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Computing topological indices of the line graphs of Banana tree graph and Firecracker graph

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Abstract

In this paper, we compute first Zagreb index (coindex), second Zagreb index (coindex), third Zagreb index, first hyper-Zagreb index, atom-bond connectivity index, fourth atom-bond connectivity index, sum connectivity index, Randić connectivity index, augmented Zagreb index, Sanskruti index, geometric-arithmetic connectivity index and fifth geometric-arithmetic connectivity index of the line graphs of Banana tree graph and Firecracker graph.

Keywords: Banana tree graph; Firecracker graph; Topological indices; line graph**AMS 2010 codes:** 05C78

1 Introduction and Preliminaries

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. We denote $p = |V(G)|$ and $q = |E(G)|$, order and size of graph G respectively. The degree d_v of a vertex v in graph G is defined as the number of first neighbors of vertex v in G . The concept of degree in graph theory is closely related to the concept of valence in chemistry. The complement of a graph G , represented through \overline{G} , is a simple graph on the similar set of vertices $V(G)$ wherein two vertices u and v are joined by an edge uv , if and only if they are not adjacent in G . Obviously, $E(G) \cup E(\overline{G}) = E(K_p)$ where K_p is complete graph of order p , and $|E(\overline{G})| = \frac{p(p-1)}{2} - q$. In the mathematically discipline of graph theory, the line graph of an undirected graph G is alternative graph $L(G)$ that denotes the adjacencies among the edges of G . Line graphs are very useful in mathematical chemistry, but in recent years they were considered very little in chemical graph theory. For further facts about the applications of line graphs in chemistry, we mention the articles [6, 25, 32].

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Topological indices are the arithmetical numbers which depends upon the construction of any simple graph. Topological indices are generally classified into three kinds: degree-based indices (see [2, 5, 7, 8, 28, 29]), distance-based indices (see [3, 16]), and spectrum-based indices (see [11, 27]). There are also certain topological indices depends upon both degrees and distances (see [9, 27]). The advantage of topological indices is in that they may be used directly as simple numerical descriptors in a comparison with, chemical, physical or biological parameters of molecules in Quantitative Structure Property Relationships (QSPR) and in Quantitative Structure Activity Relationships (QSAR). Wiener index is the oldest topological index and its mathematical properties and chemical applications have been extensively studied.

The Wiener index of graph G is defined as follows:

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u,v).$$

where (u, v) is any ordered pair of vertices in G and $d(u, v)$ is $u - v$ geodesic.

The Zagreb indices were first introduced by Gutman in [24], they are important molecular descriptors and have been closely correlated with many chemical properties (see [35]) and defined as:

$$M_1(G) = \sum_{u \in V(G)} d_u^2 \quad \text{and} \quad (1)$$

$$M_2(G) = \sum_{uv \in E(G)} d_u d_v. \quad (2)$$

In fact, one can rewrite the first Zagreb index as

$$M_1(G) = \sum_{uv \in E(G)} [d_u + d_v].$$

The third Zagreb index, introduced by Fath-Tabar in [13]. This index is defined as follows:

$$M_3(G) = \sum_{uv \in E(G)} |d_u - d_v|. \quad (3)$$

The hyper-Zagreb index was first introduced in [34]. This index is defined as follows:

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2. \quad (4)$$

In 2008, Doslic put forward the first Zagreb coindex, defined as (see [1]):

$$\overline{M}_1 = \overline{M}_1(G) = \sum_{uv \notin E(G)} [d_u + d_v].$$

The second Zagreb coindex is defined as (see [1]):

$$\overline{M}_2 = \overline{M}_2(G) = \sum_{uv \notin E(G)} d_u d_v.$$

The degree distance index for graphs developed by Dobrynin and Kochetova in [10] and Gutman in [17] as

a weighted version of the Wiener index. The degree distance of G , denoted by $DD(G)$, is defined as follows:

$$DD(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)[d_u + d_v].$$

Randić index introduced by Milan Randić in 1975 (see [30]). This index is defined as follows:

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}. \quad (5)$$

Later, this index was generalized by Bollobás and Erdős (see [4]) to the following form for any real number α , and named the general Randić index:

$$R(G) = \sum_{uv \in E(G)} [d_u d_v]^\alpha.$$

The Atom-Bond Connectivity index (ABC), introduced by Estrda et al. in [12] which has been applied up until now to study the stability of alkanes and the strain energy of cycloalkanes. The ABC index of G is defined as:

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \quad (6)$$

In 2010, the general sum-connectivity index $\chi(G)$ has been introduced in [37]. This index is defined as follows:

$$\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}}. \quad (7)$$

D. Vukicevic and B. Furtula introduced the geometric arithmetic (GA) index in [36]. The GA index for G is defined by

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \quad (8)$$

Inspired by the work on the ABC index, Furtula et al. proposed the following modified version of the ABC index and called it as Zagreb index (AZI) in [14]. This index is defined as follows:

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_u d_v}{d_u + d_v - 2} \right)^3. \quad (9)$$

The fourth member of the class of ABC index was introduced by M. Ghorbani et al. in [20, 22, 23] as:

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}. \quad (10)$$

where S_u is the summation of degrees of all neighbors of vertex u in G . In other words, $S_u = \sum_{uv \in E(G)} d_v$. Similarly for S_v .

The 5th GA index was introduced by Graovac et al. in [21] as:

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}. \quad (11)$$

The Sanskruti index $S(G)$ of a graph G is defined in [33] as follows:

$$S(G) = \sum_{uv \in E(G)} \left(\frac{S_u S_v}{S_u + S_v - 2} \right)^3. \quad (12)$$

Theorem 1. [17] Let G be a graph of order p and size q . Then

$$M_1(\overline{G}) = M_1(G) + p(p-1)^2 - 4q(p-1); \quad (13)$$

$$\overline{M}_1(G) = 2q(p-1) - M_1(G); \quad (14)$$

$$\overline{M}_1(\overline{G}) = 2q(p-1) - M_1(G). \quad (15)$$

Theorem 2. [19] Let G be a graph of order p and size q . Then

$$M_2(\overline{G}) = \frac{1}{2}p(p-1)^3 - 3q(p-1)^2 + 2q^2 + \frac{2p-3}{2}M_1(G) - M_2(G); \quad (16)$$

$$\overline{M}_2(G) = 2q^2 - \frac{1}{2}M_1(G) - M_2(G); \quad (17)$$

$$\overline{M}_2(\overline{G}) = q(p-1)^2 - (p-1)M_1(G) + M_2(G). \quad (18)$$

Theorem 3. [26] Let G be a graph of order p and size q . Then

$$\overline{M}_1(G) \geq 2W(G) - 2M_1(G) + 6q(p-1) - p^3 + p^2. \quad (19)$$

Theorem 4. [26] Let G be a nontrivial graph of diameter $d \geq 2$. Then

$$\overline{M}_1(G) \leq \frac{DD(G) - M_1(G)}{2} \quad (20)$$

with equality if and only if $d = 2$.

The following lemma is helpful for computing the degree of a vertex of line graph.

Lemma 5. Let G be a graph with $u, v \in V(G)$ and $e = uv \in E(G)$. Then:

$$d_e = d_u + d_v - 2.$$

Lemma 6. [18] Let G be a graph of order p and size q , then the line graph $L(G)$ of G is a graph of order p and size $\frac{1}{2}M_1(G) - q$.

In this paper we discuss the topological indices and co-indices of the line graphs of Banana tree graph and Firecracker graph and their complement graphs.

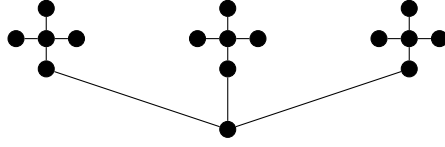
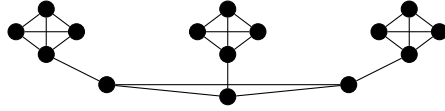
2 Topological indices of line graph of Banana tree graph

In this section we computed the topological indices of the line graph of Banana tree graph. The Banana tree graph $B_{n,k}$ is the graph obtained by connecting one leaf of each of n copies of an k -star graph with a single root vertex that is distinct from all the stars. The $B_{n,k}$ has order $nk + 1$ and size nk . $B_{3,5}$ is shown in the Fig. 1.

Theorem 7. Let G be the line graph of the Banana tree graph $B_{n,k}$. Then

$$1. M_1(G) = nk^3 - 5nk^2 + 10nk + n^3 - 7n;$$

$$2. M_2(G) = \frac{1}{2}nk^4 - \frac{7}{2}nk^3 + n^2k + 10nk^2 - 14nk + \frac{1}{2}n^4 - \frac{1}{2}n^3 - n^2 + 8n;$$

**Fig. 1** The Banana tree graph $B_{3,5}$.**Fig. 2** The line graph of Banana tree graph $B_{3,5}$.

3. $M_3(G) = -n^2 + 2nk - 3n$;
4. $HM(G) = 2n^4 + 2nk^4 - 14nk^3 + 2n^2k + 41nk^2 - 57nk - n^3 - 2n^2 + 31n$;
5. $R(G) = \frac{1}{2} \frac{n^2-n}{\sqrt{n^2}} + \frac{n}{\sqrt{(k-1)n}} + \frac{kn-2n}{\sqrt{(k-1)(k-2)}} + \frac{1}{2} \frac{nk^2+6n-5nk}{\sqrt{(k-2)^2}}$;
6. $ABC(G) = \frac{1}{2}n\sqrt{3} + \frac{2}{3}n + \frac{1}{6}n\sqrt{15} + \frac{1}{4}n\sqrt{6}$;
7. $GA(G) = \frac{2n\sqrt{(k-1)n}}{k-1+n} + \frac{2n(k-2)\sqrt{(k-1)(k-2)}}{2k-3} + \frac{1}{2}(n^2 + nk^2 - 5nk + 5n)$;
8. $\chi(G) = \frac{(n^2-n)\sqrt{2}}{4\sqrt{n}} + \frac{n}{\sqrt{k-1+n}} + \frac{kn-2n}{2k-3} + \frac{nk^2+6n-5nk}{\sqrt{2k-4}}$;
9. $AZI(G) = \frac{(n^2-n)n^6}{(2n-2)^3} + \frac{n^4(k-1)^3}{(k-3+n)^3} + \frac{(kn-2n)(k-1)^3(k-2)^3}{(2k-5)^3} + \frac{(nk^2+6n-5nk)(k-2)^6}{(2k-6)^3}$.

Proof.

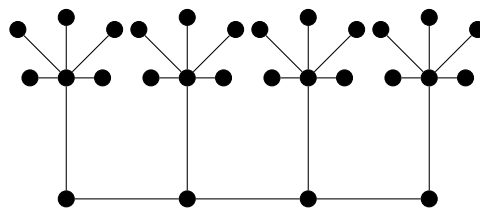
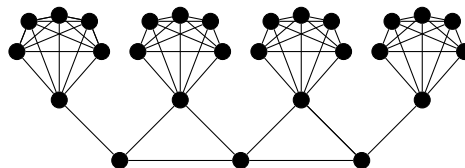
The graph G for $n = 3$ and $k = 5$ is shown in Fig. 2. By using Lemma 5, it is easy to see that the order of G is nk out of which $(k-2)n$ vertices are of degree $k-2$, n vertices are of degree $k-1$ and n vertices are of degree n . Therefore by using Lemma 6, G has size $\frac{n^2+3n+nk^2-3nk}{2}$. We partition the size of G into edges of the type $E_{(d_u, d_v)}$ where uv is an edge. In G , we get edges of the type $E_{(n,n)}$, $E_{(k-1,n)}$, $E_{(k-1,k-2)}$ and $E_{(k-2,k-2)}$. The number of edges of these types are given in the Table 1. By using Formulas (1)-(9) and Table 1, we can obtain the required results.

(d_u, d_v) where $uv \in E(G)$	(n, n)	$(k-1, n)$	$(k-1, k-2)$	$(k-2, k-2)$
Number of edges	$\frac{n(n-1)}{2}$	n	$(k-2)n$	$\frac{nk^2+6n-5nk}{2}$

Table 1 The size partition of G .

Theorem 8. Let G be the Web graph W_n . Then

1. $M_1(\overline{G}) = -2n^3k - 6n^2k + nk^2 + 5nk + n^3 + 2n^2 - n$;
2. $\overline{M}_1(G) = \overline{M}_1(\overline{G}) = -nk^3 + 3nk^2 - 7nk - n^3 + n^2 + 10n$;
3. $M_2(\overline{G}) = n^4k - \frac{1}{2}n^3k^2 - 8n^2k - \frac{3}{2}n^2k^2 + 3nk^2 + 3nk + 2n^3 + 4n^2 - 2n$;
4. $\overline{M}_2(G) = -nk^3 + 3nk^2 - 7nk - n^3 + n^2 + 10n$;

Fig. 3 The Firecracker graph $F_{4,7}$.Fig. 4 The line graph of Firecracker graph $F_{4,7}$.

5. $\overline{M}_2(\overline{G}) = -n^4k + \frac{1}{2}n^3k^2 - n^3k + \frac{3}{2}n^2k^2 + 5n^2k - \frac{3}{2}nk^2 - \frac{11}{2}nk + \frac{1}{2}n^4 + \frac{1}{2}n^3 - \frac{1}{2}n^2 + \frac{5}{2}n$;
6. $W(G) \leq \frac{(1-3k)n^3 + (-3k^3 + 6k^2 - 9k)n^2 + (4k^2 + 4k + 12)n + (nk-1)^3 - (nk-1)^2}{2}$;
7. $DD(G) \geq -n^3 + 2(-3k^2 + 1)n^2 + (-k^3 + 7k^2 - 4k + 13)n$.

(S_u, S_v) where $uv \in E(G)$	$(n^2 - n + k - 1, n^2 - n + k - 1)$	$(k^2 - 4k + n + 4, n^2 - n + k - 1)$
Number of edges	$\frac{n(n-1)}{2}$	n
(S_u, S_v) where $uv \in E(G)$	$(k^2 - 4k + n + 4, k^2 - 4k + 5)$	$(k^2 - 4k + 5, k^2 - 4k + 5)$
Number of edges	$(k-2)n$	$\frac{nk^2 + 6n - 5nk}{2}$

Table 2 The size partition of G .

Proof. By using Theorems (1)-(4) and (7), we can obtain the required results.

Theorem 9. Let G be the Banana tree graph $B_{n,k}$. Then

1. $ABC_4(G) = \frac{1}{2}(n^2 - n)\sqrt{\frac{2n^2 + 2k - 2n - 4}{(n^2 + k - n - 1)^2}} + n\sqrt{\frac{k^2 + n^2 - 3k + 1}{(k^2 - 4k + n + 4)(n^2 + k - n - 1)}} + (kn - 2n)\sqrt{\frac{2k^2 - 8k + n + 7}{(k^2 - 4k + n + 4)(k^2 - 4k + 5)}} + \frac{1}{2}(nk^2 + 6n - 5nk)\sqrt{\frac{2k^2 - 8k + 8}{(k^2 - 4k + 5)^2}}$;
2. $GA_5(G) = (n^2 - n)\frac{\sqrt{(n^2 + k - n - 1)^2}}{2n^2 + 2k - 2n - 2} + 2n\frac{\sqrt{(k^2 - 4k + n + 4)}}{k^2 + n^2 - 3k + 3} + \frac{2(kn - 2n)\sqrt{(k^2 - 4k + n + 4)(k^2 - 4k + 5)}}{2k^2 - 8k + n + 9} + \frac{(nk^2 + 6n - 5nk)\sqrt{(k^2 - 4k + 5)^2}}{2k^2 - 8k + 10}$;
3. $S(G) = \frac{(n^2 - n)(n^2 + k - n - 1)^6}{(2n^2 + 2k - 2n - 4)^3} + \frac{n(k^2 - 4k + n + 4)^3(n^2 + k - n - 1)^3}{(k^2 + n^2 - 3k + 1)^3} + \frac{(kn - 2n)(k^2 - 4k + n + 4)^3(k^2 - 4k + 5)^3}{(2k^2 - 8k + n + 7)^3} + \frac{(nk^2 + 6n - 5nk)(k^2 - 4k + 5)^6}{2(2k^2 - 8k + 8)^3}$.

Proof.

We partition the size of G into edges of the type (S_u, S_v) where $uv \in E(G)$ as shown in Table 2. Hence we get the required results by using Table 2 and Formulas (10)-(12).

3 Topological indices of line graph of Firecracker graph

In this section we computed the topological indices of the line graph of Firecracker graph. The Firecracker graph $F_{n,k}$ is the graph obtained by the concatenation of nk -stars by linking one leaf from each. The $F_{n,k}$ has order nk and size $nk - 1$. $F_{4,7}$ is shown in the Fig. 3.

Theorem 10. Let G be the line graph of the Firecracker graph $F_{n,k}$. Then

1. $M_1(G) = nk^3 - 5nk^2 + 12nk - 4k + 8n - 28$;
2. $M_2(G) = \frac{1}{2}nk^4 - \frac{7}{2}nk^3 + 11nk^2 - 10nk - 2k^2 + 28n - 4k - 54$;
3. $M_3(G) = 4nk - 4k - 12n + 16$;
4. $HM(G) = 2nk^4 + 4nk^2 - 14nk^3 - 52nk - 10k^2 - 2k + 136n - 248$;
5. $R(G) = \frac{1}{3}\sqrt{3} + \frac{2}{3}\sqrt{\frac{3}{k}} + \frac{2}{\sqrt{3(k-1)}} + \frac{1}{4}n - 1 + \frac{n-3}{\sqrt{k}} + 2\sqrt{\frac{k-2}{k-1}} + \frac{nk^2+6n-5nk}{k-2} + (n-2)\sqrt{\frac{k-2}{k}}$;
6. $ABC(G) = \frac{1}{3}\sqrt{15} + \frac{2}{3}\sqrt{3}\sqrt{\frac{k+1}{k}} + \frac{2}{3}\sqrt{3}\sqrt{\frac{k}{k-1}} + \frac{1}{4}(n-4)\sqrt{6} + \frac{1}{2}(2n-6)\sqrt{\frac{2+k}{k}} + (2k-4)\sqrt{\frac{2k-5}{(k-2)(k-1)}} + \frac{1}{2}(nk^2+6n-5nk)\sqrt{\frac{2k-6}{(k-2)^2}} + (nk-2n-2k+4)\sqrt{\frac{2k-4}{(k-2)k}}$;
7. $GA(G) = \frac{8}{7}\sqrt{3} + \frac{4\sqrt{3k}}{3+k} + \frac{4\sqrt{3(k-3)}}{2+k} + \frac{8(n-3)\sqrt{k}}{4+k} + \frac{4(k-2)\sqrt{(k-1)(k-2)}}{2k-3} + \frac{1}{2}(nk^2+8n-5nk-8) + (n-2)\sqrt{k(k-2)}$;
8. $\chi(G) = \frac{2}{\sqrt{7}} + \frac{2}{\sqrt{3+k}} + \frac{2}{\sqrt{2+k}} + \frac{2(n-3)}{\sqrt{4+k}} + \frac{2(k-2)}{\sqrt{2k-3}} + \frac{1}{2}\frac{nk^2+6n-5nk}{\sqrt{2(k-2)}} + (n-2)\sqrt{\frac{1}{2}(k-2)} + \frac{1}{4}(n-4)\sqrt{2}$;
9. $AZI(G) = -\frac{162668}{3375} + \frac{54k^3}{(1+k)^3} + \frac{128(n-3)k^3}{(k+2)^3} + \frac{2(k-2)(k-1)^3(k-2)^3}{(2k-5)^3} + \frac{(nk^2-6n-5nk)(k-2)^6}{16(k-3)^3} + \frac{1}{8}(nk-2n-2k+4)k^3$.

Proof.

The graph G for $n = 4$ and $k = 7$ is shown in Fig. 4. By using Lemma 5, it is easy to see that the order of G is $nk - 1$ out of which 2 vertices are of degree 3, 2 vertices are of degree $k - 1$, $n - 3$ vertices are of degree 4, $n(k - 2)$ vertices are of degree $k - 2$, and $n - 2$ vertices are of degree k . Therefore by using Lemma 6, G has size $\frac{nk^2-3nk+8n-8}{2}$. We partition the size of G into edges of the type $E_{(d_u, d_v)}$ where uv is an edge. In G , we get edges of the type $E_{(3,4)}$, $E_{(3,k)}$, $E_{(3,k-1)}$, $E_{(4,4)}$, $E_{(4,k)}$, $E_{(k-1,k-2)}$, $E_{(k-2,k-2)}$, and $E_{(k,k-2)}$. The number of edges of these types are given in the Table 3. By using Formulas (1)-(9) and Table 3, we can obtain the required results.

$(d_u, d_v) : uv \in E(G)$	$(3, 4)$	$(3, k)$	$(3, k - 1)$	$(4, 4)$	$(4, k)$	$(k - 1, k - 2)$	$(k - 2, k - 2)$	$(k, k - 2)$
Number of edges	2	2	2	$n - 4$	$2n - 6$	$2(k - 2)$	$\frac{nk^2-5nk+6n}{2}$	$(n - 2)(k - 2)$

Table 3 The size partition of G .

Theorem 11. Let G be the line graph of the Firecracker graph $F_{n,k}$. Then

1. $M_1(\overline{G}) = 2nk^2 - 16n^2k + 19nk - 4k + 40n - 61$;
2. $\overline{M}_1(G) = \overline{M}_1(\overline{G}) = 8n^2k - 14nk + 4k - 24n + 44$;
3. $M_2(\overline{G}) = \frac{3}{2}nk^3 + 8n^2k^2 - 15nk^2 + 32n^2k - 56nk + 2k^2 + 14k - 160n + \frac{409}{2}$;
4. $\overline{M}_2(G) = 8n^2k^2 - 24n^2k - 12nk^2 + 2k^2 + 32n^2 + 28nk - 96n + 6k + 100$;
5. $\overline{M}_2(\overline{G}) = -8n^2k^2 - 24n^2k + 9nk^2 + 52nk - 2k^2 - 12k + 60n - 126$;
6. $W(G) \leq -8n^2k + \frac{1}{2}nk^2 + \frac{21}{2}nk - 2k + 20n - 31$;
7. $DD(G) \geq nk^3 - 5nk^2 + 16n^2k - 16nk + 2k - 40n + 60$.

Proof.

By using Theorems (1)-(4) and (10), we can obtain the required results.

Theorem 12. Let G be the line graph of the Firecracker graph $F_{n,k}$. Then

1. $ABC_4(G) = \frac{1}{2}(k-2)(k-3)(n-2)\sqrt{\frac{2k^2-8k+11}{(k^2-4k+6)(k^2-4k+7)}} + (n-4)(k-2)$
 $\sqrt{\frac{2k^2-8k+16}{(k^2-4k+12)(k^2-4k+6)}} + (k^2-3k+2)\sqrt{\frac{2}{k^2-4k+7}} + 2k-4\sqrt{\frac{2k^2-8k+15}{(k^2-4k+11)(k^2-4k+6)}} + 2\sqrt{\frac{k^2-2k+12}{2k^3-5k^2+10k+33}} + 2\sqrt{\frac{k^2-2k+16}{2k^3-k^2-6k+77}} +$
 $2\sqrt{\frac{k^2-2k+17}{2k^3-k^2-4k+84}} + 2\sqrt{\frac{k^2-2k+8}{2k^3-5k^2+2k+21}} + (n-5)\sqrt{2}\sqrt{\frac{k^2-2k+18}{(k^2-4k+12)(4+k)}} + 2\sqrt{\frac{4k+13}{4k^2+30k+56}} + 4\sqrt{\frac{k+2}{4k^2+20k+21}} + \frac{1}{\sqrt{2}}(n-6)\sqrt{\frac{2k+7}{k+4}};$
2. $GA_5 = 4\sqrt{\frac{(2k+3)(k^2-4k+7)}{k^2-2k+10}} + \frac{2\sqrt{(2k+3)(2k+7)}}{2k+5} + \frac{4\sqrt{(2k+3)(k^2-4k+11)}}{k^2-2k+14} + \frac{4\sqrt{(2k+7)(k^2-4k+11)}}{k^2-2k+18} + \frac{4\sqrt{(2k+7)(k^2-4k+12)}}{k^2-2k+19} + \frac{4\sqrt{(2k+7)(2k+8)}}{4k+15} +$
 $\frac{2(k-2)\sqrt{(k^2-4k+5)(k^2-4k+7)}}{k^2-4k+6} + \frac{(n-2)(k^2-5k+6)\sqrt{(k^2-4k+6)(k^2-4k+7)}}{2k^2-8k+13} + \frac{4(k-2)\sqrt{(k^2-4k+11)(k^2-4k+6)}}{2k^2-8k+17} + \frac{(n-4)(k-2)\sqrt{(k^2-4k+12)(k^2-4k+6)}}{k^2-4k+9} +$
 $\frac{4(n-5)\sqrt{(k^2-4k+12)(2k+8)}}{k^2-2k+20} + \frac{(k^2-5k+6)\sqrt{(k^2-4k+5)(k^2-4k+7)}}{k^2-4k+6} + n-6;$
3. $S(G) = \frac{2(2k+3)^3(k^2-4k+7)^3}{(k^2-2k+8)^3} + \frac{(2k+3)^3(2k+7)^3}{32(2+k)^3} + \frac{2(2k+3)^3(k^2-4k+11)^3}{(k^2-2k+12)^3} + \frac{2(2k+7)^3(k^2-4k+11)^3}{(k^2-2k+16)^3} + \frac{2(2k+7)^3(k^2-4k+12)^3}{(k^2-2k+17)^3} + \frac{2(2k+7)^3(2k+8)^3}{(4k+13)^3} +$
 $\frac{8(n-6)(4+k)^6}{(2k+7)^3} + \frac{1}{4}(k-2)(k^2-4k+7)^3 +$
 $\frac{(n-2)(k-2)(k-3)(k^2-4k+6)^3(k^2-4k+7)^3}{2(k^2-8k+11)^3} + \frac{2(k-2)(k^2-4k+11)^3(k^2-4k+6)^3}{(2k^2-8k+15)^3} + \frac{(n-4)(k-2)(k^2-4k+12)^3(k^2-4k+6)^3}{8(k^2-4k+8)^3} + \frac{16(n-5)(k^2-4k+12)^3(4+k)^3}{(k^2-2k+18)^3} +$
 $\frac{1}{8}(k-2)(k-3)(k^2-4k+7)^3.$

Proof.

We partition the size of G into edges of the type (S_u, S_v) where $uv \in E(G)$ as shown in Table 4. Hence we get the required results by using Table 4 and Formulas (10)–(12).

$(S_u, S_v) : uv \in E(G)$	$(2k+3, k^2-4k+7)$	$(2k+3, 2k+7)$	$(2k+3, k^2-4k+11)$	$(2k+7, k^2-4k+11)$
Number of edges	2	2	2	2
$(S_u, S_v) : uv \in E(G)$	$(2k+7, k^2-4k+12)$	$(2k+7, 2k+8)$	$(2k+8, 2k+8)$	(k^2-4k+5, k^2-4k+7)
Number of edges	2	2	$n-6$	$2(k-2)$
$(S_u, S_v) : uv \in E(G)$	(k^2-4k+6, k^2-4k+7)	$(k^2-4k+11, k^2-4k+6)$	$(k^2-4k+12, k^2-4k+6)$	$(k^2-4k+12, 2k+8)$
Number of edges	$\frac{(n-2)(k^2-5k+6)}{2}$	$2(k-2)$	$(n-4)(k-2)$	$2(n-5)$
$(S_u, S_v) : uv \in E(G)$	(k^2-4k+5, k^2-4k+7)			
Number of edges	k^2-5k+6			

Table 4 The size partition of G .

4 Conclusion

In this paper, certain degree based topological indices, i.e., first Zagreb index (coindex), second Zagreb index (coindex), third Zagreb index, first hyper-Zagreb index, atom-bond connectivity index, fourth atom-bond connectivity index, sum connectivity index, Randić connectivity index, augmented Zagreb index, Sanskruti index, geometric-arithmetic connectivity index and fifth geometric-arithmetic connectivity of the line graphs of Banana tree graph and Firecracker Graph is solved here analytically.

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