocity at SOI of Jupiter at T_2
$\overrightarrow{V}_J$ is the heliocentric velocity of Jupiter at T_2.
the goal's mission is landing on Jupiter To achieve this goal we make the perigee of the hyperbola equal to the Jupiter radius ($r_{p3} = R_J = 69911$ km) and the velocity of S/C relative Jupiter equal zero at perigee. we give S/C the third impulse in the perigee of the hyperbola equal to the it's velocity and in the opposite direction. The results are summarized in Table (4),

$$\Delta\overrightarrow{V}_3 = -\overrightarrow{V}_{p3}$$

6 Conculsion

The problem of preliminary interplanetary design to outer planets has been studied using Gravity-assisted maneuvers techniques which have been introduced as a resource to get the required energy to reach far planets. Deep Space Maneuvers and impulses at the flyby periapsis have also been described as means to increase

Table 4 Capture from Jupiter at T_2

$\overrightarrow{V}_3 = \{-5.888427200446674, 3.2105733597573787, 0.225695802081451 87\}$

$\overrightarrow{V}_J = \{9.130213923868334, 9.803076409849977, -0.2456201552966279\}$

JD (Julian day No) of $T_2 = 0.21253784951961086$

$\overrightarrow{V}_{\infty 3} = \{-15.0186, -6.5925, 0.471316\}$

$V_{\infty 2}(km/s)$	$\mu_J(km^3/s^2)$	$r_{p1}(km)$	e_2	$\Delta(km)$	$V_{p3}(km/s)$	$\Delta V_3(km/s)$
16.4086	1.26675×10^8	69911	1.14859	265842	62.3951	-62.3951

the degrees of freedom in the global trajectory design process. The method is applied to transfer trajectory from the Earth to planet Jupiter making flyby with Mars to gain an extra energy to reach to the target planet (Jupiter). Lambert problem were used to find a solution for the position vectors from initial orbits in each transfer.

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