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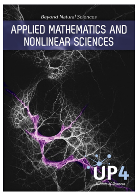

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Research on the Dynamic Evolution Behavior of Group Loitering Air Vehicles

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Abstract

In this paper we focus on integrated Reconnaissance/Strike LAV, in order to reveal the evolution regularity when group LAVs combats cooperatively. The evolution of cooperative behavior of group LAVs, which is described with finite state machine, can be regarded as a conversion process of a LAV in different task states, using the rate equation for probability analysis. Then based on the missions of integration of reconnaissance, attack, and damage effectiveness evaluation, we build the model of finite state machine based on behavior state transition. Solved with Runge-Kutta method. We can analyze how the key technology quota of LAV impact on the operational effectiveness of Group LAVs.the fractional order control approach.

Keywords: Loitering air vehicles; cooperative; Runge-Kutta; finite state automation; effectiveness
AMS 2010 codes: 68T42.

1 Introduction

LAV (loitering air vehicles) is an advanced weapon system, which can loitering in the air over the targets area, detect and acknowledge the targets, and then attack them. It comes out with the development of UAV and munitions [1]. LAVs are being used for military tasks, such as reconnaissance, classification, electronic attack or as munitions in the battle [2]. The group LAVs has better ability to accomplishing the task than group independent LAVs through synergy. Through cooperation the team can reconfigure its distribution architecture to minimize the performance degradation to such expected failures. Such cooperation should take advantage of the global information, resource management capabilities available to the group [3].

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Interest in using group LAVs as platforms for scientific instruments has been growing in the past few years. However, the current study, whether for its cooperation from automatic organization or conscious cooperation, are both carried out only for a specific problem, Lin realizes the incapability of the classical Nash strategy approach in dealing with the distributed UAV formation [4], Flight formation is investigated in reference [5–7]. Flight In reference [8], the model of multi-agents dynamic group is applied to cooperative task allocation. Cooperative task allocation problem is discussed using game theory [9]. Lanah, Ana, Herman and Albert [10] consider the online stochastic UAV mission planning problem with time windows and time-sensitive targets using a re-planning approach. Target tracking is discussed in reference [11], and many others [12–19].

In this paper we focus on integrated Reconnaissance/Strike LAV, which is designed to operate as a pack of vehicles that search, confirm and attack targets. Such as WASP, LOCAAS, Those are developed by Lockheed Martin. While much attention is given to the engineering and technological for these weapons systems, there are few researchers on operational for effectiveness and efficiency. Firstly, the collaboration behavior of the Group LAVs system is described starting from the cooperative mechanism and the basic properties. Then the differential model based on the state transition was established with the finite state machine. Finally, the model was solved with fourth-order Runge-Kutta method and analyzes how the key technology quota of LAV impact on the operational effectiveness of Group LAVs. This provides the necessary theoretical basis to design and analyze the cooperative behaviour of group LAVs system.

2 Problem formulation and analysis

2.1 The behaviors in the process of cooperative combat

In this paper we focus on integrated Reconnaissance/Strike LAV, so LAV can be in one of four possible situations: search, attack, BDA or removed. LAV is always being searching if it has not acquired a real target until it was destroy. While LAV discovers a target it attempts to identify if it is a real target. If the target is an unreal target, the LAV moves on with its search. If the target is a real target, The LAV attacks it. Once a LAV enters an attack stage, it cannot go back to the any other stage. A LAV may fail with enemy's air defense or technical failure or accident [1].

The combat mission constraints and rules of conduct of their behaviour are as follows:

(1) Judge the physical constraints of LAV [20], if the LAV meets physical characteristics and communication capability, it will form the Group LAVs, and perform a search mission;

(2) When obstacles, threats of enemy air defence weapons appear, The LAVs select obstacle avoidance, hedge bypass;

(3) When the target enters the visual field of the LAV, using information fusion and processing technology on the target judgment, can perform strike missions against the enemy targets;

(4) When perform strike missions against the enemy targets, can perform damage assessment against the enemy targets;

(5) When finish performing damage assessment task against the enemy targets , can perform the next round of decision.

2.2 Probability analysis based on state transformation

Let vector $\vec{\delta}_k(t) : k = 1,2,\ldots,K$ denote the task status of Group LAVs at time t. In order to obtain the probability of the state transition, the configuration equation of group LAVs system is derived, which is described as follows:

$$\vec{N} = \sum_{k=1}^{K} N_k \vec{\delta}_k(t),$$

where N_k is the number of LAVs at the state k and $\vec{N}$ is the distribution in each state of Group LAVs system.

Suppose that the probability distribution of group LAVs system state is $P(n,t)$ at time t. Then at time $t+\tau$, the probability distribution is $P(n,t+\tau)$. Define $W_t(\eta,n)$ as the transition probability density from state η to state n in unit time during time interval $[t,t+\tau]$. So the transition probability from state η to state n during time interval $[t,t+\tau]$ is $\tau W_t(\eta,n)$. According to [20], the primary equation model as follows:

$$\frac{dP(n,t)}{dt} = \int \{W(\eta,n)P(\eta,t) - W(n,\eta)P(n,t)\}\, d\eta.$$

In order to obtain evolution equation under the statistical average, we can get the state time evolution equation as follows:

$$\begin{aligned}
\frac{\partial}{\partial t}\langle \vec{n} \rangle &= \frac{\partial \vec{n}}{\partial t}\int \vec{n}P(\vec{n},t)d\vec{n} \\
&= \int \vec{n}\int \left\{W\left(\vec{n}',\vec{n}\right)P(\vec{n}',t) - W\left(\vec{n},\vec{n}'\right)P(\vec{n},t)\right\}d\vec{n}'d\vec{n} \\
&= \int\int \left(\vec{n}'-\vec{n}\right)W\left(\vec{n}',\vec{n}\right)P(\vec{n},t)d\vec{n}'d\vec{n} \\
&= \left\langle \int \vec{n}'W\left(\vec{n}',\vec{n}\right) - \vec{n}W\left(\vec{n}',\vec{n}\right)d\vec{n}' \right\rangle.
\end{aligned}$$

So it easy to verify that process of the state transition is a stability Markov chain. And it has some notation as following:

1. λ_{ij}–the average sustained rate of form state i to state j.
2. p_{ij}–the transition probability from state i to state j and satisfied equation $\sum_{j\neq i} P_{ij} = 1$.

$\lambda_{ij} = 0$–the state i is a transient state, once the process enters this state immediately leave.

$\lambda_{ij} = \infty$–the state i is an absorbing state; once the process enters this state will never leave.

Obviously, the last state of the LAV is an absorbing state.

Theorem 1. *Let p_{ij} be the transition probability density from task state i to state j $\forall\, i,j\in I$, where I is a set of LAVs task. Then p_{ij} is a consistent continuous function at time t.*

Proof. Generally, assuming $\forall t>0$,

$$P_{ij}(t+\Delta t) - P_{ij}(t) = \sum_{k\in I} P_{ik}(\Delta t)P_{kj}(t) - P_{ij}(t) = P_{ii}(\Delta t)P_{ij}(t) - P_{ij}(t) + \sum_{k\in I} P_{ik}(\Delta t)P_{kj}(t)$$

Thus we can obtain,

$$(P_{ii}(\Delta t)-1) \le P_{ij}(t+\Delta t) - P_{ij}(t) = (P_{ii}(\Delta t)-1)\,P_{ij}(t) + \sum_{k\in I} P_{ik}(\Delta t)P_{kj}(t) \le (1-P_{ii}(\Delta t))$$

Thus,

$$\left|P_{ij}(t+\Delta t) - P_{ij}(t)\right| \le (1-P_{ii}(\Delta t))$$

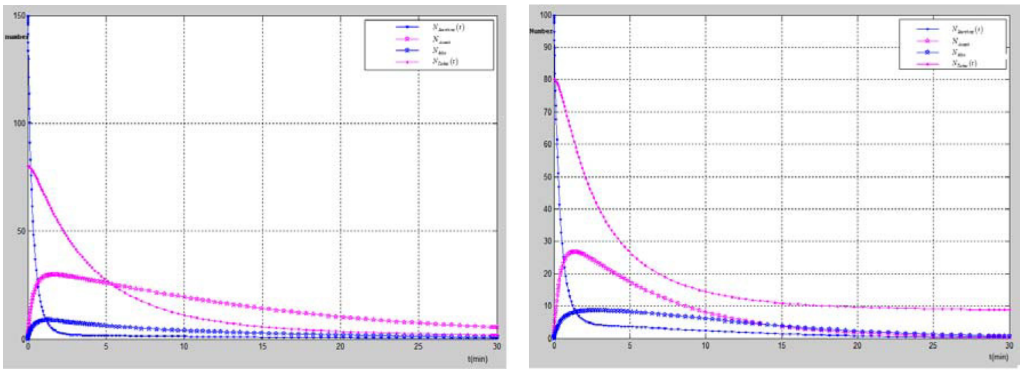
Because the state is not a transient state. Therefore, the transition probability satisfies regularity conditions:

$$\lim_{t\to 0} P_{ij}(t) = \begin{cases} 1, i=j \\ 0, i\neq j \end{cases}$$

Then we have the conclusion as follows:

$$\lim_{\Delta t\to 0}\left|P_{ij}\left(t+\Delta t\right) - P_{ij}\left(t\right)\right| = 0.$$

Accordingly, p_{ij} is a consistent continuous function at time t.



(a) $N = 150$, $\alpha_s = 1$, $\lambda_A = 30s$. (b) N=100,$\alpha_s = 1$,$\lambda_A = 30s$

Fig. 1 The number of task state with respect to time at different scale.

3 Mathematical model and analysis of system

3.1 Mathematical model of the system based on the state-transition

To the best knowledge of the authors, the finite state machine model was established under the assumptions stated in the following.

(1) LAVs are two-way information exchange via data link;

(2) The time of translation state form one task to another task is exponential distribution;

(3) The parameters of encountered obstacles during combat threats are random variable; Differential equation model was established using finite state machine automatically.

$$\begin{cases} \frac{dN_S(t)}{dt} = \frac{1}{\lambda_{Av}} N_{AS}(t) + \frac{1}{\lambda_{BDA}} N_{BDA}(t) - \alpha_s N_S(t) N_T(t) - \alpha_{As} N_S(t)(N_S(t)+C). \\ \frac{dN_{Av}(t)}{dt} = -\frac{1}{\lambda_{Av}} N_{Av}(t) + \alpha_{AA} N_A(t)(N_A(t)+C) + \alpha_{As} N_S(t)(N_S(t)+C) + \alpha_{AB} N_{BDA}(t)(N_{BDA}(t)+C). \\ \frac{dN_A(t)}{dt} = \alpha_s N_S(t) N_T(t) - \frac{1}{\lambda_A} N_A(t). \\ \frac{dN_{BDA}(t)}{dt} = \alpha_{AB} N_{AB}(t) - \frac{1}{\lambda_{BDA}} N_{BDA}(t) + \alpha_{AB} N_{BDA}(t)(N_{BDA}(t)+C). \\ \frac{dN_T(t)}{dt} = -\alpha_s N_S(t) N_T(t). \end{cases}$$

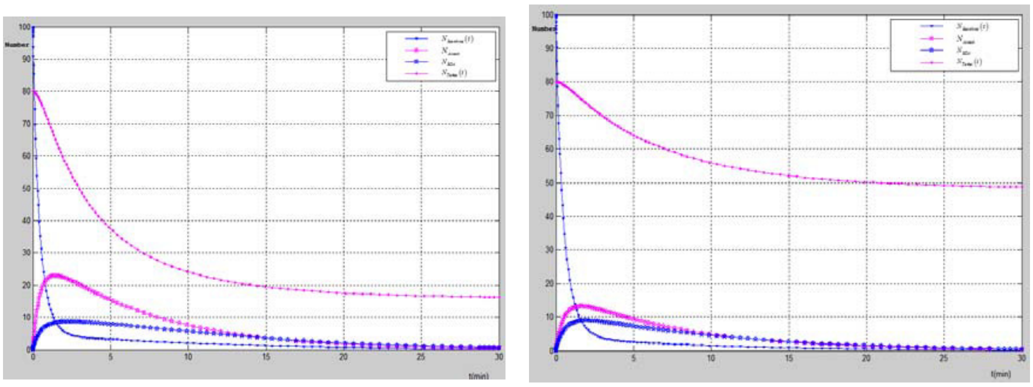
Parameters Notation: C is a constant and satisfied $C = N + B$, N is the Scale of the system, B is the scale of the obstacles, $N_s(t)$ is the scale of group in the searching state at time t, $N_{Av}(t)$ is the scale of group in the avoiding state at time t, $N_A(t)$ is the scale of group in the attacking state at time t, $N_{BDA}(t)$ is the scale of group in the BDA state at time t, $N_T(t)$ is the scale of targets, $\lambda_{Av}, \lambda_A, \lambda_{BDA}$ are the average time of Avoiding, Attacking, and BDA, resp., α_S is the rate of searching a real target, and $\alpha_{AS}, \alpha_{AA}, \alpha_{AB}$ are the rates of encounter obstacles on searching,attacking, and BDA.

3.2 Simulation and analysis

In order to illustrate the feasibility and effectiveness of the differential equation model, which was solved by Runge-Kutta method, we chose $N_T = 80$, λ_{Av}=5 min, λ_{BDA}=5 min, and $\alpha_{AS}, \alpha_{AB}, \alpha_{AA} \in [0,1]$ are random variables. In order to analysis the impact of the design and operating parameters, we have established expected relative efficacy $E_L = \frac{E(x)}{N_T}$ and efficiency $E_N = \frac{E(x)}{N}$.

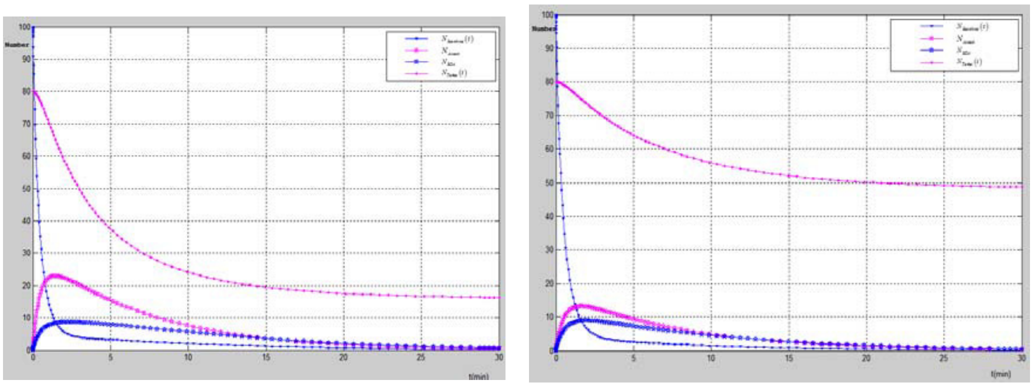
Fig. 1 describes the number of task state with respect to time under different group scale. The group LAVs system reached a state of equilibrium after a period of the game. The scale of the group will affect the operational efficacy and efficiency. With the scale of the number larger, targets decay faster, and it has higher efficacy. When the system reaches a certain scale, the efficiency is reduced.

Fig. 2 describes the number of task state with respect to time at different Probability of detecting the target



(a) $N = 100$, $\alpha_s = 0.8$, $\lambda_A = 30s$ (b) $N = 100$, $\alpha_s = 0.4$, $\lambda_A = 30s$

Fig. 2 The number of task state with respect to time at different probability of detecting the target value.



(a) $N = 100$, $\alpha_s = 0.8$, $\lambda_A = 30s$. (b) $N = 100$, $\alpha_s = 0.4$, $\lambda_A = 10s$.

Fig. 3 The number of task state with respect to time at different attack time.

value. Obviously with the success probability to detect valuable targets bigger, the number of damaging enemy targets is larger, the higher its operational effectiveness. Success probability of detecting the target value is a very important factor.

Fig. 3 displays the number of task states with respect to time at different attack time. We can see that shorten time to attack enemy targets in LAV, the operational tasks whose time is shortened accordingly, but the combat effectiveness impact is not great.

4 Conclusions

(1) The model for cooperative behavior of the group LAVs, which is described with finite state machine by the rate equation for probability analysis. It can describe the dynamics of group behavior with the time.

(2) The model of finite state machine based on behavior state transition is proposed according to the missions of integration of reconnaissance, attack, and damage effectiveness evaluation. We can analyze how the key technology quota of LAV impact on the operational effectiveness of Group LAVs.

This provides the necessary theoretical basis to design and analyze the combat efficacy of group LAVs system.

References

[1] Li Bing, Li Jie, He Guanglin and Li Dalin, (2014), *Research on Cooperative Combat for Integrated Reconnaissance-Attack-BDA of Group LAVs*, Mathematical Problems in Engineering, Article ID 123142, 6 pages. doi 10.1155/2014/123142

[2] P. B. Sujit, A. Sinha and D.Ghose, (2005), *Multi-UAV task allocation using team theory*. In: Proceedings of the 44th IEEE Conference on Decision and Control, 1497-1502. doi 10.1109/CDC.2005.1582370

[3] T. Shima and S.Rasmussen, (2009), *UAV Cooperative Decision and Control. Challenges and Practical Approaches*, Society for Industrial and Applied Mathematics, 2009.

[4] Wei Lin, (2014), *Distributed UAV formation control using differential game approach*, Aerospace Science and Technology, 35, 54-62. doi 10.1016/j.ast.2014.02.004

[5] Hyoung-seok Kim and Youdan Kim, (2014), *Trajectory optimization for unmanned aerial vehicle formation reconfiguration*, Engineering Optimization, 46, No 1, 84-106. doi 10.1080/0305215X.2012.748048

[6] I. Bayezit and B. Fidan, (2013), *Distributed Cohesive Motion Control of Flight Vehicle Formations*, IEEE Transactions on Industrial Electronics, 60, No 12, 5763-5772. doi 10.1109/TIE.2012.2235391

[7] Zhou Chao, Shao-Lei Zhou, Lei Ming and Wen-Guang Zhang, (2012), *UAV Formation Flight Based on Nonlinear Model Predictive Control*, Mathematical Problems in Engineering, Article ID 261367, 15 pages. doi 10.1155/2012/261367

[8] Jie Li, Shuangchun Peng, Honglei An, Xiaojia Xiang and Lincheng Shen, (2013), *UAVs formation rendezvous method based on differential geometry and Lie group*, Journal of National University of Defense Technology, 35, No 6, 157-164.

[9] Huimin Chen, Dan Shen, Genshe Chen, Erik P. Blasch and Khanh Pham, (2010), *Tracking evasive objects via a search allocation game*, In: Proceedings of the 2010 American Control Conference, 6981 - 6986. doi 10.1109/ACC.2010.5531343

[10] Han Quanye, Wang Xiaoming and Dang Jianwu, (2007), *Research on the task allocation model and algorithm of MAS based on game theory*, Computer Applications and Software, No 12, 51-53.

[11] Lanah Evers, Ana Isabel Barros, Herman Monsuur and Albert Wagelmans, (2014), *Online stochastic UAV mission planning with time windows and time-sensitive targets*, European Journal of Operational Research, 238, No 1, 348-362. doi 10.1016/j.ejor.2014.03.014

[12] Ma Yunhong, Jing Zhe and Zhou Deyun, (2013), *A Faster Pruning Optimization Algorithm for Task Assignment*, Journal of Northwestern Polytechnical University, 31, No 1, 40-43.

[13] Hyunjin Choi and Youdan Kim, (2014), *UAV guidance using a monocular-vision sensor for aerial target tracking*, Control Engineering Practice, 22, 10-19. doi 10.1016/j.conengprac.2013.09.006

[14] R. W. Beard and V. Stepanyan, (2003), *Information consensus in distributed multiple vehicle coordinated control*, In: Proceedings of the 42nd IEEE Conference on Decision and Control, 2, 2029-2034. doi 10.1109/CDC.2003.1272913

[15] Yoonsoo Kim, Da-Wei Gu and Ian Postlethwaite, (2007), *Real-Time Optimal Mission Scheduling and Flight Path Selection*, IEEE Transactions on Automatic Control, 52, No 6, 1119-1123. doi 10.1109/TAC.2007.899048

[16] John Tisdale, Zuwhan Kim and J. Karl Hedrick, (2009), *Autonomous UAV path planning and estimation*, IEEE Robotics & Automation Magazine, 16, No 2, 35-42. doi 10.1109/MRA.2009.932529

[17] P. B. Sujit and Randy Beard, (2007), *Cooperative Path Planning for Multiple UAVs Exploring an Unknown Region*, In: 2007 American Control Conference, 347-352. doi 10.1109/ACC.2007.4283044

[18] Marjorie Darrah, Edgar Fuller, Thilanka Munasinghe, Kristin Duling, Mridul Gautam and Mitchell Wathen, (2013), *Using Genetic Algorithms for Tasking Teams of Raven UAVs*, Journal of Intelligent & Robotic Systems, 70, No 1, 361-371. doi 10.1007/s10846-012-9696-3

[19] A. Pant, P. Seiler, T. J. Koo and K. Hedrick, (2001), *Mesh stability of unmanned aerial vehicle clusters*, In: Proceedings of the 2001 American Control Conference, 1, 62-68. doi 10.1109/ACC.2001.945514

[20] Li Bing, Li Jie and Huang KeWei, (2013), *Modeling and Flocking Consensus Analysis for Large-Scale UAV Swarms*, Mathematical Problems in Engineering, Article ID 368369, 9 pages. doi 10.1155/2013/368369