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Jittering regimes of two spiking oscillators with delayed coupling

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Abstract

A system of two oscillators with delayed pulse coupling is studied analytically and numerically. The so-called jittering regimes with non-equal inter-spike intervals are observed. The analytical conditions for the emergence of in-phase and anti-phase jittering are derived. The obtained results suggest universality of the multi-jitter instability for systems with delayed pulse coupling.

Keywords: Pulse-coupled oscillators, synchronization, phase resetting curve, delayed coupling, jittering
AMS 2010 codes: 37L10, 37L15.

1 Introduction

Pulse-coupled oscillators are popular and powerful models used widely to describe the dynamics of networks of various nature, including neuronal populations, cardiac tissues and many others [1–3]. The key features captured by these models are the periodical self-sustained activity of the individual units and the pulse-mediated interaction between them. Pulses here are brief signals which have a temporal duration much smaller than the oscillations period. In realistic systems, such pulses often have finite speed of propagation between the units, which leads to the emergence of coupling delays.

In the framework of pulse-coupled oscillators, the effect of a pulse depends on the dynamical state of the oscillator at the time of the pulse arrival. In the case of weak coupling the oscillators are usually described by their phases, and the influence of a pulse is determined with the help of the so-called phase resetting curves (PRCs, [4, 5]). A characteristic PRC of any oscillatory system corresponding to any type of stimulus can be computed numerically or measured experimentally [5–10]. Thus, pulse-coupled systems can be considered

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either as stand-alone models, or as approximations of more complex systems. Among the advantages of such models is their simplicity both for numerical implementation and theoretical analysis.

A number of important results have been obtained within the framework of pulse-coupled oscillators. The stability of synchronous [1,2] and desynchronized [11] states have been studied. Emergence of coexistent stable clusters is reported in [12–15]. Splay states and their stability in globally pulse-coupled systems are reported in [16]. Rings of pulse-coupled oscillators are investigated in [17]. Recently, a novel dynamical phenomenon has been discovered in spiking oscillators subject to delayed pulse feedback [18, 19]. In such systems, under certain conditions on the oscillator's PRC, regular spiking may destabilize and change to irregular, so-called "jittering" regimes with non-equal inter-spike intervals (ISIs). The number of different coexisting jittering regimes grows exponentially with the delay which makes the system highly multistable.

In the present paper, we make a first step to extend the concept of "jittering" to systems more complex than just a single oscillator. Namely, we discover and study jittering regimes in a system of two spiking oscillators with time-delay coupling.

2 Jittering regimes of one oscillator

Jittering regimes of a single spiking oscillator have been studied in detail in our previous papers [18, 19]. Here we reproduce the key results of those papers because the similar technique will be used in the present study. The basic model of a single spiking oscillator with delayed feedback is as follows:

$$\frac{d\varphi}{dt} = 1 + Z(\varphi)\sum_{t_p} \delta(t - t_p - \tau). \tag{1}$$

The oscillator is described by its phase $\varphi \in [0,1]$. Without the feedback, the phase grows uniformly with $d\varphi/dt = 1$. When the phase reaches unity, it resets to zero and the oscillator emits a spike. The instants when this happens are denoted by t_p, $p \in \mathbf{Z}$. The produced pulses propagate along the delay line and arrive back to the oscillator after delay τ at the instants $t_p + \tau$ (see Fig. 1(a)). When the oscillator receives a pulse, the phase instantly changes to the new value:$\varphi \mapsto \varphi + Z(\varphi)$, where the function $Z(\varphi)$ is the phase resetting curve (PRC, [3]).

Previously, the researchers concentrated on regular regimes of (1) when spikes are produced with constant inter-spike interval (ISI). However, recently we have shown that regular regimes may destabilize and give rise to irregular ones with distinct ISIs. These regimes were called "jittering", and the corresponding scenario was called "multi-jitter bifurcation". In the bifurcation point, numerous jittering regimes emerge at the same point which makes the system highly multistable. Namely, we have shown that the number of coexisting jittering regimes grows exponentially as the delay increases.

Analytical study of the multi-jitter bifurcation relies on the technique developed in [20] which allows to reduce (1) to a multidimensional discrete map. This map fully describes the dynamics of the oscillator and deals with the sequence of the ISIs denoted as T_j (Fig. 1(b)). The next ISI T_{j+1} can depends on the phase ψ_j at which the oscillator receives a pulse. As the phase grows with $d\varphi/dt = 1$ between the moments when the oscillator emits a spike and receives a pulse, this phase can be found as

$$T_{j+1} = 1 - Z\left(\tau - \sum_{k=j-P+1}^{j} T_k\right). \tag{2}$$

Here, P is the maximal number for which $\sum_{k=j-P+1}^{j} T_k < \tau$. Thus, the map (2) calculates the next ISI basing on P previous ones.

For regular spiking, when all the ISIs equal $T_j = T$, $P = [\tau/T]$. In this case the period of the regular regime is given as a solution to

$$T = 1 - Z(\tau \mod T). \tag{3}$$

The stability of the regular regime is determined by the roots of the characteristic equation

$$\lambda^P - \alpha \sum_{k=0}^{P-1} \lambda^k = 0, \tag{4}$$

where $\alpha := Z'(\psi)$ is the PRC slope at the phase $\psi = \tau \mod T$ at which pulses hit the oscillator.

The multi-jitter bifurcation occurs at points with $\alpha = -1$. One can easily see, that at such points P critical multipliers $\lambda_k = e^{i2\pi k/(P+1)}$, $k = 1, \ldots, P$, emerge simultaneously. Thus, the regular regime destabilizes. Moreover, *all* directions of perturbations in the system phase space become unstable, which means that the dimension of the unstable manifold increases abruptly from zero to P. Deeper study of this remarkable phenomenon shows that numerous jittering regimes emerge at the same point. Each of the emergent regimes presents a $(P+1)$-periodical sequence of ISIs where each term can take one of two values $\Theta_1$or Θ_2. It is possible to show that for each binary sequence of the length $P+1$ the corresponding jittering regime does exist. This means that the total number of the emergent jittering regimes grows exponentially with P.

As mentioned above, the criterion for the multi-jitter bifurcation is sufficient steepness of the PRC. Suppose there exists an interval of phases $\varphi \in [\psi_A; \psi_B]$ for which the PRC slope is less than minus one. Then, on the borders of this interval (at $\psi = \psi_A$ and $\psi = \psi_B$) multi-jitter bifurcations take place giving rise to multiple jittering regimes. These regimes exist at least in the interval $\psi \in [\psi_A; \psi_B]$, but the region of existence of some of these regimes may be even larger. It also should be noted that the intervals of stability of different jittering regimes alternate in this region.

3 Jittering regimes of two oscillators

Now let us consider a system of two pulse oscillators with mutual delayed coupling. For the sake of simplicity, let the system be symmetrical, so that the native frequencies, the coupling strength and the delays are the same. In this case the system of two oscillators is governed by the equations

$$\frac{d\varphi_j}{dt} = 1 + Z(\varphi_j) \sum_{t_p^k} \delta(t - t_p^k - \tau). \tag{5}$$

Here, $j = \overline{1,2}$ is the oscillator id, $k = 3 - j$ is the id of the peer oscillator, φ_j is the j-th oscillator's phase, and t_p^k are the instants when the k-th oscillator produces spikes. When k-th oscillator produces a spike at $t = t_p^k$, a pulse arrives to the j-th one at $t = t_p^k + \tau$ and perturbs its phase (see Fig. 1(c)). We do not introduce self-coupling in the system.

The system of two delay-coupled oscillators have been intensively studied previously [2,12,16,23]. Depending on the frequency mismatch and the coupling strength, the oscillators may either synchronize or demonstrate asynchronous behavior. If the oscillators are identical they always synchronize, but depending on the delay value two different synchronous regimes are possible: the in-phase and the antiphase synchronization. For the in-phase synchronization, the oscillators emit spikes simultaneously. In this case the stable synchronous manifold in the system phase space exists for which the states of the oscillators are equal. On this manifold, two oscillators behave as one oscillator with delayed feedback. This means that the system dynamics on the synchronous manifold is described by (1), and they should have the same properties. Thus, the in-phase jittering of two oscillators should be possible.

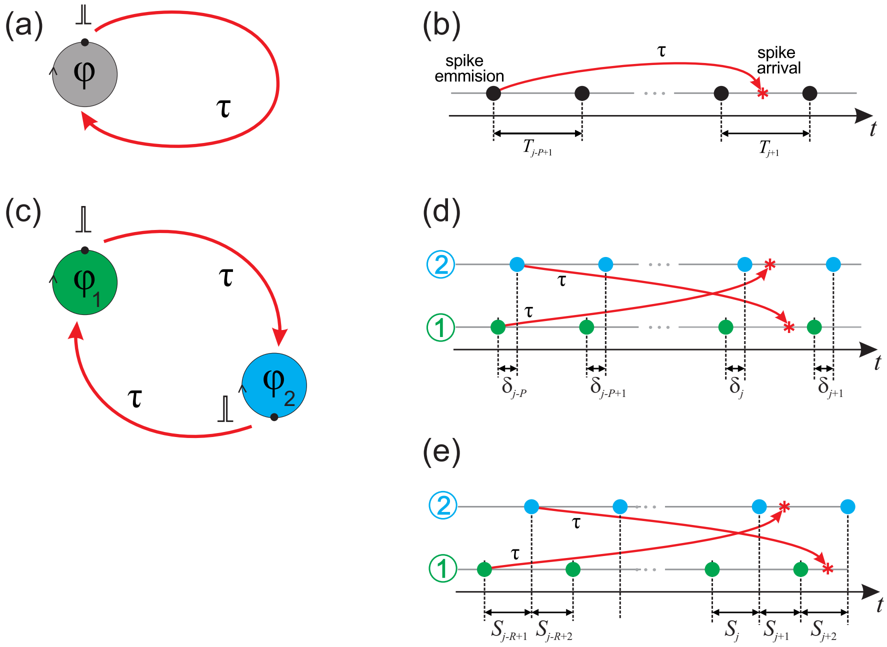

Fig. 1 Dynamics of the oscillators with pulse delayed coupling and derivation of the discrete maps. Dots depict the moments of spikes emission. (a) Single oscillator with delayed feedback and (b) its dynamics. Each spike produced arrives to the oscillator after delay time τ. T_j are the inter-spike intervals. (c) Two oscillators with mutual delayed coupling and (d-e) their dynamics. A spike produced by each oscillator arrives to another one after delay time τ. (d) Near-in-phase dynamics, δ_j are the time lags between the oscillators. (c) Near-antiphase dynamics, S_j are cross-spike intervals.

3.1 In-phase jittering

Let us study the stability of the in-phase synchronous regime. For this sake, consider small perturbation of this regime so that the spikes are emitted by the oscillators not simultaneously, but with small time lags $\delta_j \ll 1$. For the certainty suppose $\delta_j > 0$ when the first oscillator outpaces the second one (Fig. 1(d)). Consider the system dynamics during the $j+1$-st inter-spike interval. An the beginning of the interval, the lag between the oscillators is δ_j. Denote ψ the phase at which the next pulse from the first oscillator arrives to the second one. Then the phase at which the pulse from the second oscillator hits the first one equals $\psi + \delta_j + \delta_{j-P}$. Here, δ_{j-P} is the lag between the oscillators P intervals before. Given this, it is easy to show that the lag at the end of the current interval equals

$$\delta_{j+1} = \delta_j + Z(\psi + \delta_{j-P} + \delta_j) - Z(\psi). \tag{6}$$

For the in-phase synchronization, $\psi = \tau \mod T$ and $P = [\tau/T]$, just as in the case of one oscillator with feedback. Linearization of (6) leads to the characteristic equation

$$\lambda^{P+1} - (1+\alpha)\lambda^P - \alpha = 0. \tag{7}$$

Here, $\alpha := Z'(\psi)$ has the same sense as for one oscillator.

Destabilization of the in-phase synchronous regime corresponds to $|\lambda| = 1$. Under this condition (7) leads to

$$|\lambda^{P+1} - \alpha| = (1+\alpha)|\lambda^P| = 1 + \alpha, \tag{8}$$

which, for $\alpha \neq 0$, implies $\lambda^{P+1} = -1$. Substituting this into (7) gives $\lambda^P = -1$ or $\alpha = -1$. The conditions $\lambda^{P+1} = -1$ and $\lambda^P = -1$ can not be fulfilled together. Thus, the only points at which critical multipliers appear are $\alpha = 0$ and $\alpha = -1$. At $\alpha = 0$, eq. (7) transforms into $\lambda^P(\lambda - 1) = 0$, and the critical multiplier $\lambda = 1$ appears which corresponds to a saddle-node bifurcation. A much more remarkable scenario takes place at $\alpha = -1$, where $P+1$ critical multipliers

$$\lambda_k = e^{i\pi(2k+1)/(P+1)} \tag{9}$$

appear simultaneously. These points correspond to the emergence of in-phase jittering regimes of two oscillators.

3.2 Antiphase jittering

The other basic synchronous regime of two identical oscillators is antiphase synchronization when the oscillators emit spikes one after another with the same inter-spike intervals. To study the stability of the anti-phase synchronization let us first derive a discrete map governing the system dynamics in the neighborhood of this regime. For this sake we first introduce convenient notations. Denote S_j the so-called cross-spike interval (CSI) - the interval between two spikes emitted one after another irrespectively of the identity of the oscillators which produced them (Fig. 1(e)). Easy to see that the ISI between the two successive spikes of the same oscillator equals the sum of two consequent CSIs. In Fig. 1(e), $T_j = S_j + S_{j+1}$ is the ISI of the first, and $T_{j+1} = S_{j+1} + S_{j+2}$ of the second oscillator. Easy to see that, say, the ISI T_{j+1} depends on the phase ψ_{j+1} of the second oscillator at which it receives a pulse:

$$T_{j+1} = 1 - Z(\psi_{j+1}). \tag{10}$$

Similarly with the case of one oscillator, this phase can be found as

$$\psi_{j+1} = \tau - \sum_{k=j-R+1}^{j} S_k. \tag{11}$$

Here, R is the maximal *odd* number satisfying $\sum_{k=j-R+1}^{j} S_k < \tau$. These formulas allow to obtain the map for the cross-spike intervals

$$S_{j+2} = 1 - S_{j+1} - Z\left(\tau - \sum_{k=j-R+1}^{j} S_k\right), \tag{12}$$

which calculates the next CSI based on $R+1$ previous ones.

The anti-phase synchronization corresponds to the case when all the CSIs equal $S_j = S = T/2$, where T is the period. In this case $R = 2[\tau/T + 1/2] - 1$, and the period can be found as a solution to

$$T = 1 - Z((\tau + T/2) \mod T). \tag{13}$$

Linearization of (12) leads to the characteristic equation

$$\lambda^{R+1} + \lambda^R - \beta \sum_{k=0}^{R-1} \lambda^k = 0. \tag{14}$$

Here, $\beta := Z'(\xi)$, where $\xi = (\tau + T/2) \mod T$ is the phase at which oscillators receive pulses.

To study the characteristic equation, let us multiply it by $(\lambda - 1)$ and obtain

$$\lambda^{R+2} - (1+\beta)\lambda^{R+1} + \beta = 0. \tag{15}$$

The set Λ' of roots of (15) contains all roots Λ of (14) and the root $\lambda = 1$, i.e. $\Lambda = \Lambda' \backslash \{1\}$. Let us determine the conditions when critical roots of (14) with $|\lambda| = 1$ appear. For such roots (15) implies

$$|\lambda^{R+2} + \beta| = |(1+\beta)\lambda^R| = |1+\beta|.$$

For $\beta \neq 0$, this implies $\lambda^{R+2} = 1$. Substituting this into (15) leads to

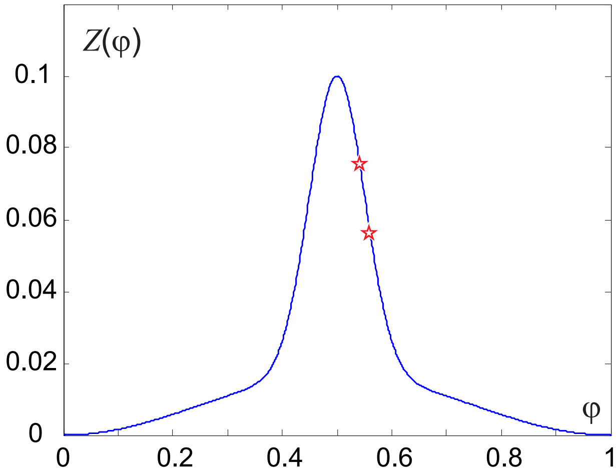

Fig. 2 The PRC for $\kappa = 0.1$, $\varepsilon = 0.1$, $q = 40$. The stars depict points with $Z'(\varphi) = -1$.

$$1+\beta = (1+\beta)\lambda^{R+1}.$$

For $\beta \neq -1$, this implies $\lambda^{R+1} = 1$ and, consequently, $\lambda = 1$. Thus, for $\beta \notin \{-1,0\}$, $\lambda = 1$ is the only critical root of (15). Substituting $\lambda = 1$ into (14) one obtains immediately $\beta = 2/R$. Thus, $\lambda = 1$ is a root of (14) only for this value of β.

Thus, the characteristic equation (14) can have critical roots only for $\beta \in \{-1, 0, 2/R\}$. For $\beta = 0$, (14) transforms to $\lambda^R(\lambda+1) = 0$, and the critical root is $\lambda = -1$ corresponding to the period-doubling bifurcation. For $\beta = -1$, eq. (14) becomes identical to eq. (4) with $\alpha = -1$ and $P = R+1$. Thus, it has the same spectrum, namely, $R+1$ critical multipliers $\lambda_k = e^{i2\pi k/(R+1)}$, $k = 1, ..., R$, emerge simultaneously. This point corresponds to the emergence of antiphase jittering regimes of two oscillators.

3.3 Numerical results

Our analytical study suggests that both in-phase and antiphase synchronous regimes of two oscillators may destabilize and give rise to jittering regimes. Now we seek to confirm our theoretical predictions by numerical simulations. For this sake we consider the PRC in the form

$$Z(\varphi) = \frac{\kappa}{1+2\varepsilon}\left((\sin\varphi)^q + \varepsilon(1-\cos 2\pi\varphi)\right). \tag{16}$$

Here, $\kappa = 0.1$ is the coupling strength, $\varepsilon > 0$ and $q > 0$ are the parameter used to control the PRC shape. We used $\varepsilon = 0.1$ and $q = 40$, for which the PRC is depicted in Fig. 2. For these parameter values two points φ exist with $Z'(\varphi) = -1$, and $Z'('\varphi) < 1$ in the interval between them. Thus, according to our theory, the multi-jitter bifurcations should be observed on the borders of this interval.

First, let us study the synchronous solutions of (5) with PRC (16). For the in-phase synchronous regimes, the period is given by (3), which can be rewritten in parametric form as

$$T = 1 - Z(\psi), \qquad \tau = PT + \psi.$$

Varying $\psi \in [0;1]$ and $P = 1, 2, ...$, one may obtain all the branches of the in-phase synchronous solutions. These branches are plotted in Fig. 3 by thin blue lines, stable parts by solid, unstable by dashed lines. The stability criterion used is $-1 < Z'(\psi) < 0$. The bifurcation points corresponding to saddle-node bifurcations ($Z'(\psi) = 0$) and multi-jitter bifurcations ($Z'(\psi) = -1$) are marked by symbols.

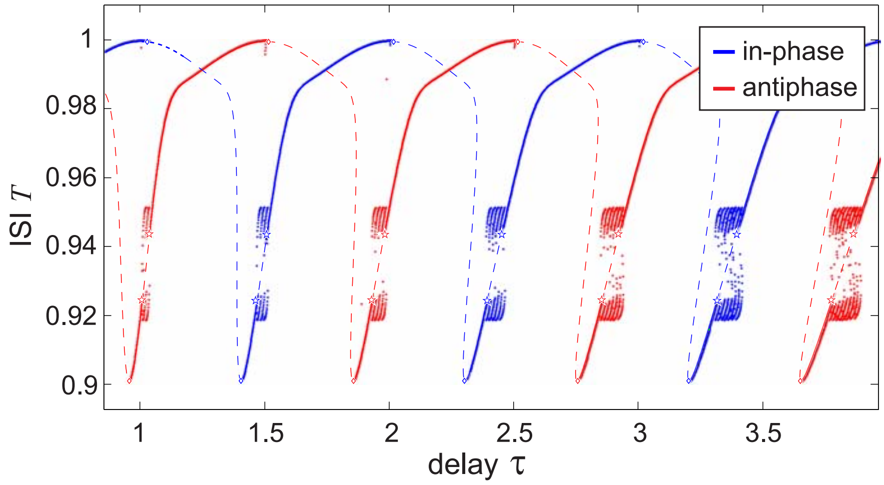

Fig. 3 The bifurcation diagram of two oscillators with mutual delayed coupling. The observed ISIs of the established dynamical regimes are plotted versus the time delay. Blue lines stand for near-in-phase, red for near-antiphase regimes. Theoretically predicted regular regimes are plotted by thin lines, solid for stable and dashed for unstable regimes. Diamonds depict saddle-node or period-doubling bifurcations, while stars depict multi-jitter bifurcations. Thick dots depict the numerical results. Note that almost everywhere the stable branches predicted by the theory are covered by numerically observed dots.

For anti-phase synchronous regimes , the period is given by (13), which also can be rewritten in parametric form as

$$T = 1 - Z(\xi), \qquad \tau = RT/2 + \xi.$$

All the branches of the antiphase synchronous regimes can be obtained by varying $\xi \in [0;1]$ and $R = 1, 2, \ldots$ These branches are plotted in Fig. 3 by thin red lines, stable parts by solid, unstable by dashed lines. The stability criterion for the anti-phase synchronization is $-1 < Z'(\xi) < 0$.

To compare the theoretical predictions with the numerical results we plot the latter in the same figure. To obtain these results we numerically integrated system (5) and plotted all the ISIs observed after the transient. For each value of τ, 20 different random initial conditions were taken. One can see that almost everywhere where the theory predicts the existence of a stable regular regime the numerical simulations confirm this prediction, and the numerical dots lie exactly on the theoretical curves. In the intervals where the regular regimes are expected to destabilize due to the multi-jitter bifurcations they do so indeed, and the jittering regimes with distinct ISIs emerge.

Let us deeper study the emergent jittering regimes. The examples of them are depicted in Fig. 4. Note that each observed regime contains just two distinct ISIs, the short and the long one. Such regimes were previously named “bipartite” and were shown to play crucial role in the development of the multi-jitter instability for one oscillator with feedback [18, 19]. The two ISIs may form various sequences giving rise to multiple jittering regimes.

Figure 4(a) shows the in-phase jittering regime observed at $\tau = 2.4$. Note that the oscillators remain in-phase, in a sense that they produce spikes simultaneously. Their ISIs are therefore synchronized and form a period-3 sequence of two short and one long intervals. In Fig. 4(b), the antiphase jittering regime is shown observed at $\tau = 1.95$. The oscillators produce pulses (almost) in antiphase, and the ISI sequences for the both oscillator are the same and consist of three short and two long intervals. Interestingly enough, these sequences are also in antiphase: when the first oscillator demonstrates long ISIs, the second one demonstrates short ISIs, and vise versa.

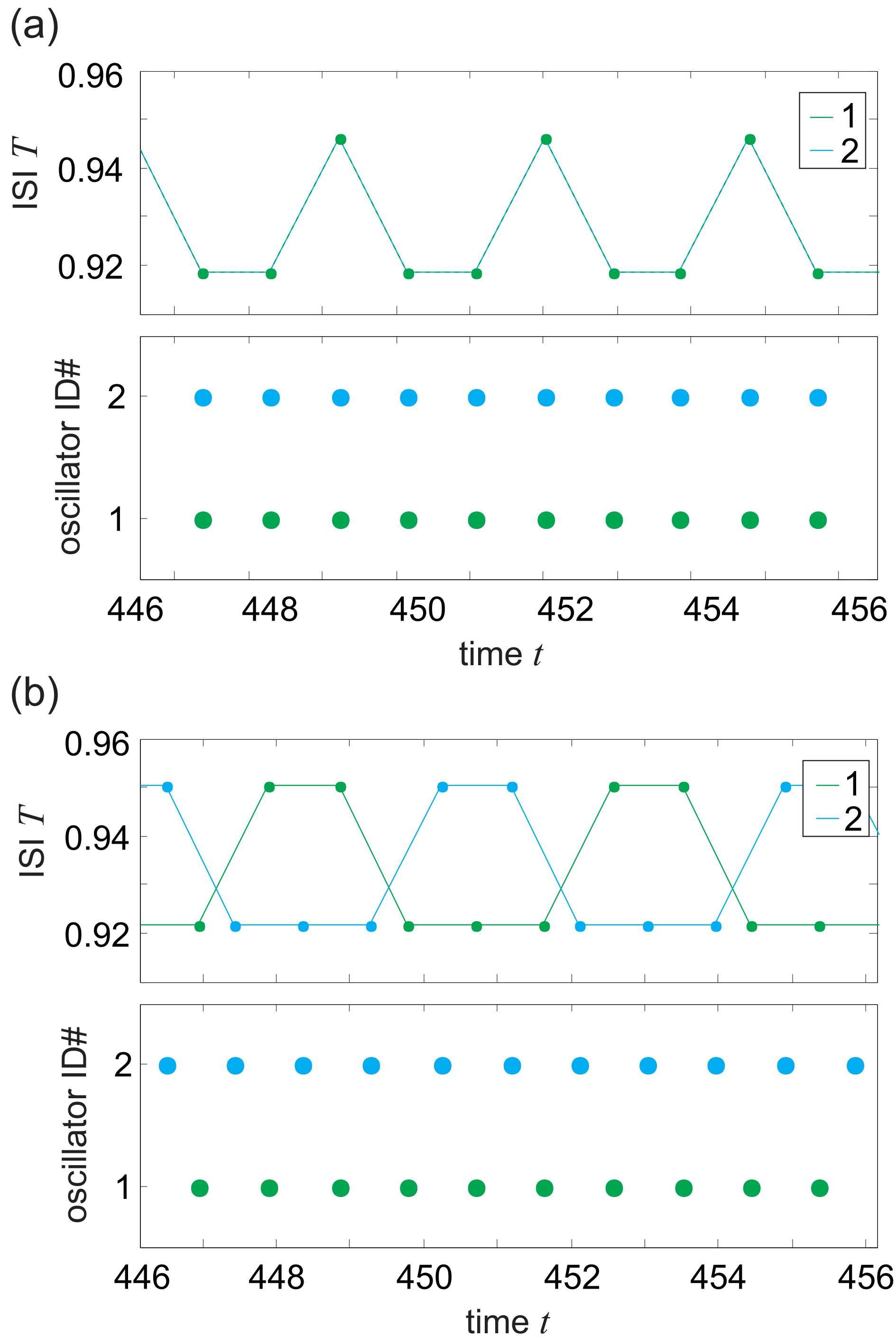

Fig. 4 Examples of jittering regimes of two oscillators: (a) antiphase jittering at $\tau = 1.95$ and (b) in-phase jittering at $\tau = 2.4$. The upper panels show the temporal dynamics of the ISIs for both oscillators, the bottom panels show the moments of spike emission. Note that in the case of in-phase jittering the ISIs for the both oscillators coincide, which means that they remain fully synchronized.

4 Discussion and conclusions

We have shown that jittering regimes can emerge in a system of two pulse oscillators with mutual delayed coupling. In the case of no frequency detuning, such the system has two basic regular regimes: in-phase and antiphase synchronous spiking. Multiple branches of these two regimes exist in various intervals of the delays. If the PRC of the oscillators has slope = -1 at certain phases, regular regimes may destabilize and give rise to jittering regimes with distinct ISIs. The bifurcation points correspond to the situations when the input pulses hit the oscillators at these critical phase.

As one may see, there is a lot in common between the scenarios of development of the multi-jitter instability in a pair of coupled oscillators and in one oscillator with feedback. First of all, the condition for the bifurcation is the same: the slope of the PRC at the phase at which the oscillators receive pulses must be equal -1. Secondly, the emergent jittering regimes are bipartite, and the corresponding bifurcation diagram has a typical form with multiple loops (Fig. 3). This similarity is not surprising in the case of in-phase jittering, when the both oscillators behave as one and their dynamics is described by map (2), the same as in the case of one oscillator with feedback. However, the behavior of the oscillators in the antiphase regime is quite different and described by map (12). Curiously enough this map also demonstrates multi-jitter bifurcation.

The similarity of the properties of different systems allows to suppose that multi-jitter bifurcation may be an universal scenario of destabilization of regular regimes of networks with delayed pulse coupling. Examination of this hypothesis and definition of the conditions for the multi-jitter bifurcation in networks with various configurations should be a direction of the further study.

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