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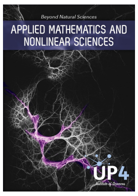

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The effect of two-temperature on thermoelastic medium with diffusion due to three phase-lag model

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Abstract

This paper is concerned on the distribution of a homogeneous isotropic elastic medium with diffusion under the effect of Three-phase-lag model. Normal mode analysis is used to express the exact expressions for temperature, displacements and stresses functions. Comparisons are made in the absence and presence of diffusion with some theories like Three-phase-lag and GNIII.

Keywords: Generalized thermoelastic diffusion, three-phase-lag model, GNIII
AMS 2010 codes: 93C20.

1 Introduction

Thermoelasticity describes the behavior of elastic bodies under the influence of non-uniform temperature fields. It represents, therefore, a generalization of the theory of elasticity. The constitutive equations, i.e., the equations characterizing the particular material, are the temperature dependent and include an additional relation connecting the heat flux in the body with the local temperature gradient. This relation was known in its simplest form as Fourier's law.

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Biot [1] develop the coupled theory of thermo-elasticity to deal with defeat of the uncoupled theory that mechanical cause has no effect on the temperature field. The theory of generalized thermoelasticity with one relaxation time was introduced by Lord and Shulman [2] which in their replace Fourier's law with Maxwell- Cattaneo law of heat conduction. Muller [3] first introduced the theory of generalized thermo-elasticity with two relaxation times. Later Green and Naghdi [4], [5] and [6] have proposed three models, labeled as types I, II, and type III. When they are linearized, type I is the same as the classical heat equation whereas the linearized versions of type-II (thermo-elasticity without energy dissipation) and type-III (thermoelasticity with energy dissipation) theories permit the propagation of thermal waves at finite speed.

Tzou [7] introduced two-phase-lag models to both the heat flux vector and the temperature gradient. According to this model, classical Fourier's law$\vec{q} = -K\nabla\vec{\theta}$ has been replaced by $\vec{q}(p,t+\tau_q) = -K\vec{\nabla}\theta(p,t+\tau_T)$, where the temperature gradient$\vec{\nabla}\theta$ at a point P of a material at time $t+\tau_T$ corresponds to the heat flux vector $\vec{q}$ at the same point in time $t+\tau_q$. Here K is the thermal conductivity of the microstructural interactions and is called the phase-lag of the temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of thermal inertial and is called the phase-lag of the heat flux. The case $\tau_q = \tau_T = 0$ corresponds to classical Fourier's law. If $\tau_q = \tau$ and $\tau_T = 0$, Tzou refers to the model as single-phase-lag model. Recently the Three-phase-lag, heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phases-lags for the heat flux vector, the temperature gradient and the thermal displacement gradient [8]. The stability of the three-phase-lag, the heat conduction is discussed [9]. M. I. Othman and S. M. Said [10] have studied the 2D problem of magneto-thermoelasticity fiber-reinforced medium under temperature dependent properties with three-phase-lag model.

Youssef [11] developed a new theory of generalized thermoelasticity by taking into account the theory of heat conduction in deformable bodies, which depends on two distinct temperatures, the conductive temperatureϕ and the thermodynamic temperature$T$. The two-temperature theory involves a material parameter $b > 0$. The limit $b \to 0$ implies that $T \to \phi$ and hence the classical theory can be recovered from two temperature theory. Sarhan Y. Atwa [12] has studied the generalized magneto-thermoelasticity with two temperature and initial stress under Green-Naghdi theory.

Diffusion is the process by which atoms move in a material. Atoms are able to move throughout solids because they are not stationary, but execute rapid, small-amplitude vibrations about their equilibrium positions, such vibrations increase with temperature and at any temperature a very small fraction of the atoms has sufficient amplitude to move from one atomic position to an adjacent one. Diffusion can be defined as the mass flow process in which atoms change their positions relative to neighbors in a given phase under the influence of thermal and a gradient, or stress gradient.

Thermodiffusion in the solids is one of a transport process that has great practical importance. Most of research associated with the presence of concentration and temperature gradients has been made with metals and alloys. The first critical review was published in the work of Oriani [13]. Sherif, et al. [14] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Othman et al. [15] have studied the effect of diffusion on the two-dimensional problem of generalized thermoelasticity with Green-Naghdi theory. Othman et al. [16] have showed the effect of fractional parameter on plane waves of generalized magneto-thermelastic diffusion with reference temperature-dependent elastic medium. Sarhan Y. Atwa [17]has studied the effect of fractional parameter on plane waves of generalized thermoelastic diffusion with temperature-dependent elastic medium.

2 Basic equations

The governing equations for an isotropic, homogeneous elastic solid with generalized thermoelastic diffusion in the absence of body forces using three-phase-lag model are:

σ_{ij}	Components of stress tensor	e_{ij}	Components of strain tensor
$e = e_{kk}$	Cubic dilatation	δ_{ij}	Kronecker's delta
u,v	Displacement vectors	T	Thermodynamic Temperature
T_o	Reference Temperature	ϕ	Conductive temperature
P	Chemical potential	C	Concentration distribution
λ, μ	Lame's constants	γ	$(3\lambda + 2\mu)\alpha_t$
α_t	Coefficient of linear thermal expansion	β_1	$(3\lambda + 2\mu)\alpha_c$
α_c	Coefficient of diffusion thermal expansion	ρ	Density
C_E	Specific heat at constant strain	K	Coefficient of thermal conductivity
K^*	Material characteristic of the theory	τ_v	Phase lag of thermal displacement gradient
τ_q	Phase lag of Heat flux	τ_T	Phase lag of temperature gradient
d	Thermoplastic diffusion constant	τ	Diffusion relaxation time
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$		

Table 1 Nomenclature

The constitutive equations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e - \gamma(T - T_o) - \beta_1 C] \tag{1}$$

$$P = -\beta_1 e + b_1 C - a(T - T_o) \tag{2}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

The equation of motion

$$\rho \ddot{u}_i = 2\mu e_{ij,j} + [\lambda e_{,j} - \gamma T_{,j} - \beta_1 C_{,j}]\delta_{ij} \tag{4}$$

The equation of mass diffusion

$$d\beta_1 \nabla^2 e + da\nabla^2 T + \dot{C} + \tau\ddot{C} - db\nabla^2 C = 0 \tag{5}$$

The equation of heat conduction

$$K^*\nabla^2\phi + \tau_v^*\nabla^2\dot{\phi} + K\tau_t\nabla^2\ddot{\phi} = (1 + \tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}[\rho C_E\ddot{T} + \gamma T_o\ddot{e} + aT_o\ddot{C}] \tag{6}$$

Where,

$$\tau_v^* = K + K^*\tau_v$$

The equation of two temperature

$$T = (1 - b\nabla^2)\phi \tag{7}$$

Where, the list of symbols is given in the nomenclature.

3 Formulation of the problem

We consider an isotropic, homogeneous elastic solid with generalized thermo-elastic diffusion. All quantities are considered as functions of the time variable t and of the coordinates x and y . We consider the normal source acting on the plane surface of generalized thermoelastic half-space under the effect of two temperature, we assume

$$\vec{u} = (u, v, 0) \tag{8}$$

The equation of motion in the absence of body force

$$\rho \ddot{u} = \mu \nabla^2 u + (\lambda + \mu) e_{,x} - \gamma(1 - b\nabla^2)\phi_{,x} - \beta_1 C_{,x} \tag{9}$$

$$\rho \ddot{v} = \mu \nabla^2 v + (\lambda + \mu) e_{,y} - \gamma(1 - b\nabla^2)\phi_{,y} - \beta_1 C_{,y} \tag{10}$$

To facilitate the solution, the following dimensionless quantities are introduced

$$(\acute{x}, \acute{y}) = c_1 \eta (x, y), (\acute{u}, \acute{v}) = c_1 \eta (u, v), (\acute{t}, \acute{\tau}, \acute{\tau_T}, \acute{\tau_q}, \acute{\tau_v}) = c_1^2 \eta (t, \tau, \tau_T, \tau_q, \tau_v), \acute{P} = \frac{P}{\beta_1}, \acute{C} = \frac{\beta_1}{\lambda + 2\mu} C$$

$$(\acute{T}, \acute{\phi}) = \frac{\gamma}{\lambda + 2\mu}(T, \phi), \acute{\sigma_i} j = \frac{\sigma_i j}{\lambda + 2\mu}, \acute{b} = c_1^2 \eta^2 b, \eta = \frac{\rho C_E}{K} and c_1^2 = \frac{\lambda + 2\mu}{\rho}. \tag{11}$$

The displacement components u (x,y,t) and v (x,y,t) may be written in terms of the potential functions $\phi(x,y,t)$ and $\psi(x,y,t)$ as

$$u = q_{,x} - \psi_{,y}, v = q_{,y} + \psi_{,x} \tag{12}$$

Using Eqs. (11) and (12), the governing equations (9), (10) and Eqs. (5), (6) recast in the following form (after suppressing the primes)

$$\ddot{q} = \nabla^2 q - (1 - b\nabla^2)\phi - C \tag{13}$$

$$\ddot{\psi} = a_1 \nabla^2 \psi \tag{14}$$

$$\nabla^4 q + a_3(1 - b\nabla^2)\nabla^2 \phi + a_4 \dot{C} + \tau a_4 \ddot{C} - a_5 \nabla^2 \ddot{C} = 0 \tag{15}$$

$$a_6 \nabla^2 \phi + (1 + a_6 \tau_v)\nabla^2 \dot{\phi} + \tau_T \nabla^2 \ddot{\phi} = (1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2})[(1 - b\nabla^2)\ddot{\phi} + a_7 \ddot{e} + a_8 \ddot{C}] \tag{16}$$

where,

$$a_1 = \frac{\mu}{\rho c_1^2}, a_2 = \frac{\lambda + \mu}{\rho c_1^2}, a_3 = \frac{(\lambda + 2\mu)a}{\beta_1 \gamma}, a_4 = \frac{\lambda + 2\mu}{d\beta_1^2 \eta}, a_5 = \frac{b(\lambda + 2\mu)}{c_1^2 \beta_1^2 \eta^2}, a_6 = \frac{K^*}{\rho C_E c_1^2}, a_7 = \frac{\gamma^2 T_o}{\eta(\lambda + 2\mu)K},$$

$$and a_8 = \frac{a\gamma T_o}{\beta_1 \eta K} \tag{17}$$

4 Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal modes in the following form

$$[u, v, \phi, \psi, q, T, C](x, y, t) = [u^*, v^*, \phi^*, \psi^*, q^*, T^*, C^*](y) e^{i(\omega t + kx)} \tag{18}$$

Where ω is the complex time constant (frequency), i is the imaginary unit, k is the wave number in the x-direction and $u^*, v^*, \phi^*, \psi^*, q^*, T^*, C^*$ are the amplitudes of the functions $u, v, \phi, \psi, q, T, C$.
Using equation (18), equations (13)-(16) become respectively

$$(D^2 - a_9)q^* + (bD^2 - a_{10})\phi^* - C^* = 0 \tag{19}$$

$$(a_1 D^2 - a_{11})\psi^* = 0 \tag{20}$$

$$(k^4 - 2k^2 D^2 + D^4)q^* - (a_3 b D^4 - a_{12} D^2 + a_{13})\phi^* + (a_{14} - a_5 D^2)C^* = 0 \tag{21}$$

$$(a_{16} D^2 - a_{17})q^* + (a_{18} D^2 - a_{19})\phi^* + a_{20} C^* = 0 \tag{22}$$

Where

$$a_9 = k^2 - \omega^2, a_{10} = 1 + bk^2, a_{11} = a_1k^2 - \omega^2, a_{12} = a_3(1 + 2bk^2), a_{13} = a_3k^2(1 + bk^2), a_{14} = a_4(i\omega - \tau\omega^2) + a_5k^2,$$

$$a_{15} = 1 + i\omega\tau_q - \frac{1}{2}\tau_q^2\omega^2, a_{16} = \omega^2 a_7 a_{15}, a_{17} = \omega^2 k^2 a_7 a_{15}, a_{18} = a_6 + i\omega(1 + a_6\tau_v) - \tau_T\omega^2 - \omega^2 b a_{15},$$

$$a_{19} = k^2(a_6 + i\omega(1 + a_6\tau_v)) - \omega^2\tau_T k^2 - a_{15}\omega^2(1 + bk^2), a_{20} = a_8 a_{15}\omega^2 and D = \frac{d}{dy}. \tag{23}$$

Eq. (20) is uncoupled differential equation while the others are coupled.
Eliminating Eqs. (19), (21) and (22) we get the following ordinary differential equations

$$(D^6 - AD^4 + BD^2 - C)\phi^*(y) = 0 \tag{24}$$

Where

$$A_1 = 1 - a_5, A_2 = a_{14} + a_5 a_9 - 2k^2, A_3 = k^4 - a_9 a_{14}, A_4 = (a_5 + a_3)b, A_5 = ba_{14} + a_5 a_{10} - a_{12}, A_6 = a_{10}a_{14} + a_{13},$$

$$A_7 = a_{20} + a_{16}, A_8 = a_9 a_{20} + a_{17}, A_9 = ba_{20} + a_{18}, A_{10} = a_{10}a_{20} + a_{19}, A_{11} = A_4 A_7 + A_1 A_9,$$

$$A_{12} = A_5 A_7 + A_4 A_8 - A_2 A_9 - A_1 A_{10}, A_{13} = A_6 A_7 + A_5 A_8 + A_3 A_9 + A_2 A_{10}, A_{14} = A_6 A_8 - A_3 A_{10},$$

$$A = \frac{A_{12}}{A_{11}}, B = \frac{A_{13}}{A_{11}}, C = \frac{A_{14}}{A_{11}} \tag{25}$$

similarly,

$$(D^6 - AD^4 + BD^2 - C)\{\phi^*(y), q^*(y), C^*(y)\} = 0 \tag{26}$$

Eq.(26) can be factorized as

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)\{\phi^*(y), q^*(y), C^*(y)\} = 0 \tag{27}$$

Where k_n^2, (n=1, 2, 3) are the roots of characteristic equation

$$k^6 - Ak^4 + Bk^2 - C = 0 \tag{28}$$

The solution of Eq. (27) bound as $y \to \infty$, is given by

$$\phi^*(y) = \sum_{n=1}^{3} M_n e^{-k_n y}, n = 1, 2, 3 \tag{29}$$

$$q^*(y) = \sum_{n=1}^{3} H_{1n} M_n e^{-k_n y}, n = 1, 2, 3 \tag{30}$$

$$C^*(y) = \sum_{n=1}^{3} H_{2n} M_n e^{-k_n y}, n = 1, 2, 3 \tag{31}$$

Where,

$$H_{1n} = \frac{-(a_{20} + a_{18})k_n^2 + a_{10}a_{20} + a_{19}}{(a_{20} + a_{16})k_n^2 - a_9 a_{20} - a_{17}} \tag{32}$$

$$H_{2n} = (k_n^2 - a_9)H_{1n} + bk_n^2 - a_{10}, n = 1, 2, 3. \tag{33}$$

The solution of Eq. (20)

$$\psi^*(y) = M_4(k, \omega)e^{-k_4 y}. \tag{34}$$

Where

$$k_4 = \sqrt{\frac{a_{11}}{a_1}} \tag{35}$$

In order to obtain the displacement components u,v using Eq. (18), Eq. (12) can be

$$u^* = ikq^* - D\psi^* \tag{36}$$

$$v^* = Dq^* - ik\psi^* \tag{37}$$

Substitution from Eqs. (29), (30),and (36) into Eq. (1)

$$\sigma^*_{xx} = \sum_{n=1}^{3} H_{3n} M_n e^{-k_n y} + H_1 M_4 e^{-k_4 y} \tag{38}$$

$$\sigma^*_{yy} = \sum_{n=1}^{3} H_{4n} M_n e^{-k_n y} + H_1 M_4 e^{-k_4 y} \tag{39}$$

$$\sigma^*_{xy} = \sum_{n=1}^{3} H_{5n} M_n e^{-k_n y} + H_2 M_4 e^{-k_4 y} \tag{40}$$

Where

$$H_{3n} = ((a_2 - a_1)k_n^2)H_{1n} + bk_n^2 - bk^2 - H_{2n} - 1, n = 1,2,3. \tag{41}$$

$$H_{4n} = (k_n^2 - k^2(a_2 - a_1))H_{1n} + bk_n^2 - bk^2 - H_{2n} - 1, n = 1,2,3. \tag{42}$$

$$H_{5n} = -2ia_1 k k_n H_{1n}, n = 1,2,3. \tag{43}$$

$$H_1 = ikk_4(a_{22} - a_1 + 1). \tag{44}$$

$$H_2 = (k^2 - k_4^2)a_1. \tag{45}$$

5 Applications

In this section, the general solutions for displacement, stresses, temperature field, concentration distribution and chemical potential, will be used to yield the response of a half-space subject to a fixed load acting with uniform constant p(x). The final solution in the original domain (x,y,t) is obtained numerically.
The boundary conditions on the plane surface y=0 are

$$\sigma_{xx} = pe^{i(\omega t + kx)}, \frac{\partial \phi}{\partial y} = 0, \frac{\partial C}{\partial y} = 0, \sigma_{xy} = 0, at y = 0. \tag{46}$$

Where p is a function in x and t. Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parrameters

$$\sum_{n=1}^{3} H_{3n} M_n + H_1 M_4 = p \tag{47}$$

$$\sum_{n=1}^{3} M_n k_n = 0 \tag{48}$$

$$\sum_{n=1}^{3} H_{2n} M_n k_n = 0 \tag{49}$$

$$\sum_{n=1}^{3} H_{5n} M_n + H_2 M_4 = 0 \tag{50}$$

Solving Eqs. (47)- (50), we get the parameters of M_n(n=1, 2, 3)

6 Particular case

Neglecting diffusion effect, by taking $C = a = b_1 = \beta_1 = 0$, we obtain the expressions for displacement components, stresses and temperature field in the generalized thermoelastic medium are obtained as

$$(D^2 - a_9)q^* + (bD^2 - a_{10})\phi^{=}0 \tag{51}$$

$$(a_{16}D^2 - a_{17})q^* + (a_{18}D^2 - a_{19})\phi^{=}0 \tag{52}$$

Eliminating $\phi^*(y)$ and $q^*(y)$ and between Eqs. (51) and (52) we get the following ordinary differential equation satisfied by

$$(D^4 - ED^2 + F)\{\phi^*(y), q^*(y)\} = 0 \tag{53}$$

Eq. (53) can be factorized as

$$(D^2 - s_1^2)(D^2 - s_2^2)\phi^*(y) = 0 \tag{54}$$

Where,$s_n^2(n = 1,2)$ are the roots of the following characteristic equation.

$$s^4 - Es^2 + F = 0 \tag{55}$$

Where

$$E = \frac{-ba_{17} - a_{10}a_{16} + a_9a_{18} + a_{19}}{a_{18} - ba_{16}} \tag{56}$$

$$F = \frac{a_{19}a_9 - a_{10}a_{17}}{a_{18} - ba_{16}} \tag{57}$$

The solution of Eq. (55) is given by

$$\phi^*(y) = \sum_{n=1}^{2} G_n(k,\omega)e^{-s_ny} \tag{58}$$

Similarly,

$$q^*(y) = \sum_{n=1}^{2} w_{1n}G_n(k,\omega)e^{-s_ny} \tag{59}$$

Where,

$$w_{1n} = \frac{a_{10} - bs_n^2}{s_n^2 - a_9} \tag{60}$$

$$\psi^* = G_3e^{-s_3y}, where s_3 = \sqrt{\frac{a_{11}}{a_1}} \tag{61}$$

$$\sigma_{xx}^*(y) = \sum_{n=1}^{2} w_{2n}G_n(k,\omega)e^{-s_ny} + w_1G_3e^{-s_3y} \tag{62}$$

$$\sigma_{yy}^*(y) = \sum_{n=1}^{2} w_{3n}G_n(k,\omega)e^{-s_ny} + w_1G_3e^{-s_3y} \tag{63}$$

Where,

$$w_{2n} = (-k^2 + (a_2 - a_1)s_n^2)w_{1n} + b(s_n^2 - k^2) - 1, n = 1,2. \tag{64}$$

$$w_{3n} = (s_n^2 - k^2(a_2 - a_1))w_{1n} + b(s_n^2 - k^2) - 1, n = 1,2. \tag{65}$$

$$w_{4n} = -2ia_1ks_nw_{1n}, n = 1,2. \tag{66}$$

$$w_1 = iks_3(1 + a_2 - a_1) \tag{67}$$

$$w_2 = a_1(k^2 - s_3^2) \tag{68}$$

In this case the boundary conditions are

$$\sigma_{xx} = pe^{i(\omega t+kx)}, \frac{\partial \phi}{\partial y} = 0, \sigma_{xy} = 0 at y = 0 \tag{69}$$

Substituting from the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters

$$\sum_{n=1}^{2} w_{2n} G_n + w_1 G_3 = p \tag{70}$$

$$\sum_{n=1}^{2} G_n s_n = 0 \tag{71}$$

$$\sum_{n=1}^{2} w_{4n} G_n + w_2 G_3 = p \tag{72}$$

7 Numerical results

With an aim to illustrate the problem, we will present some numerical results using copper as the thermoelastic material for which we take the following values of the different physical constants.

$$T_o = 293K, C_E = 383.1J/(kg.K), \alpha_t = 1.78\times 10^{-5}K^{-1}, \rho = 8954kg/m^3, \lambda = 7.76\times 10^{10}kg/(m.s^2),$$

$$\mu = 3.86\times 10^{10}kg/(m.s^2), \alpha_c = 1.98\times 10^{-4}m^3/kg, K = 300W, d = 0.85\times 10^{-8}kg.s/m^3,$$

$$a = 1.2\times 10^4 m^2/(s^2.K), b_1 = 0.9\times 10^6 m^5/(kg.s^2), \omega_o = 2.5, \xi = 0.4, \omega = \omega_o + i\xi,$$

$$x = 0.5, t = 0.2, K^* = 2.97\times 10^{13}, k = 2.4, p = 0.1, b = 0.2,$$

$$\tau_v = 0.2, \tau_T = 0.5, \tau_q = 0.8.$$

The numerical result was used to show the distribution of the displacement component u the stress components σ_{yy}, σ_{xy} the temperature distribution ϕ,the chemical potential P , and the concentration distribution C on the vertical axis with y at x=0.5 , for two theories: Three phase-lag (TPL) model and (G-N III) theory with and without diffusion (W & WD respectively). All curves reaching to zero with the increasing of y . The diffusion effect is appearing on the curves of u ,σ_{yy} and σ_{xy} as in the case of diffusion the curves of the two theories are somewhere far from each and u only start from different values other while in case of absence of diffusion are close from each other, while the effect on ϕ curve is appearing on the starting values of curves only as in presence of diffusion the two theories start from negative value while in absence of it (TPL) curve start from negative, while (G-N III) start from positive.

Fig. 1 shows that the distribution of u with the two theories in the presence and absence of diffusion starts from positive values. In the presence of diffusion u increases to a maximum value in the range 0<y<0.4 and then decreases to a minimum value in the range 0.5<y<1.4,while in the absence of diffusion u increases to a maximum value in the range 0<y<0.6 and then decreases to a minimum value in the range 0.9<y<1.4.

Fig. 2 shows the distribution of the temperature ϕ with the two theories in the presence and absence of diffusion start from the negative value except the curve belongs to (G-N III) theory without diffusion G-N (WD). The curves belong to TPL theory (W & WD) and G-N (WD) are smooth curves and they fastly reach to zero, while the curve belongs to G-N (D) acts as a wave has a minimum value in the range 0<y<0.9 and a maximum value in the range 0.9<y<2 and then become to decreasing wave until vanishing.

Fig. 3 shows that the stress distribution σ_{yy} with the two theories in the presence and absence of diffusion

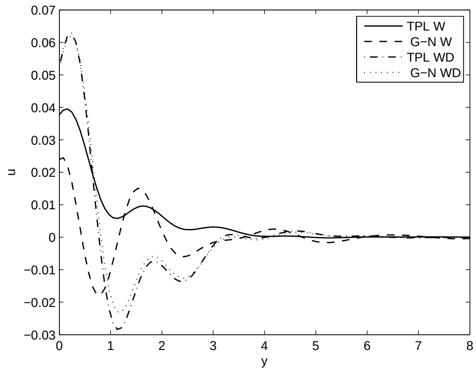

Fig. 1 Distribution the displacement component u with and without diffusion.

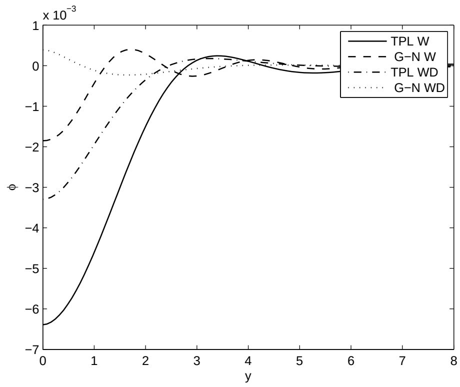

Fig. 2 Distribution the temperature function ϕ with and without diffusion.

starts from a positive value. In the case of diffusion presence they firstly decrease to a minimum value in the range 0.2<y<1, and then it increases to its maximum value in the range 1<y<1.9 while in the absence of it the minimum value is in the range 0.4<y<1 , and the maximum value in the range 1.2<y<1.9.

Fig. 4 shows that the stress distribution σ_{xy} in the two theories in the presence and absence of diffusion starts from zero and it satisfy the boundary condition. In presence of diffusion the two theories decreases to a minimum value in the range 0.2<y<0.9, and then it increase to its maximum value in the range 0.9<y<1.9, while in absence of diffusion the two theories decreases to a minimum value in the range 0.3<y<0.9, and then it increase to its maximum value in the range 1.2<y<1.9.

Fig. 5 shows that the chemical potential P in the two theories in the presence of diffusion starts from positive values. P increases to a maximum value in the range 0<y<0.6 and then decreases to a minimum value in the range 0.7<y<1.2.

Fig. 6 shows that the concentration distribution C in the two theories in the presence of diffusion starts from positive values, in TPL theory the curve is smooth and directly reaches to zero, while in (G-N III) theory, it acts as a wave with maximum value in the range 0.1<y<1.2 and a minimum value in the range 1.5<y<2.9 .

3D curves 7-12 are giving 3D surface curves for the physical quantities, under (TPL) model with diffusion. These figures are important to study the dependence of these physical quantities on the vertical component of the distance. All physical quantities are satisfied the boundary conditions.

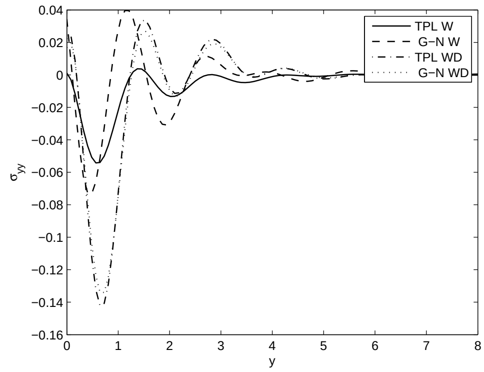

Fig. 3 Distribution the stress function σ_{yy} with and without diffusion.

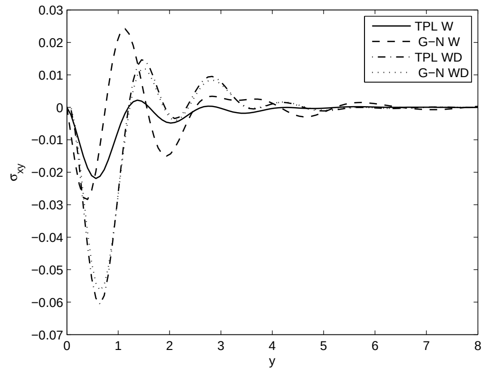

Fig. 4 Distribution the stress function σ_{xy} with and without diffusion.

8 Conclusions

1. The analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed.
2. The method that was used in the present article is applicable to a wide range of problems in thermodynamics and thermoelasticity.
3. The comparison of the two theories of thermoelasticity, (G-N) and (TPL) theories is carried out.
4. The physical quantities are very depending on the vertical distance and horizontal distance.

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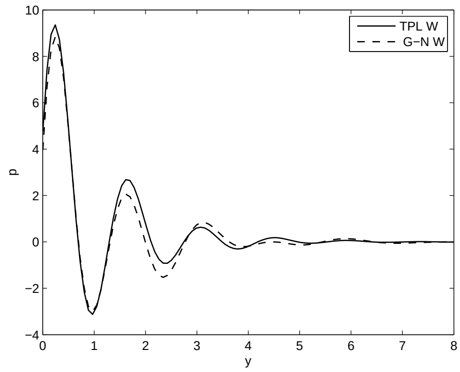

Fig. 5 Distribution the chemical potential p with and without diffusion.

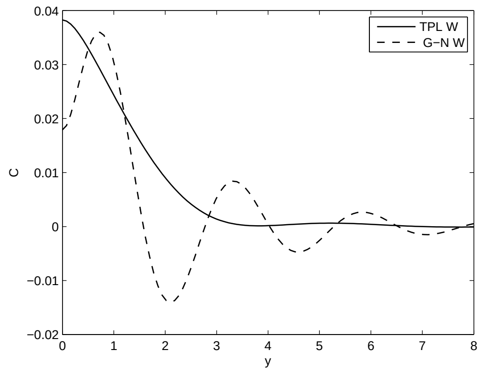

Fig. 6 Distribution the concentration C with and without diffusion.

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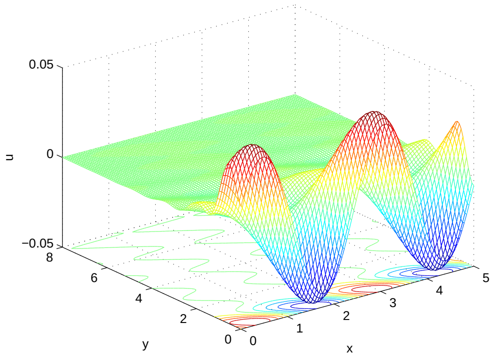

Fig. 7 Distribution the displacement component u against both components of distance in the TPL model with diffusion.

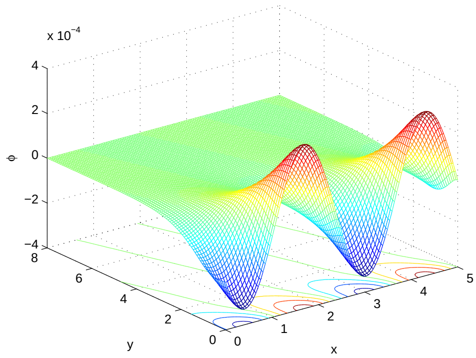

Fig. 8 Distribution the temperature component ϕagainst both components of distance in the TPL model with diffusion.

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