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Some new standard graphs labeled by 3−total edge product cordial labeling

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Abstract

In this paper, we study 3−total edge product cordial (3−TEPC) labeling which is a variant of edge product cordial labeling. We discuss Web, Helm, Ladder and Gear graphs in this context of 3−TEPC labeling. We also discuss 3−TEPC labeling of some particular examples with corona graph.

Keywords: 3-TEPC labeling; Web graph; Helm graph; Ladder graph and Gear graph
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1 Introduction and Preliminaries

Let G be finite, simple and undirected graph. Let V_G be the vertex set and E_G be the edge set of graph G. We follow the standard notations and terminology of graph theory as in [15]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition (s). If the domain of mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling) proved as in [6] .
To define some different types of cordial labelings h we have the following notations:
(1) The number of vertices labeled by x is $v_h(x)$;
(2) The number of edges labeled by x is $e_h(x)$;
(3) $v_h(x,y) = v_h(x) - v_h(y)$;
(4) $e_h(x,y) = e_h(x) - e_h(y)$;
(5) $s(x) = v_h(x) + e_h(x)$;
Now we will define different types of cordial labeling.

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Definition 1.

1. A vertex labeling function $h : V_G \to \{0,1\}$ can induce an edge labeling $h^* : E_G \to \{0,1\}$ as for each edge uv, $h^*(uv) = |h(u) - h(v)|$ if it satisfies $|v_h(1,0)| \leq 1$ and $|e_h(1,0)| \leq 1$ holds then it is called cordial. A graph G is cordial if it admits a cordial labeling as proved in [2].

2. A vertex labeling function $h : V_G \to \{0,1\}$ can induce an edge labeling $h^* : E_G \to \{0,1\}$ as for each edge uv, $h^*(uv) = h(u)h(v)$ if it satisfies $|v_h(1,0)| \leq 1$ and $|e_h(1,0)| \leq 1$ holds then it is called product cordial labeling. A graph G is product cordial if it admits a product cordial labeling as proved in [8].

3. For a graph G a function $h : V_G \to \{0,1,2,...,k-1\}$ such that k is an integer $2 \leq k \leq |E_G|$ gives label to each edge uv with $h(u)h(v)$. If $|s(x) - s(y)| \leq 1$ for $x,y \in \{0,1,...,k-1\}$ holds then it is called total product cordial(TPC) labeling. A graph G is TPC if it admits a TPC labeling as proved in [9].

4. A edge labeling is a function $h : E_G \to \{0,1\}$ can induce a vertex labeling $h^* : V_G \to \{0,1\}$ such that $h^*(v) = h(e_1)h(e_2)...h(e_n)$ for edge $e_1,e_2,...,e_n$ that are incident to v. If it satisfies $|v_h(1,0)| \leq 1$ and $|e_h(1,0)| \leq 1$ holds then it is called edge product cordial(EPC) labeling. A graph G is EPC if it admits a EPC labeling as proved in [10].

5. A edge labeling is a function $h : E_G \to \{0,1,2,...,k-1\}$ can induce a vertex labeling $h^* : V_G \to \{0,1,2,...,k-1\}$ such that $h^*(v) = h(e_1)h(e_2)...h(e_n)(mod k)$ for edge $e_1,e_2,...,e_n$ that are incident to v. If $|s(x)-s(y)| \leq 1$ for $x,y \in \{0,1,...,k-1\}$ holds then it is called k-edge product cordial(k-EPC) labeling. A graph G is k-EPC if it admits a k-EPC labeling as proved in [10].

6. For a graph G a function $h : E_G \to \{0,1,2,...,k-1\}$ such that k is an integer $2 \leq k \leq |E_G|$. For each vertex v assign the label by $h(e_1)h(e_2)...h(e_n)(mod k)$ such that $e_1,e_2,...,e_n$ are edges incident to v. If $|s(x) - s(y)| \leq 1$ for $x,y \in \{0,1,...,k-1\}$ holds then it is called k-total edge product cordial(k-TEPC) labeling. A graph G is k-TEPC if it admits a k-TEPC labeling as proved in [1, 11].

Now we will define different family of graphs.

Definition 2.

1. The web Wb_n is the graph obtained by joining the pendant vertices of a helm H_n to form a cycle and then adding a pendant edge to each of the vertices of outer cycle. An example of web graph Wb_4 are given in Figure 1.

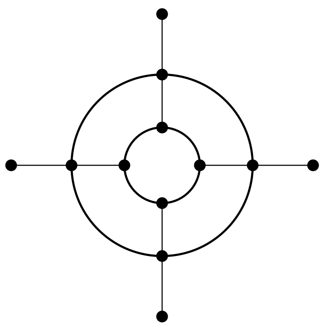

Fig. 1 Web graph Wb_4

2. The helm H_n is the graph obtained from a wheel W_n by attaching a pendent edge at each vertex of the n-cycle. An example of helm graph H_5 are given in Figure 2.

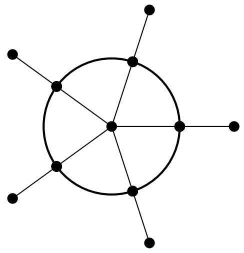

Fig. 2 Helm graph H_5

3. A gear graph G_n is obtained from the wheel graph W_n by adding a vertex between every pair of adjacent vertices of the n-cycle. An example of gear graph G_6 are given in Figure 3.

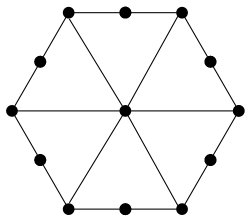

Fig. 3 Gear graph G_6

4. A ladder graph L_n of order n is a planer undirected graph with $2n$ vertices and $n+2(n-1)$ edges. An example of ladder graph L_6 are given in Figure 4.

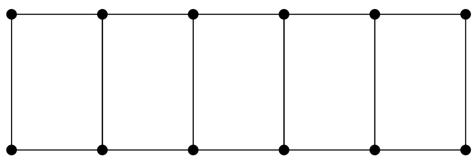

Fig. 4 Ladder graph L_6

Now we will define corona of two graphs.

Definition 3. The corona $G \odot H$ of two graphs G with n vertices and a graph H with n copies can be obtained by connecting i-th vertex of G to each vertex of i-th copy of H with an edge.

Azaizeh at al in [1] introduced the concept of 3-TEPC labeling and they proved many results on this newly concept. They discussed 3-TEPC labeling of Path, Circle and Star graphs. Madiha at el in [7] has discussed 3-TEPC labeling of Dutch Windmill and copies of n-cycle graphs. This paper is structured as follows. In Section 2, we discussed 3-total edge product cordial (3-TEPC) labeling of web graph and helm graph. 3-TEPC labeling of gear graph and a class of corona graph are discussed in Section 3.

2 3-TEPC labeling of Web graph and Helm graph

In this section we will study 3-TEPC labeling of web graph and helm graph.

2.1 3-TEPC labeling of Web graph

Theorem 1. *Let G be a Web graph Wb_n of $n+1$ vertices then G admits 3-TEPC labeling.*

Proof. : Let $V_G = \{u, v_x, w_x, 1 \le x \le n\}$ and $E_G = \{u_x v_x, 1 \le x \le n\} \cup \{v_x v_{x+1}, 1 \le x \le n-1\} \cup \{v_x w_x, 1 \le x \le n\} \cup \{u_x u_{x+1}, 1 \le x \le n-1\}$. We consider three cases as follows:

Case 1 Let $n \equiv 0 \pmod 3$ which implies $n = 3t$, for some integer $t \ge 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(u_x v_x) = \begin{cases} 0, & \text{if } 1 \le x \le t; \\ 1, & \text{if } t+1 \le x \le 3t. \end{cases}$$

$$h(v_x w_x) = \begin{cases} 1, & \text{if } 1 \le x \le t+1; \\ 2, & \text{if } t+2 \le x \le 3t. \end{cases}$$

$$h(u_x u_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le 2t; \\ 1, & \text{if } 2t+1 \le x \le 3t-1; \end{cases} \quad \text{and } h(u_{3t} u_1) = 2.$$

$$h(v_x v_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le t-1; \\ 2, & \text{if } t \le x \le 2t+1; \\ 1, & \text{if } 2t+2 \le x \le 3t-1; \end{cases} \quad \text{and } h(v_{3t} v_1) = 1.$$

In this case we have $s(0) = s(1) = s(2) = 7t$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 5.

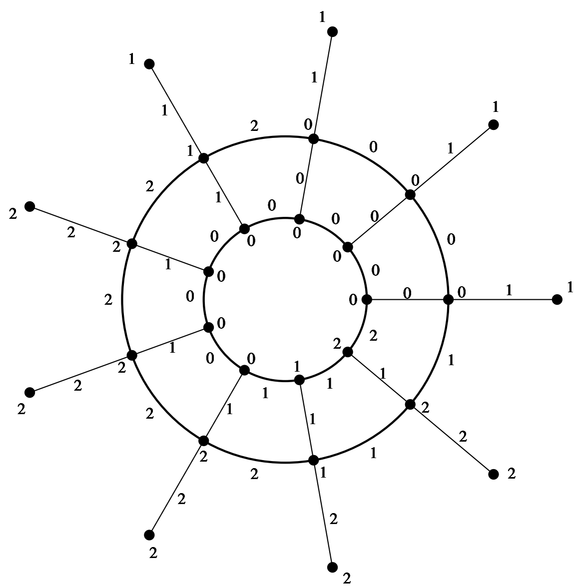

Fig. 5 3-TEPC labeling of W_9

Case 2 Let $n \equiv 1 \pmod 3$ which implies $n = 3t+1$, for some integer $t \ge 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(u_x v_x) = \begin{cases} 0, & \text{if } 1 \le x \le t-1; \\ 1, & \text{if } t \le x \le 2t; \\ 2, & \text{if } 2t+1 \le x \le 3t. \end{cases}$$

$$h(v_x w_x) = \begin{cases} 2, & \text{if } 1 \le x \le t; \\ 1, & \text{if } t+1 \le x \le 3t+1. \end{cases}$$

$$h(u_xu_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le 2t; \\ 2, & \text{if } 2t+1 \le x \le 3t; \end{cases} \text{ and } h(u_{3t+1}u_1) = 2.$$

$$h(v_xv_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le t+1; \\ 1, & \text{if } t+2 \le x \le 2t+1; \\ 2, & \text{if } 2t+2 \le x \le 3t; \end{cases} \text{ and } h(v_{3t+1}v_1) = 2.$$

In this case we have $s(0) = 7t+3$, $s(1) = s(2) = 7t$. Therefore $|s(x)-s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 6.

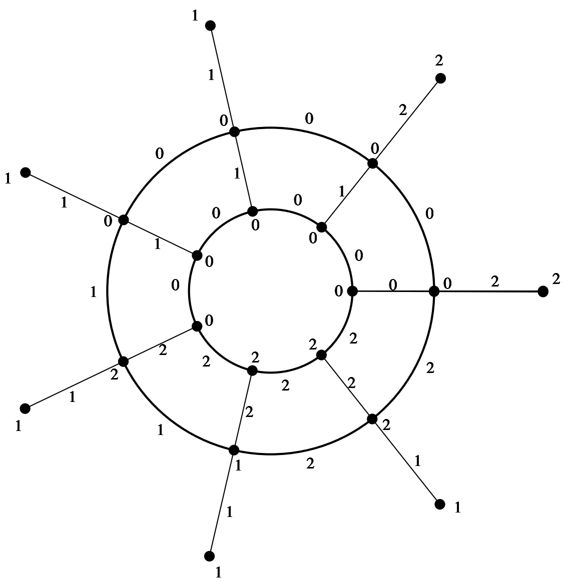

Fig. 6 3-TEPC labeling of W_7

Case 3 Let $n \equiv 2 \pmod 3$ which implies $n = 3t+2$, for some integer $t \ge 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(u_xv_x) = \begin{cases} 0, & \text{if } 1 \le x \le t; \\ 1, & \text{if } t+1 \le x \le 2t; \\ 2, & \text{if } 2t+1 \le x \le 3t+2. \end{cases}$$

$$h(v_xw_x) = \begin{cases} 2, & \text{if } 1 \le x \le t-1; \\ 1, & \text{if } t \le x \le 3t+2. \end{cases}$$

$$h(u_xu_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le 2t; \\ 2, & \text{if } 2t+1 \le x \le 3t+1; \end{cases} \text{ and } h(u_{3t+2}u_1) = 2.$$

$$h(v_xv_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le t+1; \\ 1, & \text{if } t+2 \le x \le 2t+1; \\ 2, & \text{if } 2t+2 \le x \le 3t+1; \end{cases} \text{ and } h(v_{3t+2}v_1) = 2.$$

In this case we have $s(0) = 7t+4$, $s(1) = s(2) = 7t+5$. Therefore $|s(x)-s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 7.

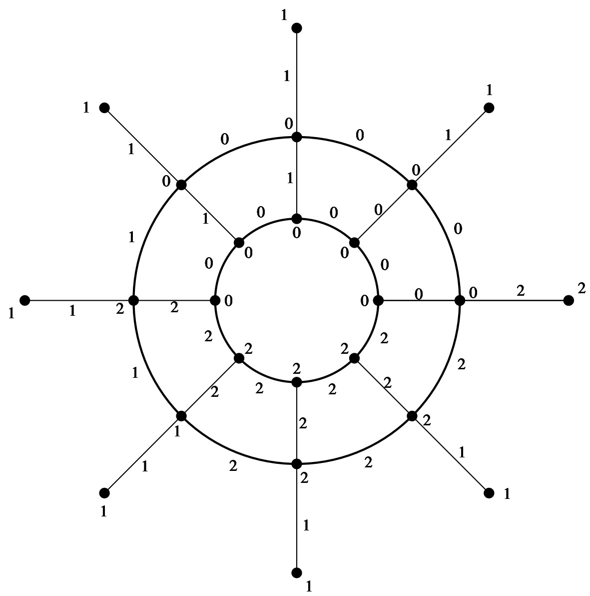

Fig. 7 3-TEPC labeling of W_8

2.2 3-TEPC labeling of Helm graph

Theorem 2. *Let G be a Helm graph H_n of $n+1$ vertices then G admits 3-TEPC labeling.*

Proof. Let $V_G = \{u, v_x, w_x, 1 \leq x \leq n\}$ and $E_G = \{uv_x, 1 \leq x \leq n\} \cup \{v_x v_{x+1}, 1 \leq x \leq n-1\} \cup \{v_x w_x, 1 \leq x \leq n\}$. We consider three cases as follows:

Case 1 Let $n \equiv 0 \pmod 3$ which implies $n = 3t$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as $h(uv_x) = 1$, for $1 \leq x \leq 3t$.

$$h(v_x v_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq t; \\ 1, & \text{if } t+1 \leq x \leq 3t-1; \end{cases} \text{ and } h(v_{3t} v_1) = 2.$$

$$h(v_x w_x) = \begin{cases} 2, & \text{if } 1 \leq x \leq 2t; \\ 0, & \text{if } 2t+1 \leq x \leq 3t. \end{cases}$$

In this case we have $s(0) = 5t+1$, $s(1) = s(2) = 5t$. Therefore $|s(x) - s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 8.

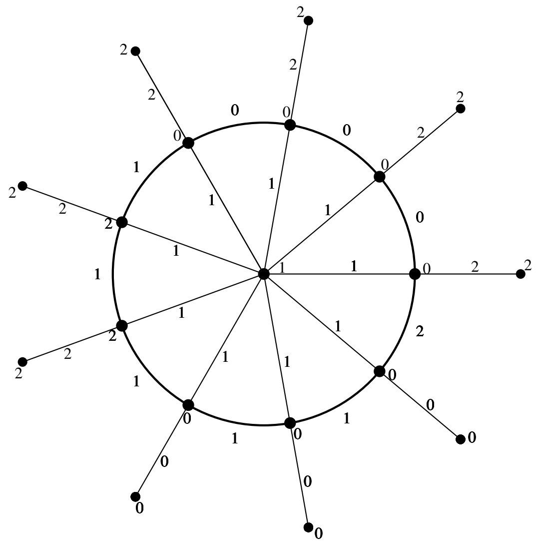

Fig. 8 3-TEPC labeling of H_9

Case 2 Let $n \equiv 1 \pmod 3$ which implies $n = 3t+1$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(uv_x) = \begin{cases} 0, & \text{if } 1 \leq x \leq t-1; \\ 1, & \text{if } t \leq x \leq 3t+1. \end{cases}$$

$$h(v_xw_x) = \begin{cases} 1, & \text{if } 1 \leq x \leq t-1; \\ 0, & \text{if } t \leq x \leq 2t; \\ 2, & \text{if } 2t+1 \leq x \leq 3t+1. \end{cases}$$

$$h(v_xv_{x+1}) = \begin{cases} 2, & \text{if } 1 \leq x \leq 2t-1; \\ 1, & \text{if } 2t \leq x \leq 3t; \end{cases} \text{ and } h(v_{3t+1}v_1) = 1.$$

In this case we have $s(0) = s(1) = s(2) = 5t+1$. Therefore $|s(x) - s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 9.

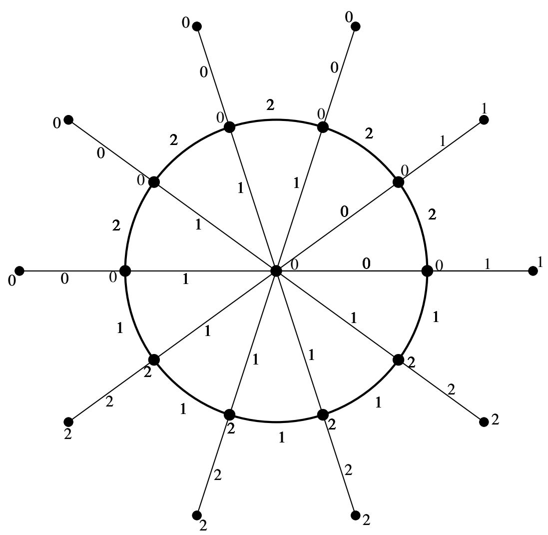

Fig. 9 3-TEPC labeling of H_{10}

Case 3 Let $n \equiv 2 \pmod 3$ which implies $n = 3t+2$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(uv_x) = \begin{cases} 0, & \text{if } 1 \leq x \leq t+1; \\ 1, & \text{if } t+2 \leq x \leq 3t+2. \end{cases}$$

$$h(v_xw_x) = \begin{cases} 1, & \text{if } 1 \leq x \leq t+1; \\ 0, & \text{if } t+2 \leq x \leq 2t+1; \\ 2, & \text{if } 2t+2 \leq x \leq 3t+2. \end{cases}$$

$$h(v_xv_{x+1}) = \begin{cases} 1, & \text{if } 1 \leq x \leq t+1; \\ 2, & \text{if } t+2 \leq x \leq 3t+1; \end{cases} \text{ and } h(v_{3t+2}v_1) = 2.$$

In this case we have $s(0) = 5t+3$, $s(1) = s(2) = 5t+4$. Therefore $|s(x) - s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 10.

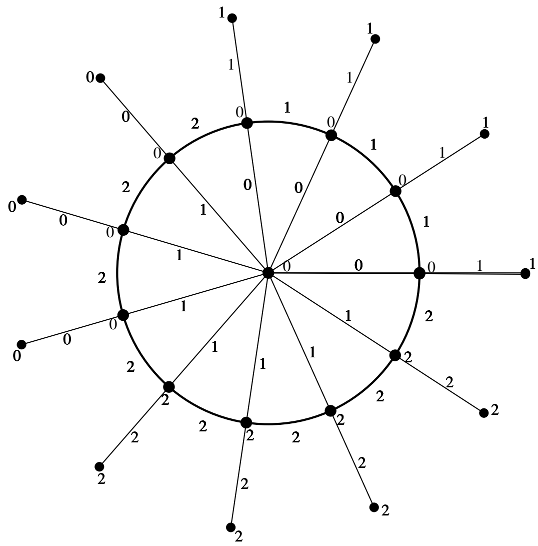

Fig. 10 3-TEPC labeling of H_{11}

3 3−TEPC Labeling of Gear graph and some classes of corona graph

In this section we will study 3-TEPC labeling of Gear graph and a class of corona graph.

3.1 3-TEPC Labeling of Gear graph

Theorem 3. *Let G be the Gear graph G_n, then G is 3-TEPC.*

Proof. Let $V_G = \{u, v_x, 1 \leq x \leq n\}$ and $E_G = \{uv_{2x-1}, 1 \leq x \leq n\} \cup \{v_xv_{x+1}, 1 \leq x \leq n-1\}$. We consider three cases as follows:

Case 1 Let $n \equiv 0 (\bmod 3)$ which implies $n = 3t$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(uv_{2x-1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq t-1; \\ 1, & \text{if } t \leq x \leq 2t; \\ 2, & \text{if } 2t+1 \leq x \leq 3t. \end{cases}$$

$$h(v_xv_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq 2t; \\ 1, & \text{if } 2t+1 \leq x \leq 3t+1; \\ 2, & \text{if } 3t+2 \leq x \leq 6t-1; \end{cases} \text{ and } h(v_{6t}v_1) = 2.$$

In this case we have $s(0) = 5t+1, s(1) = s(2) = 5t$. Therefore $|s(x)-s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 11.

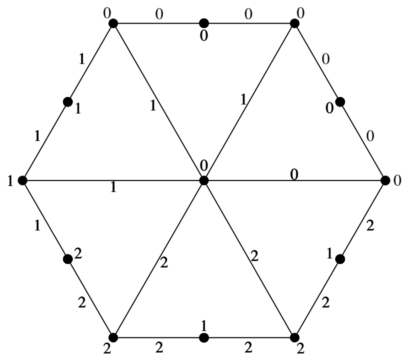

Fig. 11 3-TEPC labeling of G_6

Case 2 Let $n \equiv 1(\bmod 3)$ which implies $n = 3t+1$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(uv_{2x-1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq t; \\ 1, & \text{if } t+1 \leq x \leq 2t+1; \\ 2, & \text{if } 2t+2 \leq x \leq 3t+1. \end{cases}$$

$$h(v_xv_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq 2t; \\ 1, & \text{if } 2t+1 \leq x \leq 3t+1; \\ 2, & \text{if } 3t+2 \leq x \leq 6t+1; \end{cases} \text{ and } h(v_{6t+2}v_1) = 2.$$

So we have $s(0) = s(1) = s(2) = 5t+2$. Therefore $|s(x)-s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 12.

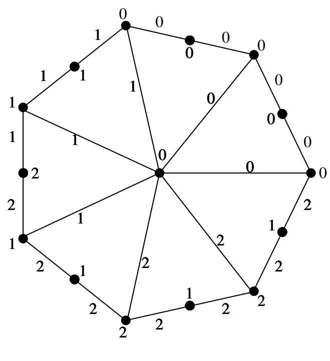

Fig. 12 3-TEPC labeling of G_7

Case 3 Let $n \equiv 2(\bmod 3)$ which implies $n = 3t+2$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$$h(uv_{2x-1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq t+1; \\ 1, & \text{if } t+2 \leq x \leq 2t+1; \\ 2, & \text{if } 2t+2 \leq x \leq 3t+2. \end{cases}$$

$$h(v_xv_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le 2t; \\ 1, & \text{if } 2t+1 \le x \le 3t+3; \\ 2, & \text{if } 3t+4 \le x \le 6t+3; \end{cases} \text{ and } h(v_{6t+4}v_1) = 2.$$

In this case we have $s(0) = 5t+3, s(1) = s(2) = 5t+4$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence h is a 3-TEPC labeling as elaborated in Figure 13.

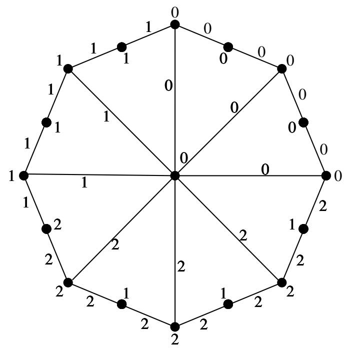

Fig. 13 3-TEPC labeling of G_8

3.2 3-TEPC of Corona $L_n \odot 2K_1$

Theorem 4. *Let G be a corona of $L_n \odot 2K_1$ then G admits 3-TEPC labeling.*

Proof. Let $V_G = \{u_x, v_x, w_x, w'_x, 1 \le x \le n\}$ and $E_G = \{u_xv_x, 1 \le x \le n\} \cup \{v_xv_{x+1}, 1 \le x \le n-1\} \cup \{u_xu_{x+1}, 1 \le x \le n-1\} \cup \{u_xw_x, 1 \le x \le n\} \cup \{v_xw'_x, 1 \le x \le n\}$. we consider three cases as follows:

Case 1 Let $n \equiv 0(\bmod 3)$ then $n = 3t$, for some integer $t \ge 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as $h(v_xk_1) = 2$, for $1 \le x \le 3t$.

$$h(u_xu_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le 2t; \\ 1, & \text{if } 2t \le x \le 3t-1. \end{cases}$$

$$h(v_xv_{x+1}) = \begin{cases} 0, & \text{if } 1 \le x \le 2t; \\ 1, & \text{if } 2t+1 \le x \le 3t-1. \end{cases}$$

$$h(u_xk_2) = \begin{cases} 2, & \text{if } 1 \le x \le t; \\ 1, & \text{if } t+1 \le x \le 3t. \end{cases}$$

$$h(u_xv_x) = \begin{cases} 0, & \text{if } 1 \le x \le t-2; \\ 1, & \text{if } t-1 \le x \le 3t. \end{cases}$$

In this case we have $s(0) = 9t$, $s(1) = s(2) = 9t-1$. Therefore $|s(x) - s(y)| \le 1$ for $0 \le x < y \le 2$. Hence g is a 3-TEPC labeling as elaborated in Figure 14.

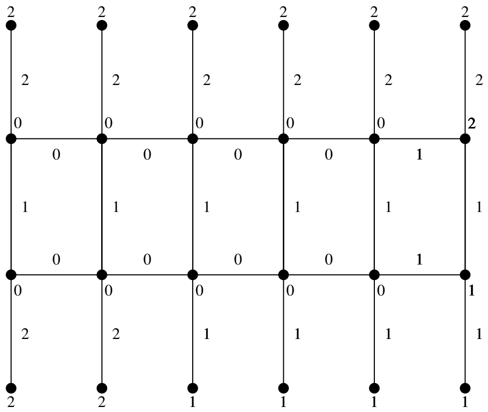

Fig. 14 3-TEPC labeling of graph $L_6 \odot 2K_1$

Case 2 Let $n \equiv 1(\bmod 3)$ then $n = 3t+1$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$h(v_x k_1) = 2$, for $1 \leq x \leq 3t+1$.

$$h(u_x u_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq 2t+1; \\ 1, & \text{if } 2t+2 \leq x \leq 3t. \end{cases}$$

$$h(v_x v_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq 2t; \\ 1, & \text{if } 2t+1 \leq x \leq 3t. \end{cases}$$

$$h(u_x k_2) = \begin{cases} 2, & \text{if } 1 \leq x \leq t; \\ 1, & \text{if } t+1 \leq x \leq 3t+1. \end{cases}$$

$$h(u_x v_x) = \begin{cases} 0, & \text{if } 1 \leq x \leq t-1; \\ 1, & \text{if } t \leq x \leq 3t+1. \end{cases}$$

In this case we have $s(0) = 9t+3$, $s(1) = s(2) = 9t+2$. Therefore $|s(x) - s(y)| \leq 1$ for $0 \leq x < y \leq 2$. Hence g is a 3-TEPC labeling as elaborated in Figure 15.

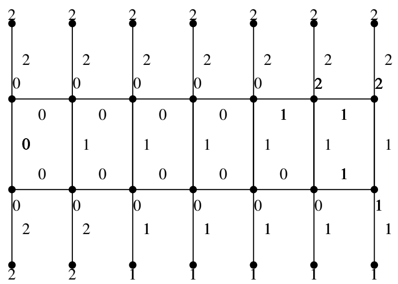

Fig. 15 3-TEPC labeling of graph $L_7 \odot 2K_1$

Case 3 Let $n \equiv 2(\bmod 3)$ then $n = 3t+2$, for some integer $t \geq 1$. We define the edge labeling $h : E_G \to \{0,1,2\}$ as

$h(v_x k_1) = 2$, for $1 \leq x \leq 3t+2$.

$$h(u_x u_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq 2t+2; \\ 1, & \text{if } 2t+3 \leq x \leq 3t+1. \end{cases}$$

$$h(v_x v_{x+1}) = \begin{cases} 0, & \text{if } 1 \leq x \leq 2t; \\ 1, & \text{if } 2t+1 \leq x \leq 3t+1. \end{cases}$$

$$h(u_xk_2)=\begin{cases}2, & \text{if } 1\le x\le t;\\ 1, & \text{if } t+1\le x\le 3t+2.\end{cases}$$
$$h(u_xv_x)=\begin{cases}0, & \text{if } 1\le x\le t;\\ 1, & \text{if } t+1\le x\le 3t+2.\end{cases}$$
In this case we have $s(0)=9t+6$, $s(1)=s(2)=9t+5$. Therefore $|s(x)-s(y)|\le 1$ for $0\le x<y\le 2$. Hence g is a 3-TEPC labeling as elaborated in Figure 16.

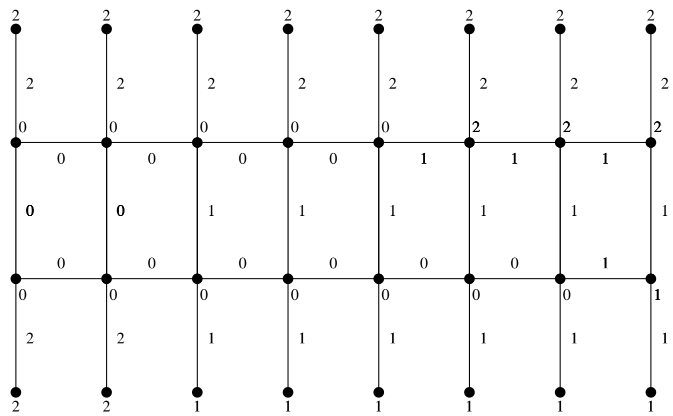

Fig. 16 3-TEPC labeling of graph $L_8\odot 2K_1$

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